1. (25 points) Show that HALT is $\text{NP}$-hard. Is it $\text{NP}$-complete?

2. (25 points) Let $\phi$ be a 3CNF. An $\neq$-assignment to the variables of $\phi$ is one where each clause contains two literals with unequal truth values.

   (a) Show that any $\neq$-assignment automatically satisfies $\phi$, and the negation of any $\neq$-assignment to $\phi$ is also an $\neq$-assignment.

   (b) Let $\neq$SAT be the collection of 3CNFs that have an $\neq$-assignment. Show that we obtain a polynomial time reduction from 3SAT to $\neq$SAT by replacing each clause

   $$c_i = (y_1 \lor y_2 \lor y_3)$$

   with the two clauses

   $$(y_1 \lor y_2 \lor z_i) \text{ and } (\overline{z_i} \lor y_3 \lor b),$$

   where $z_i$ is a new variable for each clause $c_i$ and $b$ is a single additional new variable.

   (c) Conclude that $\neq$SAT is $\text{NP}$-complete.

3. (25 points) Let DOUBLE-SAT = \{ $\phi$ | $\phi$ is a CNF that has at least two satisfying assignments \}. Show that DOUBLE-SAT is NP-complete.

4. (25 points) A subset of the nodes of a simple, undirected graph $G$ is a dominating set if every other node of $G$ is adjacent to some node in the subset. Let

   $$\text{DOMINATING-SET} = \{ (G, k) | G \text{ has a dominating set with } k \text{ nodes} \}.$$  

   Show that it is $\text{NP}$-complete by giving a reduction from VERTEX-COVER. You can assume that $G$ has no vertex with degree 0.

   **Hint:** First figure out the difference between a vertex cover and a dominating set. For example, is a vertex cover always a dominating set? Is a dominating set always a vertex-cover? Then, try to modify the first graph (e.g., add some corresponding vertices) so that a vertex cover in the first graph implies a dominating set in the second graph, and vice versa.