Reconstruction and Representation of 3D Objects with Radial Basis Functions

Carr, et.al. SIGGRAPH 2001

Presented by Matthew Bolitho

Problem

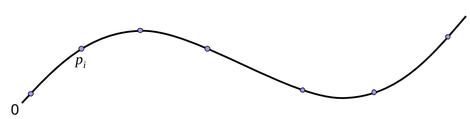
- Given a set of points $\{p_i\}$ $i = 1..n$
- Define an implicit function $f : R^3 \rightarrow R$, such that:
  $f(p_i) = 0, \ i = 1..n$
- Surface is defined by the zero set of f

Zero-Everywhere Solution

- $f(x) = 0, \forall x \in R^3$ satisfies our initial constraint
- Add off-surface normal points to constrain the solution further

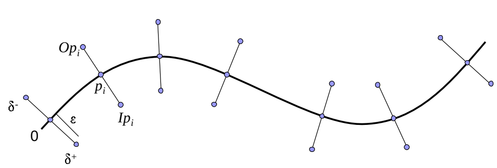
Off-Surface Normal Points

- For each p_i, add two more points defining the inside and outside



Off-Surface Normal Points

- For each p_i, add two more points defining the inside and outside

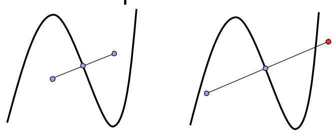


Off-Surface Normal Points

- Create Points:
  $Op_i = p_i + \varepsilon n_i$
  $Ip_i = p_i - \varepsilon n_i$
- New constraints:
  $f(p_i) = 0$,
  $f(Ip_i) = -\delta$,
  $f(Op_i) = +\delta$, $\quad i = 1..n$

Choice of Parameters

- δ is chosen as the distance of the new point from p_i : signed distance function
- ε must be chosen such that the displacement to the new points doesn't intersect other parts of the surface:



Choice of ε

Figure from Carr01

Generating Normals

- Input is a point cloud: no normals
- Generate normals as per [Hoppe92]
  - Estimation from plane fitted to neighbourhood
- Additionally, use knowledge of scanner position to infer ambiguous cases
- If that fails, don't define off-surface normal points

Interpolation Problem

- We now have an interpolation problem:
- Given a set of unstructured points with values, generate a function that interpolates all input points

$BL^{(2)}(R^3)$

- The interpolant is chosen from the "*Beppo-Levi space of distributions on R^3 with square integral second derivatives*"
- Square integral means $\int_{-\infty}^{\infty} |f(x)|^2 dx < \infty$
- So, the space of functions ☞ in R^3 with second derivatives that fall off quickly

Radial Basis Function

- All functions in ☞ have a (rotationally invariant) semi-norm.
  - The semi-norm defines the smoothness of the function
- Paper shows that the functions with small semi-norms are radial basis functions:

$$s(x) = p(x) + \sum_{i=1}^{N} \lambda_i \varphi(x - x_i)$$

- The radial basis functions are the most smooth, compactly supported functions in R^3

Radial Basis Functions

$$s(x) = p(x) + \sum_{i=1}^{N} \lambda_i \varphi(x - x_i)$$

- $p(x)$ is a low degree polynomial
- λ_i are the coefficents
- $\varphi(x\text{-}x_i)$ is a function of the Euclidean distance between two points
  - φ is chosen to fall off with distance. May have compact support

Radial Basis Functions

- RBFs are good for interpolating scattered data
  - A RBF's linear equations are always invertible at the location of data points (with some conditions)
- Use RBFs to solve this data interpolation problem

Solving RBF

- Solve a system of linear equations:

$$\begin{pmatrix} A & P \\ P^T & 0 \end{pmatrix}\begin{pmatrix} \lambda \\ c \end{pmatrix} = B\begin{pmatrix} \lambda \\ c \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix}$$

- $A_{i,j} = \varphi(|x_i\text{-}x_j|)$
- $P_{i,j} = p_j(x_i)$

- Note that B is symmetric
- Need to solve for λ, c

Solving RBF

- Once λ and c are known, we can evaluate the signed distance function anywhere
- Can extract zero-set surface via 'surface following' – use an input point as a seed

Sparsity and Support

- If φ is chosen with compact support, then B will be sparse (easier to solve)
  - Since φ will only include nearby points (more local)
- However, compact support limits performance with irregular sampling

Efficiency

- Solving with compact φ is OK (ish)
- Scales with number of input points
  - Practical for up to 1000's of points

Fast Methods

- Use of approximation
- Treat clusters of far points as a single point
- *Fitting accuracy* controls how close the approximate RBF is to the actual RBF

Fast Methods

- Reduce number of RDH centers
  - The same RBF may be represented with fewer, well placed centers
- Greedy Algorithm
  - Choose a subset of centers
    - Evaluate residual at all nodes
    - If residual is < fitting accuracy Then Done
    - Else add new centers where residual is large
    - Rinse and Repeat

Noise

- Interpolating methods perform poorly with positional noise
- Add a parameter ρ that controls interpolation vs. smoothness:

$$\min_{s \in BL^{(2)}(R^3)} \rho \|s\|^2 + \frac{1}{N} \sum_{i=1}^{N} \left(s(x_i) - f_i \right)^2$$

- Paper shows solution $s*$ changes B to:

$$\begin{pmatrix} A - 8N\pi\rho I & P \\ P^T & 0 \end{pmatrix}$$

Hole Filling

- The input is a partial mesh
- Normals are extracted from mesh
- Reconstruction will fill holes

Contributions

- Apply the fast method of solving
- Applicable to Surface Reconstruction and Hole Filling
- Smooth, Manifold surfaces
- Interpolation vs. Approximation is adjustable
- Kiwis can write SIGGRAPH papers

Figures - Iterations

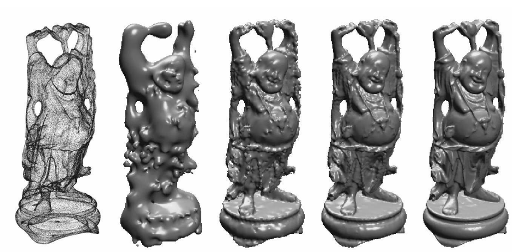


Figure from Carr01

Figures – 594,000 Centers

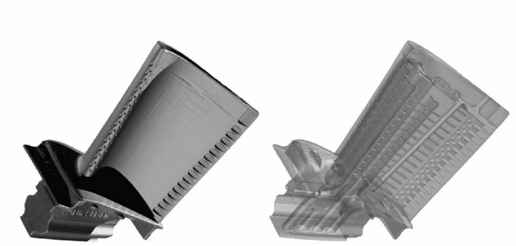


Figure from Carr01