601.436/636 Algorithmic Game Theory **Spring 2022**
Homework #4 **Due:** March 31, 2022, 4:30pm

Remember: you may work in groups of up to three people, but must write up your solution entirely on your own. Collaboration is limited to discussing the problems – you may not look at, compare, reuse, etc. any text from anyone else in the class. Please include your list of collaborators on the first page of your submission. You may use the internet to look up formulas, definitions, etc., but may not simply look up the answers online.

Please include proofs with all of your answers, unless stated otherwise. Your solution must by typeset (*not* handwritten), and must be submitted by gradescope.

1 Upper Bound on PoA of Connection Games (50 points)

In class we showed an example of the connection game in which the price of anarchy of pure Nash equilibria was equal to k (the number of players). Prove that in any connection game, the price of anarchy of pure Nash equilibria is at most k. In other words, prove that any pure Nash has cost at most k times the optimal cost.

2 Location Game Matching Bound (50 points)

In class we proved that the location game (i.e. "competitive facility location with price-taking markets and profit-maximizing firms") is $(1, 1)$-smooth, and thus the the global value (i.e. social surplus) $V(s)$ of any coarse correlated equilibrium s is at least $1/2$ the global value of the optimal solution. Give an example which shows that this is tight in the strongest sense: in your example there should be a *pure Nash equilibrium* which has social surplus that is $1/2$ the social surplus of the optimal solution.