1. (25 points) Prove that the following languages are not regular.
   (a) \( \{0^n1^m0^m | m, n \geq 0 \} \).
   (b) \( \{w | w \in \{0, 1\}^* \text{ is not a palindrome} \} \). Here a palindrome is a string that reads the same forward and backward, i.e., \( w \) is a palindrome if \( w = w^R \).

2. (25 points)
   (a) Let \( B = \{1^k y | y \in \{0, 1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for any } k \geq 1 \} \). Is \( B \) a regular language? Prove your answer.
   (b) Let \( C = \{1^k y | y \in \{0, 1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for any } k \geq 1 \} \). Is \( C \) a regular language? Prove your answer.

3. (25 points) Consider the language \( F = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k \} \), where \( \Sigma = \{a, b, c\} \).
   (a) Show that \( F \) is not regular. **Hint:** Assume \( F \) is regular, use the closure properties of regular languages to convert \( F \) into another language, and show that language is not regular.
   (b) Show that \( F \) acts like a regular language in the pumping lemma. In other words, give a pumping length \( p \) and demonstrate that \( F \) satisfies the three conditions of the pumping lemma for this value of \( p \).
   (c) Explain why parts (a) and (b) do not contradict the pumping lemma.

4. (25 points) Let the alphabet \( \Sigma = \{0, 1\} \). For any string \( w \in \Sigma^* \) with length at least 2, define the operation \( C_2(w) \) to be a cyclic shift of size 2 on \( w \). That is, let \( w = w_1 w_2 \cdots w_n \) with \( n \geq 2 \) and each \( w_i \in \Sigma \), then \( C_2(w) = w_3 \cdots w_n w_1 w_2 \) if \( n \geq 3 \), and \( C_2(w) = w \) if \( n = 2 \). Recall that \( w^R \) means \( w \) written backwards.
   Give a context-free grammar that generates the following language:
   \[ \{w | w \text{ has length at least 2 and } C_2(w) = w^R \} \].
   Briefly explain (informally) why your grammar generates the correct language. You don’t need a formal proof here (in particular, you don’t need to prove both directions).