

# Probability review for sequence modeling

Ben Langmead



For original Keynote files, email me ([ben.langmead@gmail.com](mailto:ben.langmead@gmail.com))

# TODO

*This video*

Sample space

Event

Joint probability

Conditional probability

Multiplication rule

Independence

*Next...*

Conditional independence

Markov assumption

# Probability

*Sample space* ( $\Omega$ ) is **set** of all possible outcomes

E.g.  $\Omega = \{ \text{all possible rolls of 2 dice} \}$



An *event* is a **subset** of  $\Omega$

$A = \{ \text{rolls where 1}^{\text{st}} \text{ die is odd} \}$

$B = \{ \text{rolls where 2}^{\text{nd}} \text{ die is even} \}$

$P(A)$ : fraction of outcomes that are in  $A$  \*

$$P(A) = |A| / |\Omega| = 18/36 = 0.5$$

Die 2

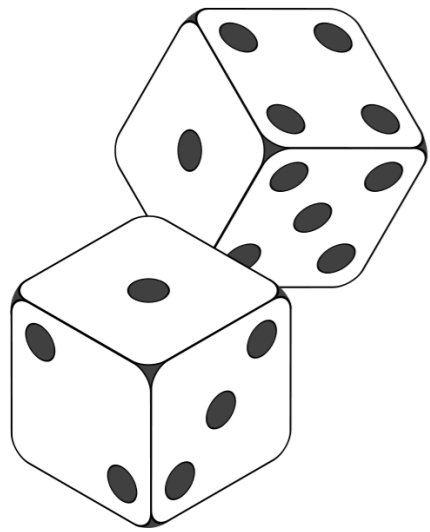
	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Die 1

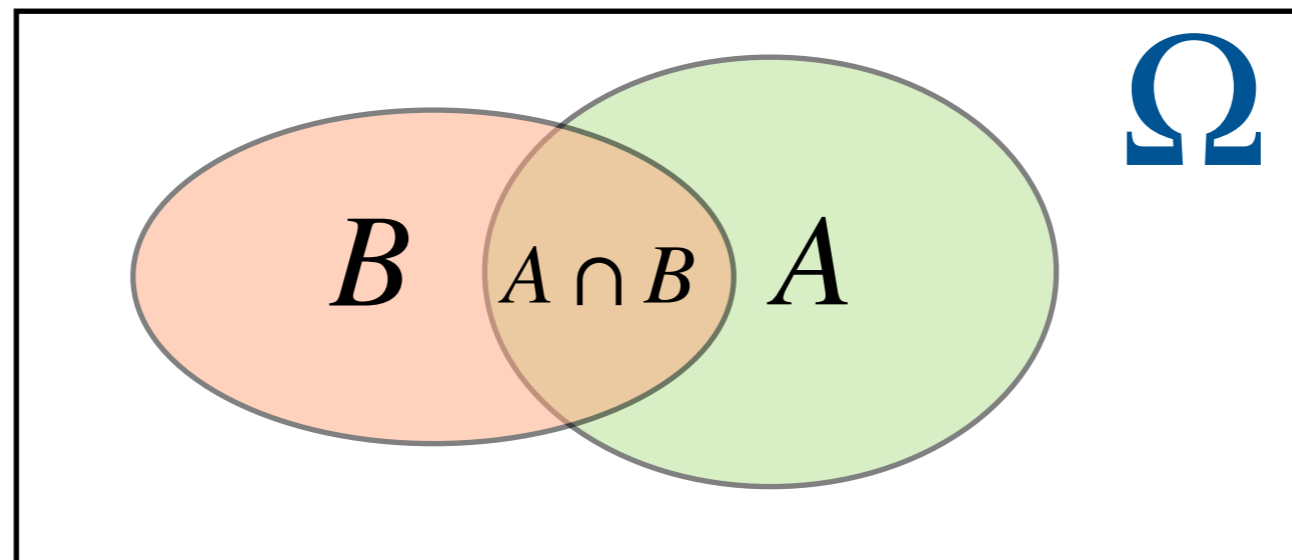
\* Naively assuming all outcomes are equiprobable

