Computing and Rendering Point Set Surfaces

Marc Alexa, et.al.

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Presented by Matthew Bolitho

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Manifold Surfaces

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- Around every point there is a local neighbourhood that is topologically the same as an open unit ball.
- We can create local parameterised neighbourhoods that cover the entire surface.

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- Define a map π that projects a point near the surface onto the surface.
- The surface is formed from all points s.t. $\pi(p) = p$.
- The map π is created from Moving Least Squares.

Moving Least Squares

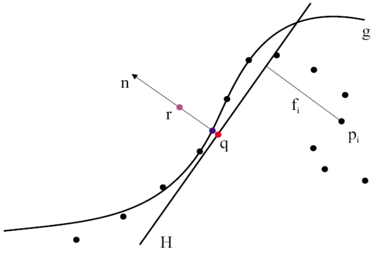


Figure from Alexa03

- A reference domain H is formed from the neighbourhood around r.

Moving Least Squares

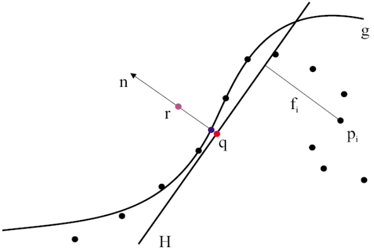


Figure from Alexa03

- Point q is the projection of r onto the plane H.

Moving Least Squares

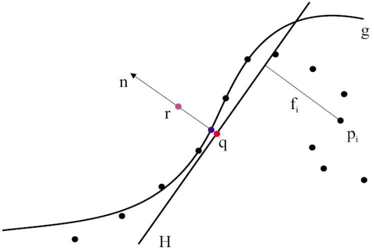


Figure from Alexa03

- A polynomial g approximates the height function f.

Moving Least Squares

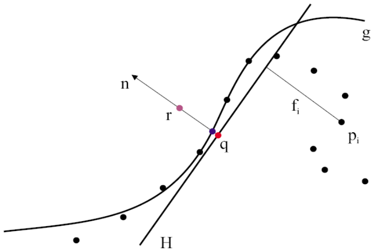
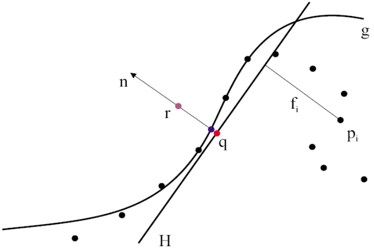


Figure from Alexa03

- $f_i = dist(p_i, H)$ weighted by $dist(p_i, q)$

Moving Least Squares



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- The map $\pi(p)$ projects p onto g

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Question

Is this resonable?

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  - Points can be removed to decimate the point set (remove the point that contributes least to g)
  - Points can be added by placing them near the surface, then mapping them onto the surface with π

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- When computing the polynomial g, each point is weighted as:
  $\theta(d) = e^{\frac{-d^2}{h^2}}$

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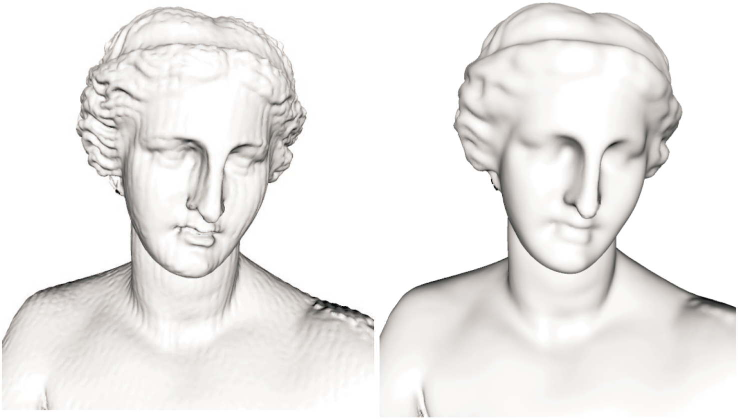
  Where $d = dist(p_i, q)$ and h is the variance.
- Note that d is defined from q not r.
- If h is small, then the neighbourhood of points in H is also small, and g captures more detail.

Weighting Function Example



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- Generates a C^∞ manifold.
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- h doesn't have to be a global paramter

Disadvatages

- Hard to assess the time requirements: 1500-3500 points per second on a "standard Pentium PC"

  But 10-30 seconds for the 36,000 point bunny seems slow.