1. We define an operation \textit{BACKANDFORWARD} such that:

\[
\text{BACKANDFORWARD}(A) = \{w \in A | w \in A \text{ and } w^R \in A\}.
\]

That is, given any language \( A \), \textit{BACKANDFORWARD}(A) is a new language containing all elements in \( A \) whose reverse is also in \( A \). Prove that the class of regular languages is closed under the operation \textit{BACKANDFORWARD}.

2. (a) Prove that the class of regular languages is closed under set difference. That is, if \( A \) is regular and \( B \) is regular, show that \( A \setminus B = \{w \in A | w \notin B\} \) is regular.

(b) Show that the class of regular languages is closed under symmetric set difference, defined as \( A\Delta B = \{w | w \in A \text{ or } w \in B \text{ but not both}\} \).

3. Prove that the class of regular languages is closed under \textit{PREFIX} where \textit{PREFIX}(A) = \{w | \exists x \in \Sigma^* \text{ such that } wx \in A\}.

4. Let \( \Sigma = \{0, 1\} \). Give an equivalent regular expression for each of the languages below.

(a) \( L_1 = \{w | w \text{ begins with a } 1 \text{ and ends with a } 0\} \)
(b) \( L_2 = \{w | w \text{ contains at least three } 1\text{s}\} \)
(c) \( L_3 = \{w | w \text{ contains the substring } 0101\} \)
(d) \( L_4 = \{w | w \text{ has length at least three and its third symbol is } 0\} \)
(e) \( L_5 = \{w | w \text{ starts with a } 0 \text{ and has odd length, or starts with a } 1 \text{ and has even length}\} \)