Spectral Surface Reconstruction from Noisy Point Clouds

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Spectral Surface Reconstruction

Some Definitions

- ***S*** = sample points
- ***S***$^+$ = S augmented with bounding box vertices
- ***T*** = Delauney Tetrahedralization of S^+
- ***Q*** = Voronoi Diagram of S^+

For each tetrahedron *t* in ***T***, there is a dual voronoi vertex *v* in ***Q***

Spectral Surface Reconstruction

Goal:

Label each tetrahedron
–or equivalently, each voronoi vertex—
inside or outside

Spectral Surface Reconstruction

Identify the special Voronoi vertices called poles.

Recall poles are voronoi vertices that are likely to lie near the medial axis of the surface being recovered. A sample point S can have 2 poles.

Spectral Surface Reconstruction

Form graph G = (V, E)
V = voronoi vertices.
E = edges.

For each point ***s*** with poles ***u***, ***v***, ***(u, v)*** is an edge in E.

For pairs of samples ***s*** and ***s'*** such that (s, s') is edge in T, where s has poles u, v and s' has poles u', v', then include edges (u, u'), (u, v'), (v, u') and (v, v').

Spectral Surface Reconstruction

assigning edge weights

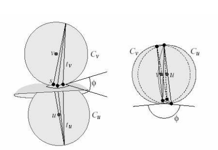


Figure 4: *(a) Small angles of intersection between circumscribing spheres may indicate that two tetrahedra are on opposite sides of the surface being recovered. (b) Large angles of intersection usually indicate that two tetrahedra are on the same side of the surface.*

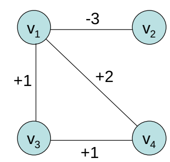
Spectral Surface Reconstruction

Form graph G' = (V, E)

Collapse certain poles into single node which are known to be "outside".

- Any tetrahedron with a vertex of the bounding box
- Direction from laser range finder
- Visual inspection

Spectral Based Partitioning



L =

	V_1	V_2	V_3	V_4
v_1	σ	-3	+1	+2
v_2	-3	σ	0	0
v_3	+1	0	σ	+1
v_4	+2	0	+1	σ

σ = $L_{ii} = \sum_{j \neq i} |L_{ij}|$.

From G', construct a Laplacian Matrix L

Review: eigenvalues and eigenvectors

$$\underset{\mathbf{L}}{\begin{pmatrix} L_{11} & L_{12} & \cdots & L_{1n} \\ \cdots & & & \\ L_{n1} & L_{n2} & \cdots & L_{nn} \end{pmatrix}} \underset{\underline{\mathbf{x}}}{\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}} = \underset{\mathbf{L}\,\underline{\mathbf{x}}}{\begin{pmatrix} L_{11}x_1 + L_{12}x_2 + \dots + L_{1n}x_n \\ \vdots \\ L_{n1}x_1 + L_{n2}x_2 + \dots + L_{nn}x_n \end{pmatrix}}$$

If L$\underline{\mathbf{x}}$=λ$\underline{\mathbf{x}}$

then λ is an eigenvalue of L

$\underline{\mathbf{x}}$ is an eignevector of L with a corresponding eigenvalue λ

note: that D$\underline{\mathbf{x}}$ is also a eigenvector, for any constant D

Laplacian Spectrum Partitioning

Define x as a partitioning vector of all of the nodes

$$\underline{x} = (x_1, x_2, \cdots, x_n) \text{ s.t. } x_i = \begin{cases} -1 & i \in X \\ 1 & i \in X' \end{cases}$$

That is, the sign of x will determine if a node is "inside" or "outside".

So we have

$$\underline{\mathbf{x}}^T \mathbf{L} \underline{\mathbf{x}} = (x_1, \dots, x_n) \begin{pmatrix} L_{11} & \cdots & L_{1n} \\ & \vdots & \\ L_{n1} & \cdots & L_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \left(\sum_{i=1}^{n} x_i L_{i1}, \quad \cdots \quad \sum_{i=1}^{n} x_i L_{in} \right) \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \sum_{i,j} x_i x_j L_{ij}$$

x which minimizes this (which is the same as the maximization) will also give us the separation between inside and outside

Spectral Based Partitioning

Solving the generalized eigensystem

Lx = λDx

And taking x for the smallest eigenvalue yields the solution.

Spectral Based Partitioning

Recall: The pole matrix is positive definite.

Typically spectral based partitioning involves positive indefinite matrices, so that the smallest eigenvalue is 0, corresponding to x with all the same sign. This paper allows one to use the smallest eigenvalue, not second smallest because pole matrix is positive definite, because of $L_{ii} > 0$.