Automata and Computation Theory Homework 3 — Due by 7pm on Feb. 25
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Note: For problems 2 and 3 you need to have a *formal* proof. Simply giving a state diagram or a description of a DFA/NFA is not enough.

1. (25 points) Give an NFA (both a state diagram and a formal description) recognizing the language $0^*1^*0^+$ with three states. The alphabet is $\{0, 1\}$. You don't need to have a formal proof.

2. (25 points) This question studies the number of states in a DFA equivalent to an NFA. Recall that in class we showed an NFA with 4 states that recognizes the language which consists of all binary strings that have a 1 in the third position from the end. For any integer k, it is easy to generalize this construction to an NFA with $k + 1$ states that recognizes the language which consists of all binary strings that have a 1 in the k'th position from the end. The general transformation from an NFA to a DFA will give us a DFA with at most 2^{k+1} states recognizing the same language.

   Show that, any DFA that recognizes the same language must have at least 2^k states.

   Hint: start by looking at the following two strings: 10^{k-1} and 0^k. Observe that when a DFA takes them as inputs, it must end up at different states, since one string is accepted and the other is rejected.

3. (20 points) Say that a string x is a *prefix* of string y if a string z exists where $xz = y$ and that x is a proper prefix of y if in addition $x \neq y$. Define the operation $\mathsf{NOPREFIX}$ on a language A to be

$$\mathsf{NOPREFIX}(A) = \{w \in A | \text{ no proper prefix of } w \text{ is a member of A.}\}$$

   Show that the class of regular languages is closed under this operation.

   Hint: Think about when a string $w \in A$ can have a proper prefix which is in A, then modify the states/transitions of the machine to avoid this.

4. (25 points) Let $\Sigma = \{0, 1\}$.

   (a) Write a regular expression for the language L consisting of all strings in Σ^* with exactly one occurrence of the substring 111.

   (b) Write a regular expression for the language L consisting of all strings in Σ^* that do not end with 00.

   You don't need to have a formal proof.