# Probability

$P(A, B)$ : fraction of outcomes in **both**  $A$  and  $B$

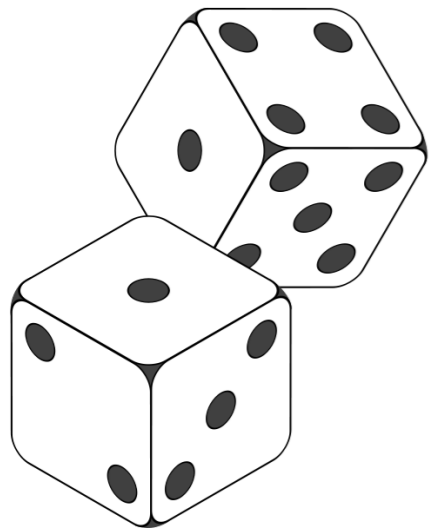


$$P(A, B) =$$



# Probability

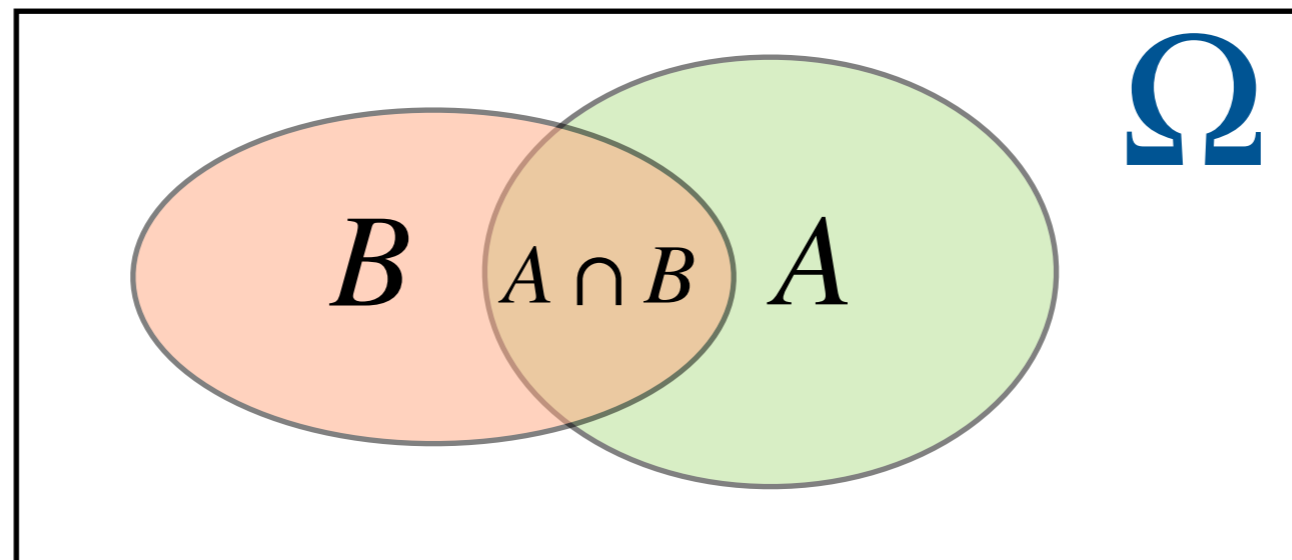
$P(A, B)$ : fraction of outcomes in **both**  $A$  and  $B$



$$P(A, B) = |A \cap B| / |\Omega| = 9 / 36 = 0.25$$

Also written:  $P(A \cap B)$  or  $P(AB)$

*Joint probability* of  $A$  and  $B$

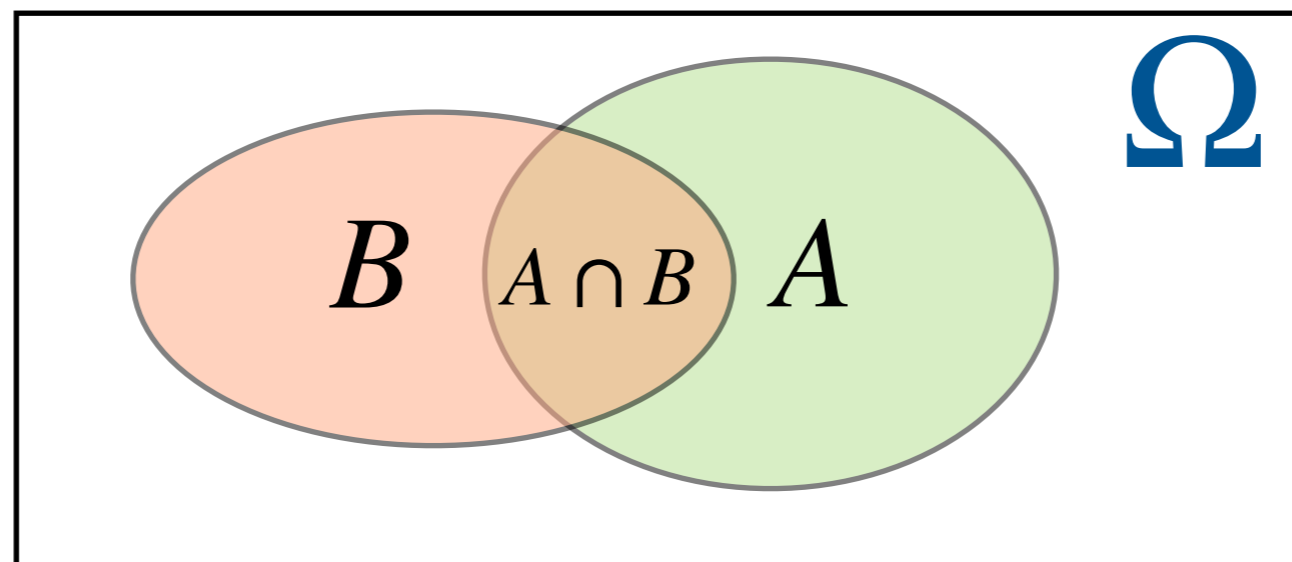
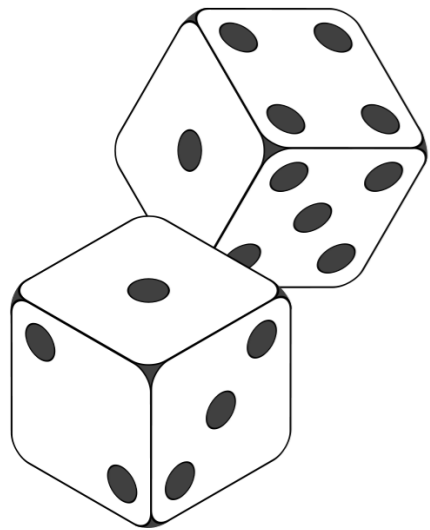


