1. (25 points) Show that HALT is \( \text{NP} \)-hard. Is it \( \text{NP} \)-complete?

2. (25 points) Let \( \phi \) be a 3CNF. An \( \neq \)-assignment to the variables of \( \phi \) is one where each clause contains two literals with unequal truth values.

   (a) Show that any \( \neq \)-assignment automatically satisfies \( \phi \), and the negation of any \( \neq \)-assignment to \( \phi \) is also an \( \neq \)-assignment.

   (b) Let \( \neq \text{SAT} \) be the collection of 3CNFs that have an \( \neq \)-assignment. Show that we obtain a polynomial time reduction from 3SAT to \( \neq \text{SAT} \) by replacing each clause

   \[ c_i = (y_1 \lor y_2 \lor y_3) \]

   with the two clauses

   \[ (y_1 \lor y_2 \lor z_i) \quad \text{and} \quad (\bar{z}_i \lor y_3 \lor b), \]

   where \( z_i \) is a new variable for each clause \( c_i \) and \( b \) is a single additional new variable.

   (c) Conclude that \( \neq \text{SAT} \) is \( \text{NP} \)-complete.

3. (25 points) Let DOUBLE-SAT = \{\( \phi \) | \( \phi \) is a CNF that has at least two satisfying assignments\}. Show that DOUBLE-SAT is \( \text{NP} \)-complete.

4. (25 points) A subset of the nodes of a simple, undirected graph \( G \) is a dominating set if every other node of \( G \) is adjacent to some node in the subset. Let

   \[ \text{DOMINATING-SET} = \{(G, k) | G \text{ has a dominating set with } k \text{ nodes}\}. \]

   Show that it is \( \text{NP} \)-complete by giving a reduction from VERTEX-COVER. You can assume that \( G \) has no vertex with degree 0.

   **Hint:** First figure out the difference between a vertex cover and a dominating set. For example, is a vertex cover always a dominating set? Is a dominating set always a vertex-cover?