Automata and Computation Theory Homework 7 Due by 7pm Apr. 29
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1. (25 points) Show that HALT is **NP**-hard. Is it **NP**-complete?

2. (25 points) Let ϕ be a 3CNF. An $\neq$-assignment to the variables of ϕ is one where each clause contains two literals with unequal truth values.

   (a) Show that any $\neq$-assignment automatically satisfies ϕ, and the negation of any $\neq$-assignment to ϕ is also an $\neq$-assignment.

   (b) Let $\neq$SAT be the collection of 3CNFs that have an $\neq$-assignment. Show that we obtain a polynomial time reduction from 3SAT to $\neq$SAT by replacing each clause

   $$c_i = (y_1 \vee y_2 \vee y_3)$$

   with the two clauses

   $$(y_1 \vee y_2 \vee z_i) \text{ and } (\bar{z}_i \vee y_3 \vee b),$$

   where z_i is a new variable for each clause c_i and b is a single additional new variable.

   (c) Conclude that $\neq$SAT is **NP**-complete.

3. (25 points) Let DOUBLE-SAT $= \{\phi | \phi$ is a CNF that has at least two satisfying assignments$\}$. Show that DOUBLE-SAT is NP-complete.

4. (25 points) A subset of the nodes of a simple, undirected graph G is a dominating set if every other node of G is adjacent to some node in the subset. Let

   $$\text{DOMINATING-SET} = \{\langle G, k\rangle | G \text{ has a dominating set with } k \text{ nodes}\}.$$

   Show that it is **NP**-complete by giving a reduction from VERTEX-COVER. You can assume that G has no vertex with degree 0.

   Hint: First figure out the difference between a vertex cover and a dominating set. For example, is a vertex cover always a dominating set? Is a dominating set always a vertex-cover? Then, try to modify the first graph (e.g., add some corresponding vertices) so that a vertex cover in the first graph implies a dominating set in the second graph, and vice versa.