1. We define an operation BACKANDFORWARD such that:

$$BACKANDFORWARD(A) = \{w \in A | w \in A \text{ and } w^R \in A\}.$$

That is, given any language A, $BACKANDFORWARD(A)$ is a new language containing all elements in A whose reverse is also in A. Prove that the class of regular languages is closed under the operation $BACKANDFORWARD$.

2. (a) Prove that the class of regular languages is closed under set difference. That is, if A is regular and B is regular, show that $A \setminus B = \{w \in A | w \notin B\}$ is regular.

   (b) show that the class of regular languages is closed under symmetric set difference, defined as $A \Delta B = \{w | w \in A$ or $w \in B$ but not both$\}$.

3. Prove that the class of regular languages is closed under $PREFIX$ where $PREFIX(A) = \{w | \exists x \in \Sigma^*$ such that $wx \in A\}$.

4. Let $\Sigma = \{0, 1\}$. Give an equivalent regular expression for each of the languages below.

   (a) $L_1 = \{w | w$ begins with a 1 and ends with a $0\}$

   (b) $L_2 = \{w | w$ contains at least three 1s$\}$

   (c) $L_3 = \{w | w$ contains the substring 0101$\}$

   (d) $L_4 = \{w | w$ has length at least three and its third symbol is $0\}$

   (e) $L_5 = \{w | w$ starts with a 0 and has odd length, or starts with a 1 and has even length$\}$