# Probability

$P(A | B)$ : fraction of outcomes in  $B$  that are also in  $A$

*conditional probability of  $A$  given  $B$*

$$P(A | B) =$$

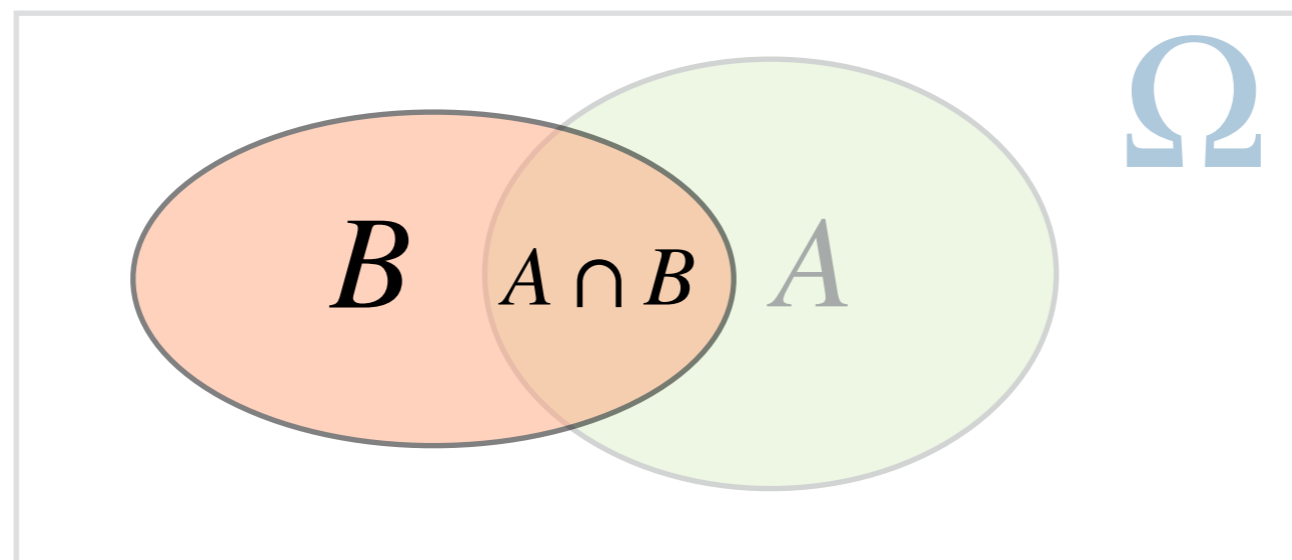
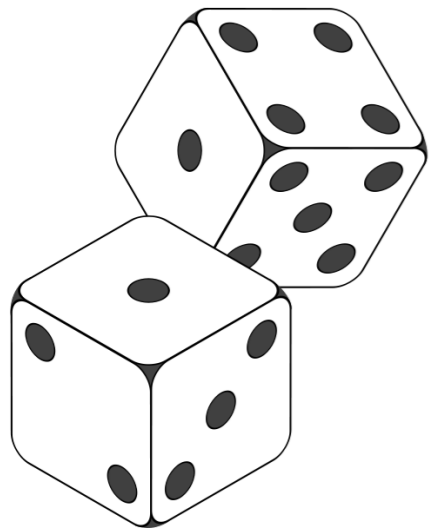


