3/15/22: Intro to Mechanism Design

- Final section of course
- Also very important part of econ, traditionally separate from game theory
- AGT: next step past inefficiency of equilibria
  - How can we design "games" where selfish players still result in "good" behavior?
- Today: intro, not super technical

Single-Item Sealed-bid Auction

- We (auctioneer) selling 1 item
- bidders (players) $[n]$
  - bidder i has private valuation $v_i \geq 0$
- Each bidder sends bid $b_i \geq 0$ to us (privately)
- We decide:
  1) Who gets item 2) Price p they pay
- Utility of player i = $\begin{cases} 0 & \text{if } i \text{ does not get item} \\ v_i - p & \text{if } i \text{ does get item} \end{cases}$

- Our goal (informal): give to bidder with highest valuation

Option 1: Give to $\underset{i \in [n]}{\operatorname{argmax}}\, b_i$ for $p = 0$

- Name highest number!

Option 2: Give to $\underset{i \in [n]}{\operatorname{argmax}}\, b_i$ for $\max_{i \in [n]} b_i$

- "First-Price Auction"
- If $b_i = v_i$, utility 0 no matter what
  $\Rightarrow$ bidders trying to bid as low as possible while still getting item
- Hard for us to predict what bidders will do

Option 3: "Second-Price Auction"

- Give to $\underset{i \in [n]}{\operatorname{argmax}}\, b_i$ for $p = \max_{j \neq i} b_j$

Thm: Setting $b_i = v_i$ is a dominant strategy $\forall i \in [n]$

Pf: Let b_{-i} be all bids other than i's

Let $B = \max_{j \neq i} b_j$

Case 1: $v_i < B$

If $b_i = v_i$: utility 0

$b_i < v_i$: utility 0

$b_i > v_i$: utility ≤ 0

Case 2: $v_i \geq B$

If $b_i = v_i$: utility $v_i - B$

$b_i > v_i$: utility $v_i - B$

$b_i < v_i$: utility $\leq v_i - B$

Thm: In second-price auction: utility of any truthful bidder is nonnegative

Pf: ✓

Def: A mechanism (auction) is incentive compatible (or DSIC, or truthful) if

1) Truthful bidding is dominant strategy
2) No truthful player ever has negative utility

Thm: Second-price auctions have following properties:

1) Incentive compatible
2) If all players bid truthfully, maximizes social surplus:

$$\max \sum_{i=1}^{n} v_i x_i, \text{ where } x \in \{0,1\}^n, \sum_{i=1}^{n} x_i = 1$$

(social surplus = social welfare if also include auctioneer: $\underbrace{v_i - p_i}_{\text{winning bidder}} + \underbrace{p_i}_{\text{auctioneer}} = v_i$. Like location game!)

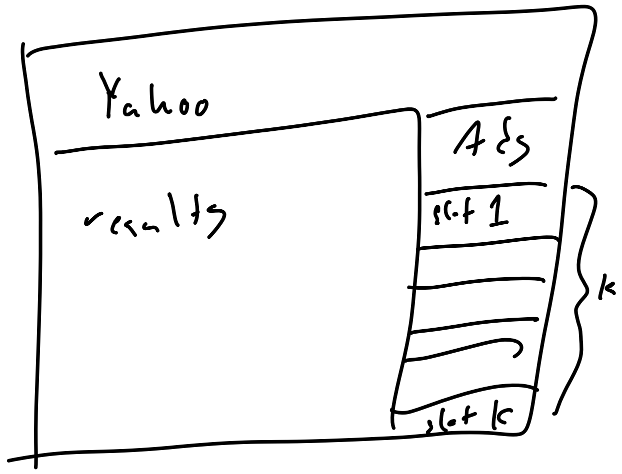
3) Computationally efficient

Roughgarden: "Awesome auctions"

Classical econ mechanism design: ignore computational efficiency

- What if we require efficiency?

Sponsored Search Auctions



- k items $[k]$; ad slots
- Each slot has click through rate (CTR) α_i

  $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_k$ (public)
- n bidders
- bidder i gets v_i per click (private)

  $\Rightarrow$ gets $\alpha_j v_i$ utility from slot j

- More complicated setting!
- Important similarity: every bidder has a single private parameter

- Goal: Awesome auction
  1) Incentive-compatible: bidders telling us v_i is dominant strategy
  2) Surplus maximizing: out of all injective functions $f : [k] \to [n]$, maximize $\sum_{j=1}^{k} \alpha_j v_{f(j)}$

3) Computationally efficient

Approach: Two-phase process

1) Assuming truthful bids ($b_i = v_i$), find surplus maximizing assignment

2) Given assignment rule from previous part, find prices to make incentive compatible

3) Do all of the above efficiently

Part 1: Given v_i's, what is optimal assignment?

Greedy! Assign slot j to j'th highest bidder

Part 2: What prices make this incentive compatible?

Not obvious!

"Generalized Second Price":

j'th highest bidder pays $(j+1)$-highest bid

Not incentive-compatible!

Next class: Awesome auction for sponsored search,

and all single-parameter environments where possible!