

Reading: Chapter 0 of the textbook.

1. (25 points) For any three sets  $A, B, C$ , prove that

$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C).$$

2. (25 points) In the textbook it is shown that  $\sqrt{2}$  is an irrational number. Use this fact to show that the following statement is true: there exist two irrational numbers  $p$  and  $q$ , such that  $q^p/p^2$  is a rational number.
3. (25 points) Show that there exist no integers  $x, y, z$  such that  $3x^2 + 3y^2 = z^2$ , except  $x = y = z = 0$ .
4. (25 points) Let  $r$  be a number such that  $r + 1/r$  is an integer. Use induction to show that for every positive integer  $n$ ,  $r^n + 1/r^n$  is an integer.