**601.436/636 Algorithmic Game Theory** **Spring 2022**
**Homework #2** **Due:** February 24, 2022, 4:30pm

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Remember: you may work in groups of up to three people, but must write up your solution entirely on your own. Collaboration is limited to discussing the problems – you may not look at, compare, reuse, etc. any text from anyone else in the class. Please include your list of collaborators on the first page of your submission. You may use the internet to look up formulas, definitions, etc., but may not simply look up the answers online.

Please include proofs with all of your answers, unless stated otherwise. Your solution must by typeset (*not* handwritten), and must be submitted by gradescope.

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## 1 Potential Games (50 points)

A team game is a game in which all players have the same utility function: $u_1(\boldsymbol{s}) = \cdots = u_n(\boldsymbol{s})$ for every strategy profile $\boldsymbol{s}$. In a dummy game, the utility of every player $i$ is independent of its strategy: $u_i(\boldsymbol{s}) = u_i(\boldsymbol{s}_{-i}, s'_i)$ for every $\boldsymbol{s} \in S$ and $s'_i \in S_i$.

Prove that a game with utilities $u_1 \ldots, u_n$ is a potential game (i.e., admits a potential function $\Phi$) if and only if it is the sum of a team game $u_1^t, \ldots, u_n^t$ and a dummy game $u_1^d, \ldots, u_n^d$ (i.e., $u_i(\boldsymbol{s}) = u_i^t(\boldsymbol{s}) + u_i^d(\boldsymbol{s})$ for all $i$ and $\boldsymbol{s}$)

## 2 No-Regret and Coarse Correlated Equilibria (50 points)

We proved that if every player uses a no-regret algorithm, then the time-averaged distribution converges to a coarse correlated equilibrium. Let's prove *almost* the converse. Consider a $k$-player cost-minimization game in which $C_i(\boldsymbol{s}) \neq C_i(\boldsymbol{s}')$ for every agent $i$ and every pair of strategy profiles $\boldsymbol{s}, \boldsymbol{s}' \in S$, (i.e., no agent has the same cost for two different profiles). Let $\sigma$ be a coarse correlated equilibrium for this game. Prove that there exist no-regret algorithms $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_k$ for the agents so that the time-averaged history of the corresponding no-regret dynamics converges to $\sigma$ as $T$ tends to infinity.

Hint: Pre-program $\sigma$ into $\mathcal{A}_1, \ldots, \mathcal{A}_k$. To make sure that each algorithm $\mathcal{A}_i$ is a no-regret algorithm, switch to your favorite no-regret algorithm (e.g., multiplicative weights) if some other agent $j$ fails to use the agreed-upon algorithm $\mathcal{A}_j$.