Automata and Computation Theory Homework 4 — Due by 7pm Mar. 18
Instructor: Xin Li — TA: Yu Zheng

1. (25 points) Prove that the following languages are not regular.
   (a) $\{0^n 1^m 0^{nm} | m, n \geq 0\}$.
   (b) $\{w | w \in \{0,1\}^*$ is not a palindrome$\}$. Here a palindrome is a string that reads the same forward and backward, i.e., w is a palindrome if $w = w^R$.

2. (25 points)
   (a) Let $B = \{1^k y | y \in \{0,1\}^*$ and y contains at least k 1s, for any $k \geq 1\}$. Is B a regular language? Prove your answer.
   (b) Let $C = \{1^k y | y \in \{0,1\}^*$ and y contains at most k 1s, for any $k \geq 1\}$. Is C a regular language? Prove your answer.

3. (25 points) Consider the language $F = \{a^i b^j c^k | i, j, k \geq 0$ and if $i = 1$ then $j = k\}$, where $\Sigma = \{a, b, c\}$.
   (a) Show that F is not regular. **Hint:** Assume F is regular, use the closure properties of regular languages to convert F into another language, and show that language is not regular.
   (b) Show that F acts like a regular language in the pumping lemma. In other words, give a pumping length p and demonstrate that F satisfies the three conditions of the pumping lemma for this value of p.
   (c) Explain why parts (a) and (b) do not contradict the pumping lemma.

4. (25 points) Let the alphabet $\Sigma = \{0, 1\}$. For any string $w \in \Sigma^*$ with length at least 2, define the operation $C_2(w)$ to be a cyclic shift of size 2 on w. That is, let $w = w_1 w_2 \cdots w_n$ with $n \geq 2$ and each $w_i \in \Sigma$, then $C_2(w) = w_3 \cdots w_n w_1 w_2$ if $n \geq 3$, and $C_2(w) = w$ if $n = 2$. Recall that w^R means w written backwards.

   Give a context-free grammar that generates the following language:

$$\{w | w \text{ has length at least 2 and } C_2(w) = w^R\}.$$

   Briefly explain (informally) why your grammar generates the correct language. You don't need a formal proof here (in particular, you don't need to prove both directions).