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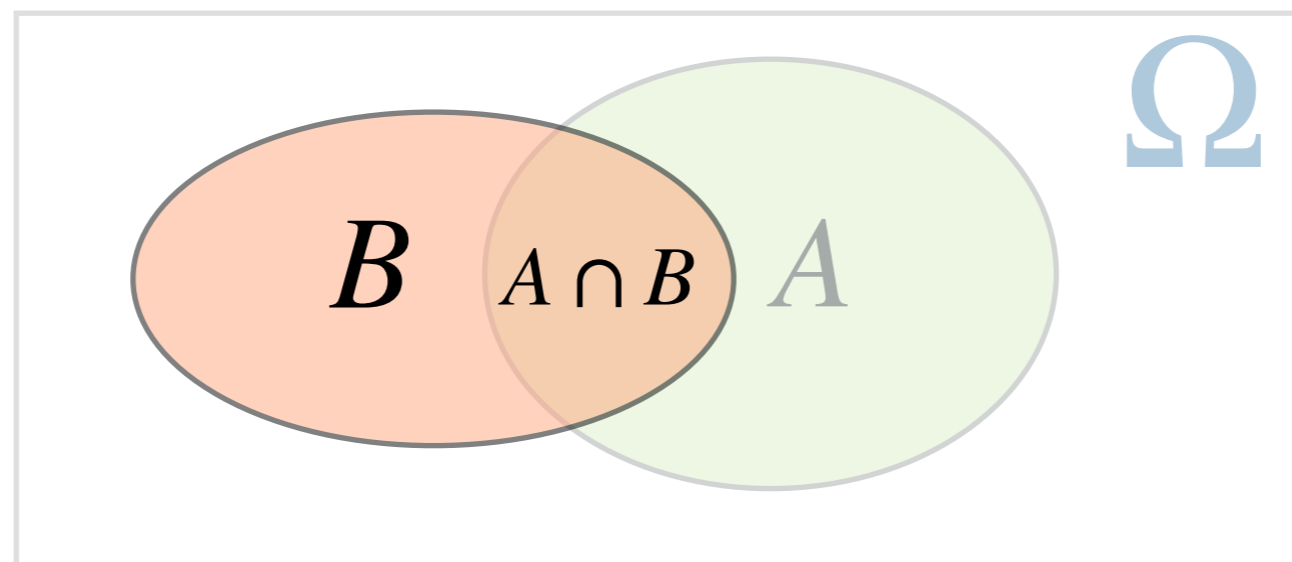
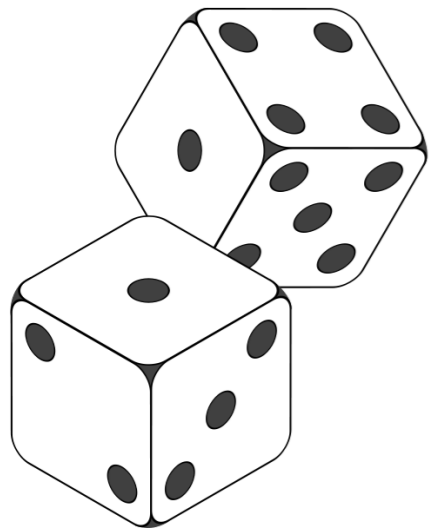


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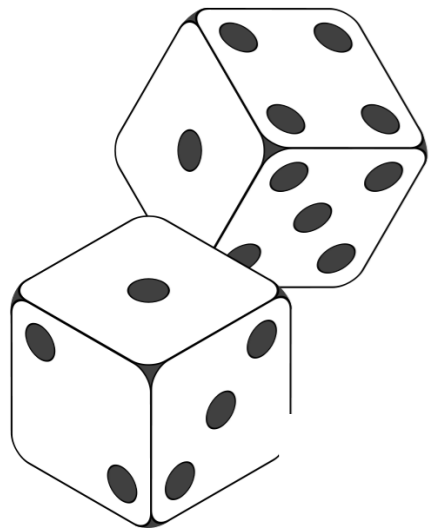
*conditional probability of  $A$  given  $B$*

