Automata and Computation Theory Homework 5 — Due by 7pm April 1

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1. (25 points) In class we showed that the class of context free languages is closed under union (how about concatenation and star? Think about it). This question studies the intersection and complementation of context free languages. Consider the languages $A = \{a^m b^n c^n | m, n \geq 0\}$ and $B = \{a^n b^n c^m | m, n \geq 0\}$.

   (a) (15 points) Give a context-free grammar for each of A and B (you don't need to prove this). Then, use A and B to show that the class of context free languages is not closed under intersection.

   (b) (10 points) Use (a) and DeMorgan's Law (Textbook Theorem 0.20) to show that he class of context-free languages is not closed under complementation.

2. (25 points) Let $D = \{xy | x, y \in \{0,1\}^*$ and $|x| = |y|$ but $x \neq y^{\mathcal{R}}\}$ (recall $y^{\mathcal{R}}$ means y writing backwards). Give a context-free grammar for D, and formally prove that your grammar generates the given language (using the two directions argument).

3. (25 points) Prove that the following language over the alphabet $\Sigma = \{0, 1, 2\}$ is not context free

$$C = \{0^a 1^b 2^c | a, b, c \geq 0 \text{ and } b < a < c.\}$$

4. (25 points) Let B be the language of all palindromes over $\{0, 1\}$ containing an unequal number of 0s and 1s. Show that B is not context-free.