1/25/20: Intro, what is AGT?

- Prof: Mike Dinitz
- TA: Ama Koranteng
- Prereq: Intro Algorithms 601.433/633
- Class Logistics:
  - Lectures: in-person, but also zoom. Also recorded
  - Course webpage:
    www.cs.jhu.edu/~mdinitz/classes/AGT/Spring2022/
  - Discussion: Campuswire
  - HW: Gradescope
  - Textbooks
- Course info:
  - Designed to be a PhD-level course!
  - HW, final project, participation
    - HW: - about 6, ≈ every 2 weeks
      - Groups of ≤3, individual writeups
    - Final Project: - research, survey, summary, ...
      - Talk to me!
      - Groups of ≤3
    - Participation: be reasonable

AGT Overview:

Three Subareas:

1) Computing equilibria
   - Classical game theory ⇒ equilibria exist.
   - Compute them efficiently? Distributedly?
   - With self-interested agents?

2) Inefficiency of equilibria
   - If we're at an equilibrium, is it close to optimal?

3) Algorithmic Mechanism Design
   - Can we design games that are guaranteed to have good outcomes?
   - Auctions where agents incentivized to tell truth?

Game Theory Basics, Computation (pseudo-formal)

Game:

- Players P
- Action set A_i for each $i \in P$
  - Notation: Let $S = \bigtimes_{i \in P} A_i$

    (S is set of strategy vectors: one action for each player)
- Utility function for each player: $u_i : S \to \mathbb{R}$

  or

  Cost function for each player: $c_i : S \to \mathbb{R}$

Ex: Prisoner's Dilemma

	confess	silent
confess	(4, 4)	(1, 5)
silent	(5, 1)	(2, 2)

row player cost

column player cost

what should row player do?

Confess!

Dominant Strategy: $c_i(\underbrace{a, s_{-i}}) \le c_i(s) \quad \forall s \in S$

replace i'th coordinate of s by a

$$u_i(a, s_{-i}) \ge u_i(s) \quad \forall s \in S$$

If every player has dominant strategy, at an equilibrium:
no one has incentive to deviate

Ex: Where to eat?

utilities:

	pizza	burger
pizza	(5, 6)	(1, 1)
burger	(1, 1)	(6, 5)

No dominant strategies!

Equilibria:

(P, P) (B, B)

Pure Nash equilibria: $s \in S$ is a PNE if

$$u_i(a, s_{-i}) \leq u_i(s) \quad \forall i \in P, \; \forall a \in A_i$$

Ex: Rock-Paper-Scissors

	R	P	S
R	(0,0)	(-1,1)	(1,-1)
P	(1,-1)	(0,0)	(-1,1)
S	(-1,1)	(1,-1)	(0,0)

No pure Nash equilibria!

Randomize! Uniform over R, P, S ($\frac{1}{3}$ each)

Should I deviate to p_R, p_P, p_S (other player does not deviate)?

$$E[\text{utility}] = \underbrace{\tfrac{1}{3}(p_P - p_S)}_{\text{opp. plays rock}} + \underbrace{\tfrac{1}{3}(p_S - p_R)}_{\text{opp plays paper}} + \underbrace{\tfrac{1}{3}(p_R - p_S)}_{\text{opp plays scissors}} = 0$$

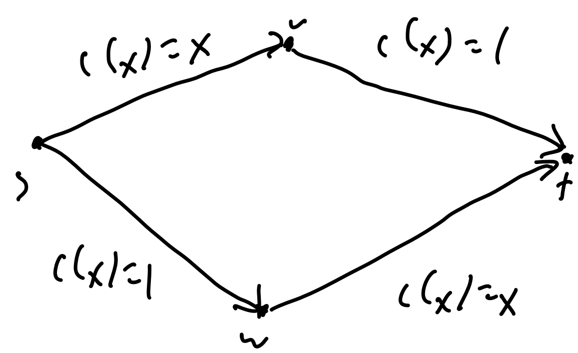
(mixed) Nash equilibrium: distribution for each player s.t. no player has incentive to deviate

Thm [Nash '51]: Every game has a Nash equilibrium!

Q: Can we compute it???

Inefficency of Equilibria:

Many drivers ($N \approx \infty$) trying to get from s to t

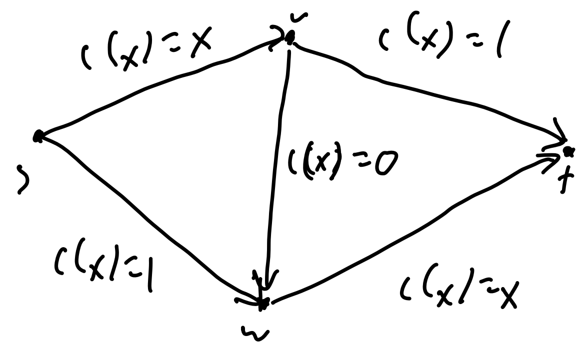


$c(x)$: cost of using edge if x fraction of drivers use it

Equilibrium:

half top, half bottom

$\Rightarrow$ Each driver has cost $\frac{3}{2}$



what happens?

- Only equilibrium is all $s-v-w-t$!

- each driver has cost 2

cost at equilibrium

"Cost of selfishness": $\dfrac{2}{3/2} = \dfrac{4}{3}$

optimal cost

Price of Anarchy: $\dfrac{\text{cost of worst Nash}}{\text{cost of OPT}}$

Price of Stability: $\dfrac{\text{cost of best Nash}}{\text{cost of OPT}}$

Mechanism Design:

Simple auction:

- selling one item
- n bidders
- bidder i values item at $v_i \geq 0$
- If bidder i gets item at price p, gets utility $v_i - p$
- If bidder i does not get item, utility 0

We get to design game!

- Each bidder gives us bid b_i
- We decide who gets item, at what price

Obvious approach: highest bid, price they bid!

winner is $\operatorname{argmax}_{i=1}^{n} b_i$

price is $\max_{i=1}^{n} b_i$

$\Rightarrow$ utility of winner i^* is $v_{i^*} - b_{i^*}$

Q: What should players bid?

Two issues: - players don't know what to bid

- we don't know what's going to happen

New approach: second-price auction

- Give item to highest bidder
- Set price to second-highest bid

Thm: For every bidder i, bidding v_i is a dominant strategy

Pf: Every other player j bids b_j (unknown to i)

How should i set b_i?

Case 1: $v_i < b_j$ for some $j \neq i$

$\Rightarrow$ setting $b_i = v_i \Rightarrow 0$ utility (don't get item)

setting $b_i < v_i \Rightarrow 0$ utility setting $b_i > v_i \Rightarrow \leq 0$ utility

<u>Case 2</u>: $v_i > b_j \quad \forall j \neq i$

Let i' be highest bidder other than i

$\Rightarrow$ setting $b_i = v_i \Rightarrow$ utility $v_i - b_{i'}$

setting $b_i > v_i \Rightarrow$ utility $v_i - b_{i'}$

setting $b_i < v_i \Rightarrow$ utility $\leq v_i - b_{i'}$