Bitvectors and RSA queries

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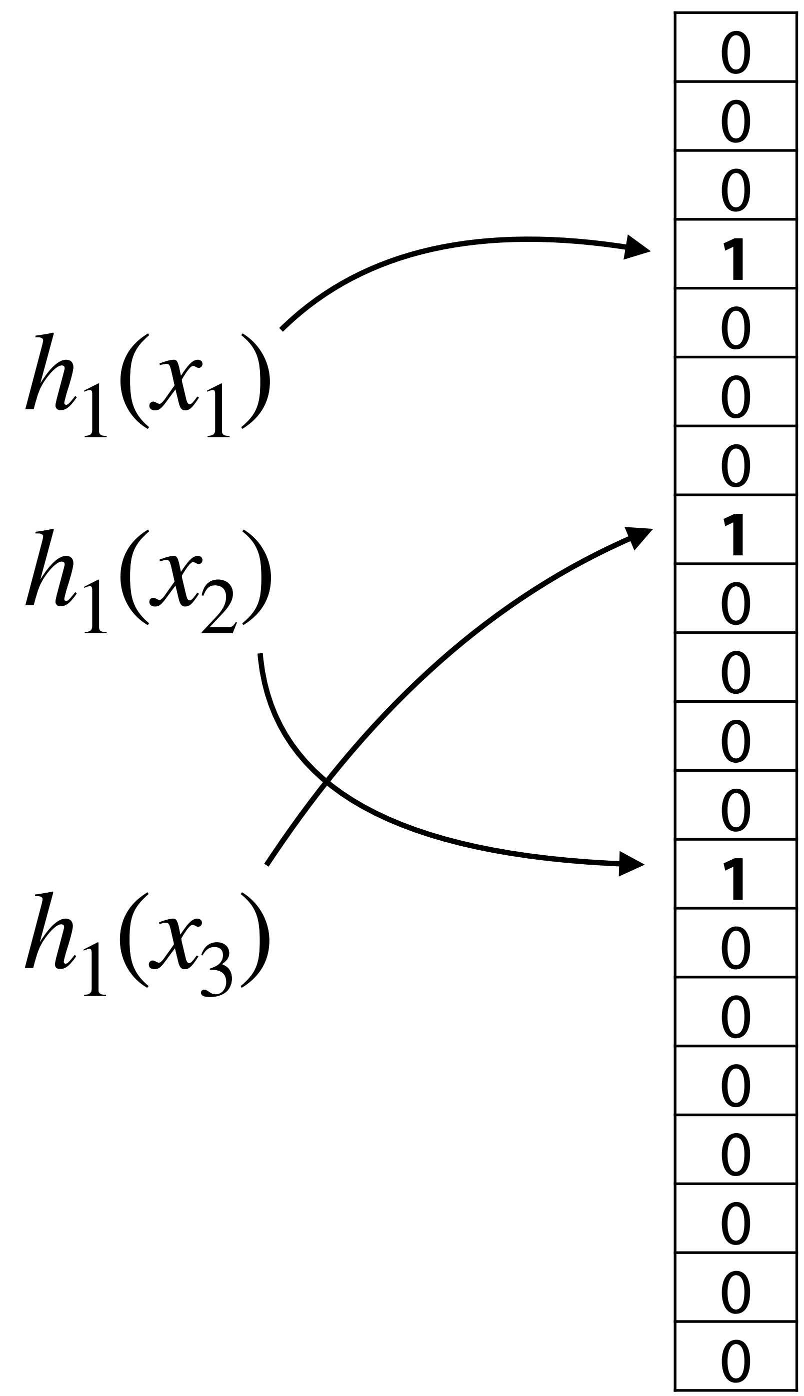
Bitvectors

Bitvectors are no stranger to us; Bloom filters!

Now we consider bitvectors where slots have *meaning*

Navigating between slots also meaningful



Bitvectors

Does this bitvector have a "meaning?"

What if its name was `is_prime`? 😉

How might we query it?

E.g. next-highest-prime

E.g. designing a 2-universal hash, we want smallest prime (leftmost 1) greater than some number

0
1
1
0
1
0
1
0
0
0
1
0
1

⋮

Bitvectors

What if the vector really was a Bloom filter?

Why might want to "navigate" it?

Say we are counting 1s to estimate cardinality

Might want to "jump" between 1s, asking how spaced out they are (like k^{th} minimum value)

0
0
0
1
0
0
0
1
0
0
0
0
1
0
0
0
0
0
0
0

⋮

Bitvectors

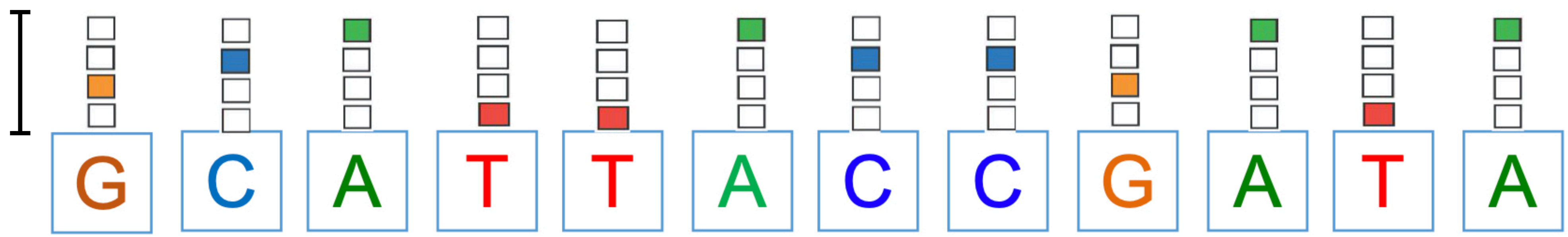
Might represent set of all words in a document

0	abinoam
1	abiogenesis
0	abiological
0	abiosis
0	abiotic
0	abiotically
0	abiotrophy
0	abirritate
0	abishag
0	abit
0	abitibi
0	abiu
1	abject
⋮	

Bitvectors

Could represent "one-hot" encoding of string

4 parallel
bitvectors



Navigating bitvectors = navigating the occurrences of characters in the string

https://www.encodeproject.org/documents/5284a1d9-7eb6-44d9-8979-b90408332b12/@@download/attachment/Anshul_slides.pdf

Bitvectors

How do we navigate / query bitvectors?

