1. True/False. For each statement below, decide whether it is TRUE or FALSE.
   (a) Any subset of a decidable language is decidable.
   (b) If A is Turing recognizable, and $\overline{B}$ is Turing recognizable, then $A \setminus B$ is Turing recognizable.
   (c) Any subset of a regular language is Turing recognizable.
   (d) If A and B are context free languages, then $A \cap B$ is context free.
   (e) If A and B are context free languages, then $A \cap B$ is never context free.
   (f) If $A \leq_p B$ and A is not Turing recognizable, then neither is B.
   (g) If A is Turing recognizable, then its complement $\bar{A}$ is also Turing recognizable.
2. Show that the class of Turing recognizable languages is close under union and intersection.
3. Define the language

$$INFINITE_{TM} = \{\langle M \rangle | M \text{ is a Turing machine and the language } L(M) \text{ is infinite.}\}$$

   Show that the language $INFINITE_{TM}$ is undecidable.
4. For any connected (undirected) graph $G = (V, E)$, a spanning tree T of G is a subgraph of G on V with no cycles, and every pair of nodes $x, y \in V$ are connected by exactly one path in T. A k-bounded spanning tree is a spanning tree where the maximum degree of any vertex in T is at most k. Consider a decision problem where given any input graph G and an integer k, return TRUE if G has a k-bounded spanning tree, and return FALSE otherwise.
   (a) Write down (using set notation) a formal language to describe the TRUE instances of this decision problem.
   (b) Prove that this language is **NP**-complete (Hint: consider HAMPATH).