Automata and Computation Theory Homework 6 | Due by 7pm Apr. 21
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1. (25 points) Define a two-dimensional Turing machine to be a TM where each of its tapes is an infinite grid (and the read/write head can move not only Left and Right but also Up and Down). Show that for every $T : \mathbb{N} \to \mathbb{N}$, any two-dimensional TM that runs in time $T(n)$ can be simulated by a standard (one-dimensional) TM in time $O(T(n)^2)$. **Note**: You may assume that the tapes of the two-dimensional TM start at $(0, 0)$ and can only access points with non-negative integer coordinates. The function $T(n)$ is not known in advance.

2. (25 points) Define a *RAM Turing machine* to be a Turing machine that has *random access memory*. We formalize this as follows: The machine has an infinite array A that is initialized to all blanks. It accesses this array as follows. One of the machine's work tapes is designated as the *address tape*. Also the machine has two special alphabet symbols denoted by R and W and an additional state we denote by q_{access}. Whenever the machine enters q_{access}, if its address tape contains $[i]R$ (where $[i]$ denotes the binary representation of i) then the value $A[i]$ is written in the cell next to the R symbol. If its tape contains $[i]W\sigma$ (where σ is some symbol in the machine's alphabet) then $A[i]$ is set to the value σ.

   Show that for every $T : \mathbb{N} \to \mathbb{N}$, any RAM TM that runs in time $T(n)$ can be simulated by a standard TM in time $O(T(n)^3)$.

3. (25 points) Let $T = \{\langle M \rangle | M$ is a TM that accepts $\alpha^{\mathcal{R}}$ whenever it accepts $\alpha\}$. Show that T is undecidable. Note: $\alpha^{\mathcal{R}}$ is the reverse string of α. You may assume the alphabet is $\{0, 1\}$.

4. (25 points) Define the language

   $C_{TM} = \{\langle M_1, M_2 \rangle | M_1, M_2$ are two Turing machines such that $L(M_1) \cap L(M_2) = \emptyset.\}$

   Show that C_{TM} is unrecognizable.