$$P(A | B) = |A \cap B| / |B| = 9 / 18 = 0.5$$

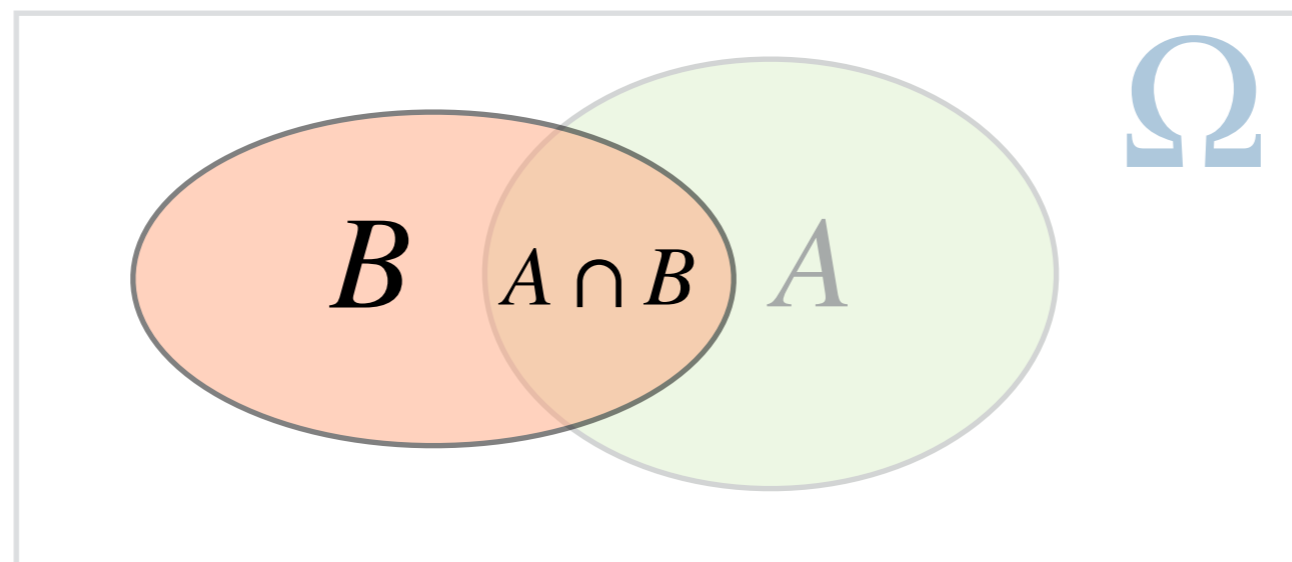


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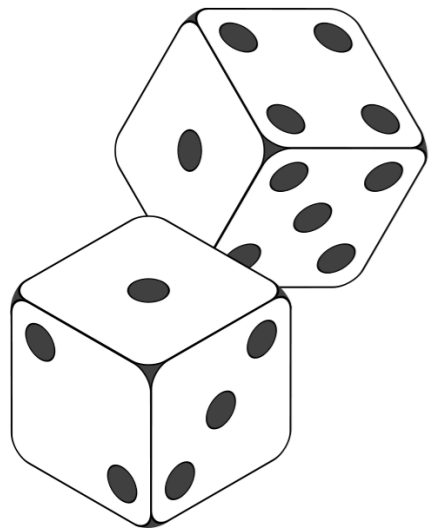


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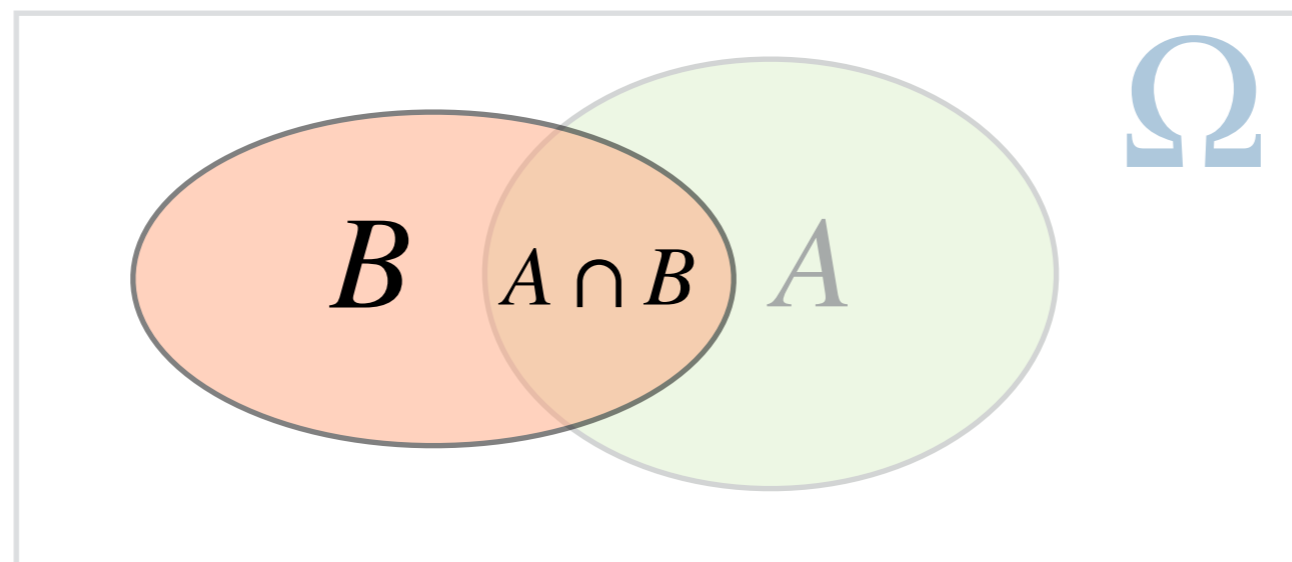
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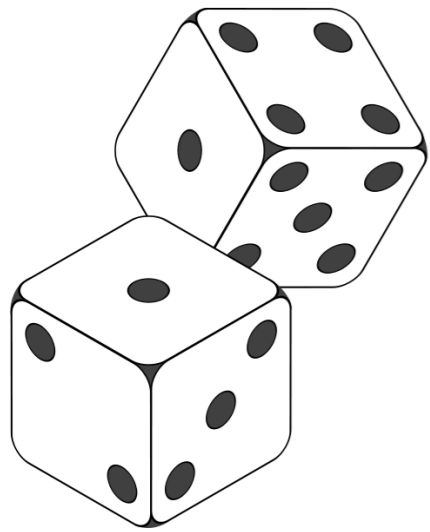
$$P(A | B) = P(A, B) / P(B)$$

