4/21/22 : Online Auctions

Ex: Selling two identical items with time windows, unit demands

	bidder 1	bidder 2	bidder 3
value:	100	80	60
arrival:	12pm	12pm	1pm
departure:	2pm	2pm	2pm

with no time windows:

Allocation: bidder 1 and 2

charged 60 charged 60

How to sell online?

Natural idea: sell 1 item at 12:59 pm,
1 item at 1:59 pm

$\Rightarrow$ at 12:59, bidder 1 gets item for 80
1:59, bidder 2 gets item for 60

Incentive Compatible?

- Say bidder 1 lies, bids 65

  $\Rightarrow$ loses first auction but wins second, pays 60!

- Say bidder 1 lies about arrival, shows up at 1 pm

  $\Rightarrow$ wins second auction, pays 60!

<u>Setup</u>:

- Each agent $i \in [n]$ has private:
  - arrival time $a_i \in \mathbb{R}_{\geq 0}$
  - departure time $d_i \in \mathbb{R}_{\geq 0}$
  - valuation $v_i \in \mathbb{R}_{\geq 0}$
- Selling 1 item
- Bid: triple (a_i', d_i', b_i)
  - Revealed to auctioneer at time a_i'
  - $a_i' \geq a_i$
- Mechanism: online allocation rule + price (collected when item sold)
- Utility of i: $v_i -$ price paid (if gets item in $[a_i, d_i]$)
- Incentive compatibility: bidding truth is dominant

Goals:

- Welfare maximization. Close to $\max_{i \in (a)} v_i$?

- Revenue maximization:
  - All valuations drawn from same unknown F (prior-free)
  - Close to second-largest valuation?

<u>Warmup</u>:

- all $[a_i, d_i]$ intervals disjoint
- no incentives

Almost like prophet inequality!

- In PI, distributions different but known
- Here distributions same but unknown

Instead, use <u>secretary problem</u>:

- interview n job applicants in <u>random order</u>
- when interview applicant, find their value
- Irrevocable decision whether to hire
- Good enough for our setting: draw n valuations from F, randomly permute = draw one at a time from F

Thm [Dynkin '62]: There is an algorithm which gets max value with probability $\frac{1}{e}$

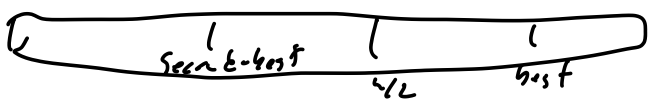
Today: prob. $\frac{1}{4}$

Pf:

Alg: - let $\frac{n}{2}$ applicants go by, p = max value

- Hire next applicant with value $\geq p$

If best in last $\frac{n}{2}$, second best in first $\frac{n}{2}$:



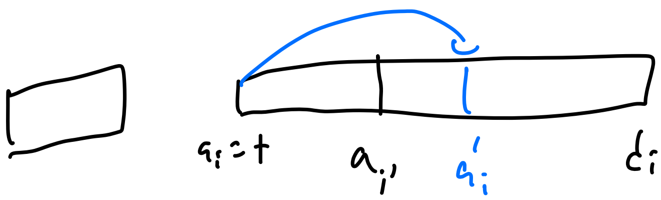
Pr[best in second half, second best in first half] $\geq \frac{1}{4}$

Back to real setting (incentives, overlapping intervals)

Attempt 2: - wait until receive $\frac{n}{2}$ bids (time t)

- set p largest so far
- sell to next bidder above p

Not IC!



$v_i = \arg\max_{j \in [n]} v_j$

Fix:

- Let t be time receive $\frac{n}{2}$ bid
  - $p \geq q$ two largest bids so far
  - If $\exists$ active bidder w/ bid p, sell at price q
- Sell to next agent above p at price p

Incentive Compatibility:

Departures:

Leaving early can't help, leaving late makes difference only if get item after $d_i \Rightarrow 0$ value

Values:

Critical bid: conditioned on getting item, bid does not affect price

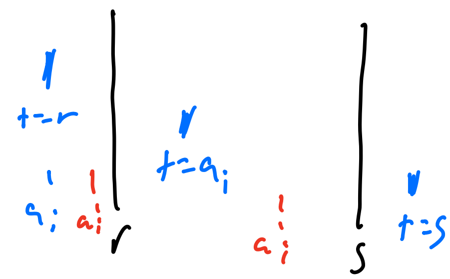
$\Rightarrow$ If v_i would win, overbidding does not affect price, underbidding either makes no difference or doesn't win

- If v_i would lose, underbidding makes no difference, overbidding either makes no difference or $\Rightarrow$ negative utility

Arrivals: Fix all other bids.

r be $\frac{n}{2}-1$ arrival time

s be $\frac{n}{2}$ arrival time



WLOG, win item with a_i' (or else a_i at least as good)

$a_i > s$:

- a_i' pushes i even later $\Rightarrow$ since wins at a_i', wins at a_i, same price

$r < a_i < s$: $a_i' < s$:

No difference b/w a_i, a_i'

$a_i' > s$:

Get item in both, but price p at a_i' instead of q at a_i

$a_i < r$: $a_i' < r$:

No difference

$r < a_i' < s$:

$t = a_i'$ instead of $t = r$, but i wins at price q in both

$a_i' > s$:

wins at price p, but a_i wins at price q

Quality:

Thm (Social welfare): Agent with max value wins the item with probability $\geq \frac{1}{4}$

Pf: Case 1: item sold at time t

$\Rightarrow$ item sold to highest bid in first $\frac{1}{2}$

$\Rightarrow$ highest overall with probability $\frac{1}{2} > \frac{1}{4}$

Case 2: item sold after time t

$\Rightarrow$ secretary analysis!

Thm (Revenue): $E[\text{Revenue}] \geq \frac{1}{4} \cdot E[\text{Revenue of Vickrey}]$

Pf: $E[\text{Revenue of Vickrey}] = E[\text{2nd highest valuation}]$

Case 1: item sold at time t

$\Rightarrow$ sold to highest bidder in first $\frac{1}{2}$

$\Rightarrow$ with prob. $\frac{1}{4}$, highest and second highest bidders overall are in first $\frac{1}{2}$

$\Rightarrow$ get same revenue as Vickrey

<u>Case 2</u>: Item sold after time t

$\Rightarrow$ just like secretary!

<u>Generalization to k items:</u>

Technically difficult, conceptually easy

1) "Learning": do nothing until time t, arrival of $\frac{n}{2}$ agent

2) "Transition": Sell up to $\lceil k/3 \rceil$ items at time t to active agents with bids above $(\lceil \frac{k}{3} \rceil + 1)$'st highest so far

3) "Accepting": Set p to $\lceil \frac{k}{3} \rceil$ highest bid in first half.

Sell to any bidder above p at price p while supply lasts