Automata and Computation Theory Homework 2 — Due by 7pm on Feb. 11
Instructor: Xin Li — TA: Yu Zheng

Reading: Chapter 1.1 of the textbook.

Note: For problems 3, 4 you need to *formally* prove that your construction works, i.e., show that your construction recognizes the desired language by proving both directions.

1. (25 points) Give the state diagram of a deterministic finite automaton recognizing the following language. The alphabet is $\{0, 1\}$.

$$\{w | w \text{ has length exactly 3 and its last symbol is different from its first symbol}\}$$

2. (25 points) Give a deterministic finite automaton (both a state diagram and a formal description) recognizing the following language. The alphabet is $\{0, 1\}$.

$$\{w | w \text{ is not the empty string and every even position of } w \text{ is a } 1\}$$

3. (25 points) Show that the following language is regular, where the alphabet is $\{0, 1\}$.

$$\{w | w \text{ contains an equal number of occurrences of the substrings } 01 \text{ and } 10\}$$

   For example, 010 is in this language since it has one occurrence of each substring, but 0101 is not in this language since it has two 01's but only one 10.

   Hint: First find an equivalent and simpler characterization of the language. You don't need to write the formal description of the finite automata, but you should at least give a state diagram.

4. (25 points) For any string w with even length, $w = w_1 w_2 \cdots w_{2n}$ where $n \geq 0$, let $w_{odd} = w_1 w_3 \cdots w_{2n-1}$ (symbols from odd positions) and $w_{even} = w_2 w_4 \cdots w_{2n}$ (symbols from even positions). For any two regular languages A, B over the same alphabet Σ, show that the following language C over alphabet Σ is also regular:

$$C = \{w || w| = 2n \text{ where } n \geq 0, \text{ such that } w_{odd} \in A \text{ and } w_{even} \in B\}.$$