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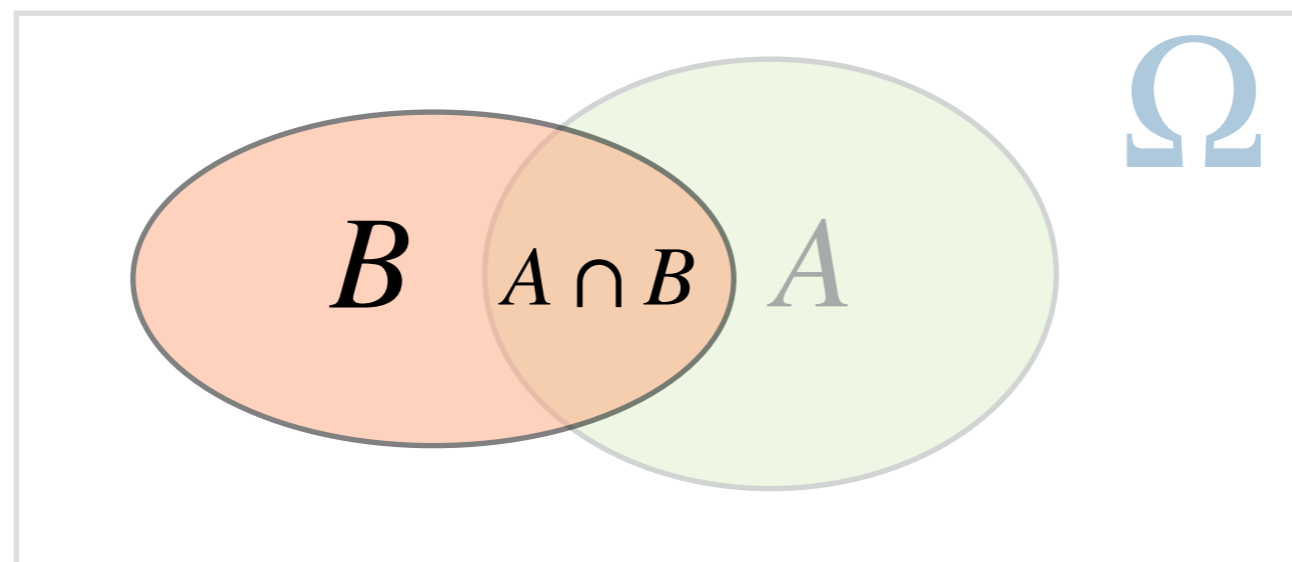
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$$P(A | B) = P(A, B) / P(B)$$

$$P(A, B) = P(A | B) \cdot P(B) \leftarrow \text{multiplication rule}$$



# Probability

Multiplication rule for joint prob with many variables:

$$P(A, B, C, D) =$$

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Multiplication rule for joint prob with many variables:

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**joint** probability

**conditional** probabilities

**marginal**  
prob

# Probability

When  $A$  &  $B$  are independent *events*

$\Omega$

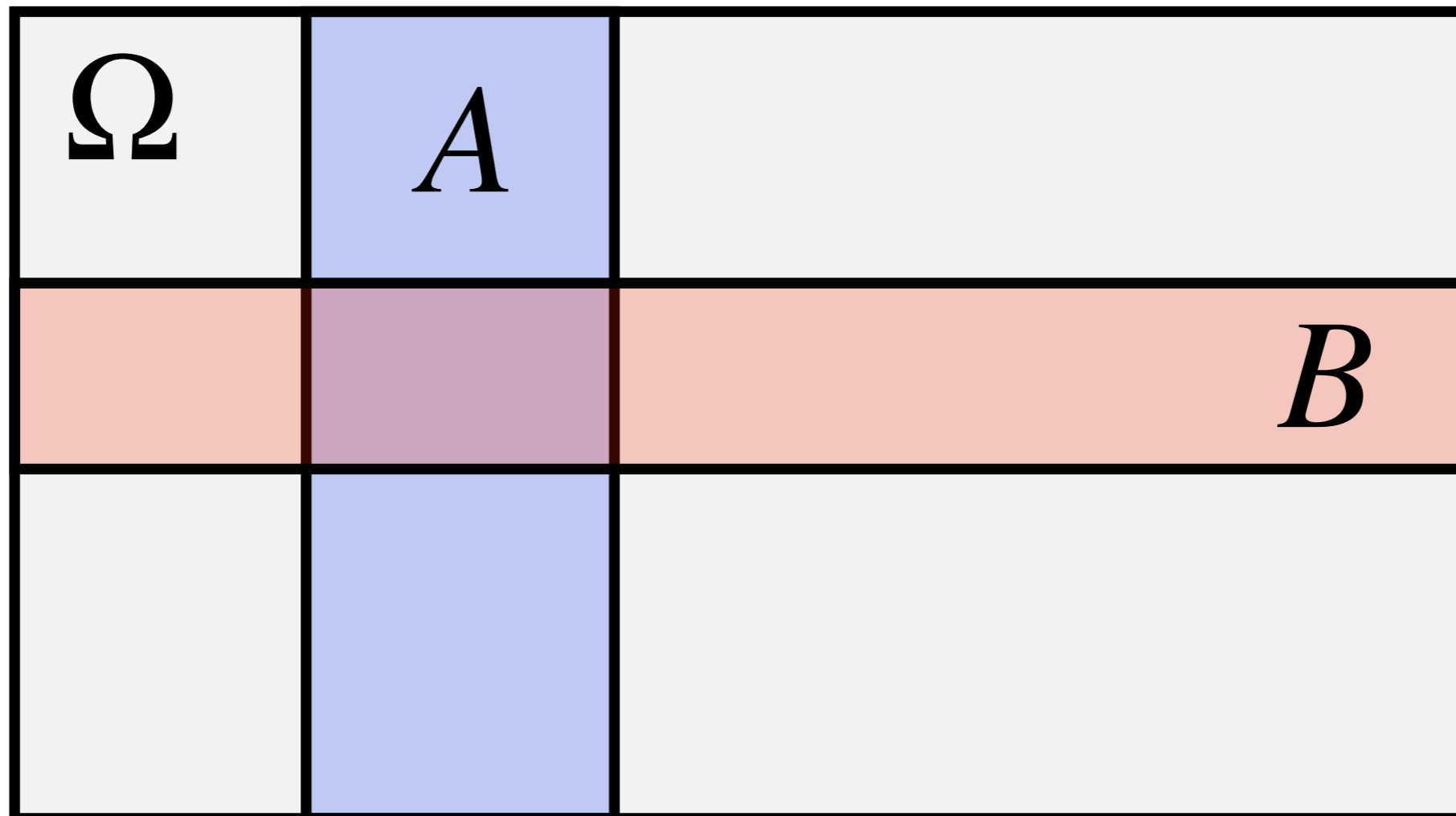
# Probability

When  $A$  &  $B$  are independent *events*

$\Omega$	$A$	
		$B$

# Probability

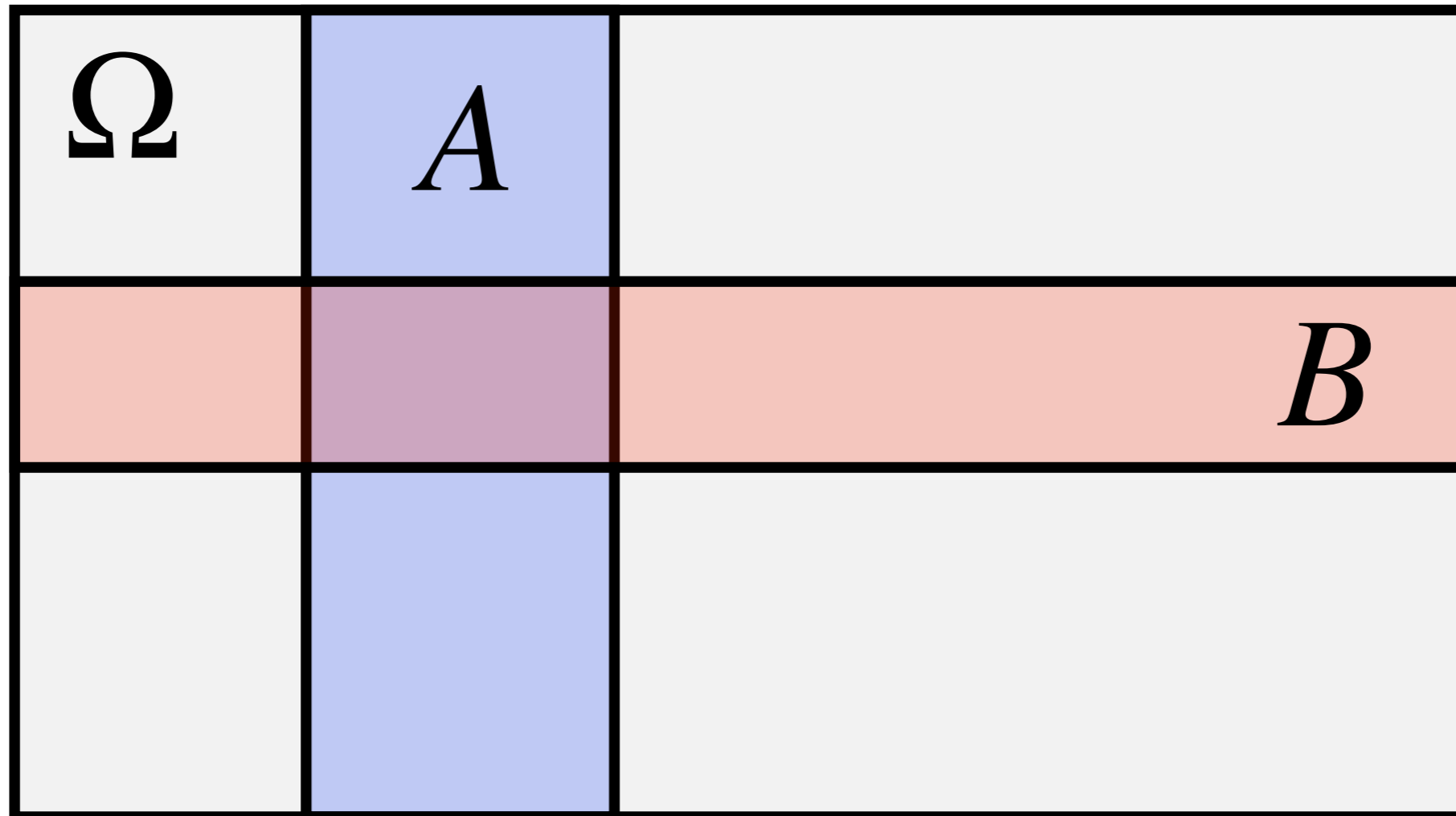
When  $A$  &  $B$  are independent *events*



$$P(A, B) = P(A) \cdot P(B)$$

# Probability

When  $A$  &  $B$  are independent *events*



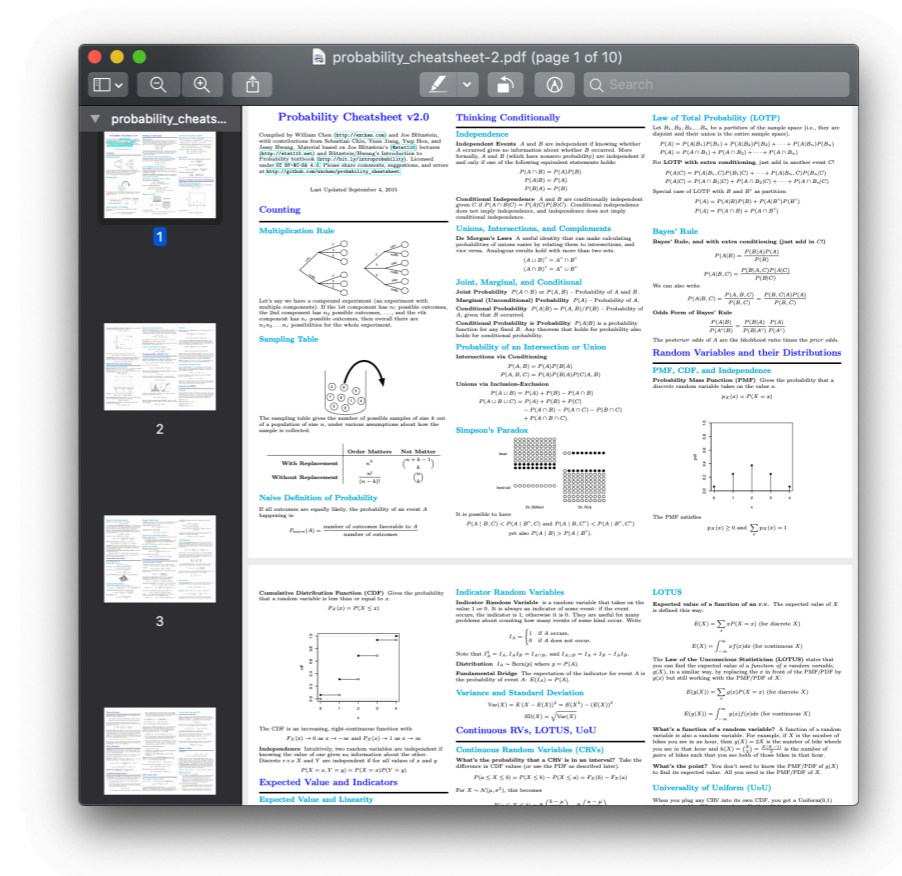
$$P(A, B) = P(A) \cdot P(B)$$

$$P(A | B) = \frac{P(A, B)}{P(B)} = P(A)$$

# Probability



Blitzstein: Probability  
(Harvard Stat 110)



Blitzstein & Chen's "cheat sheet"