Proposal: "RSA" (Rank, Select, Access)

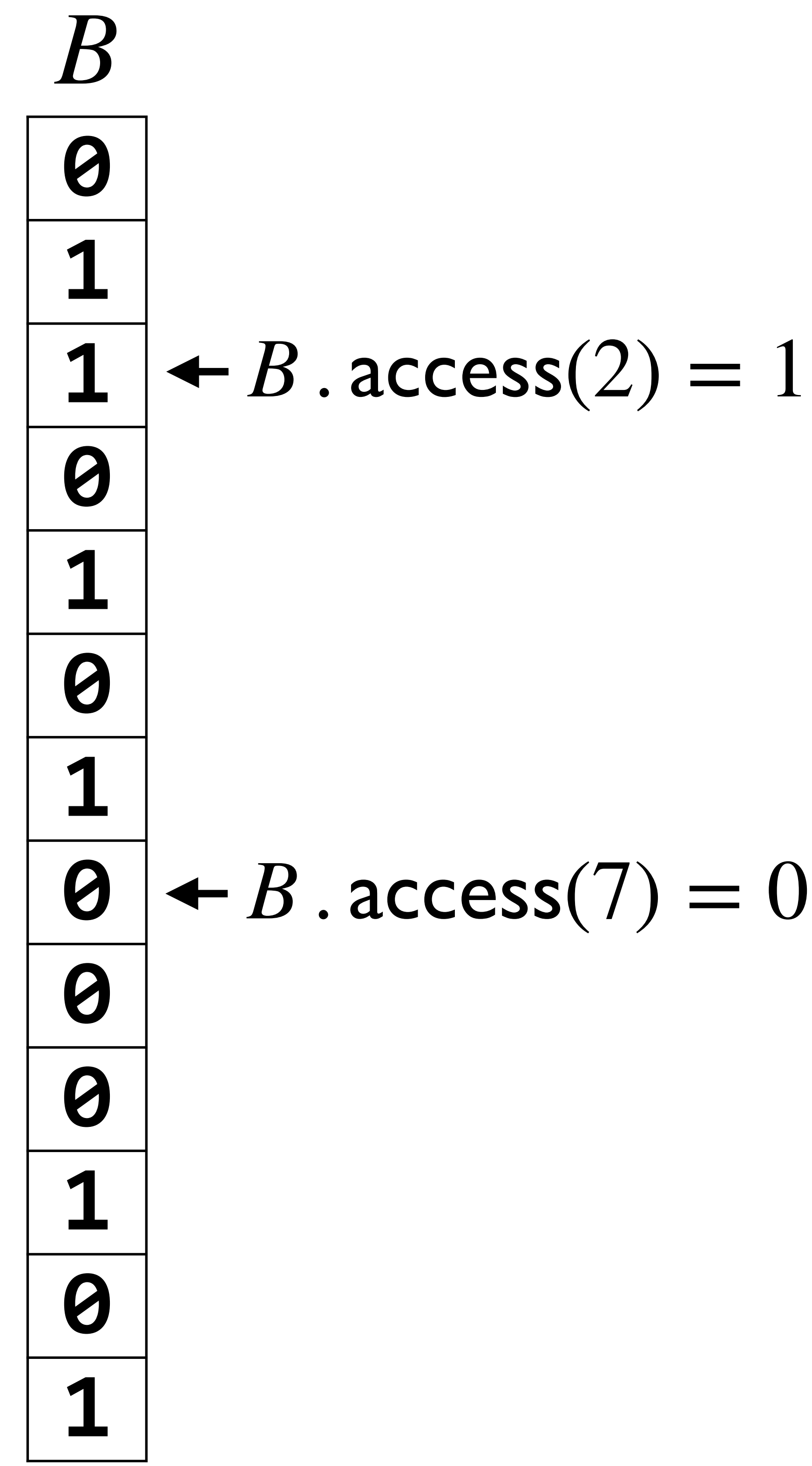
0
1
1
0
1
0
1
0
0
0
1
0
1

Bitvectors

$B\,.\,\text{access}(i) = B[i]$

Idea is easy; can be hard in practice if we compress B

We'll index starting at 0



Bitvectors

$$B\,.\,\mathrm{rank}_1(i) = \sum_{j=0}^{\color{green}{i-1}} B[j]$$

$$B\,.\,\mathrm{rank}_0(i) = i - B\,.\,\mathrm{rank}_1(i)$$

Note that rank is counting 0s/1s up to *but not including* offset i

B

0
1
1
0
1
0
1
0
0
0
1
0
1

$B\,.\,\mathrm{rank}_{\color{blue}{1}}(2) = 1$

$B\,.\,\mathrm{rank}_{\color{red}{0}}(5) = 2$

$B\,.\,\mathrm{rank}_{\color{blue}{1}}(8) = 4$

Bitvectors

$$B\,.\,\mathsf{select}_1(i) = \max\{\, j \mid B\,.\,\mathsf{rank}_1(j) = i \,\}$$

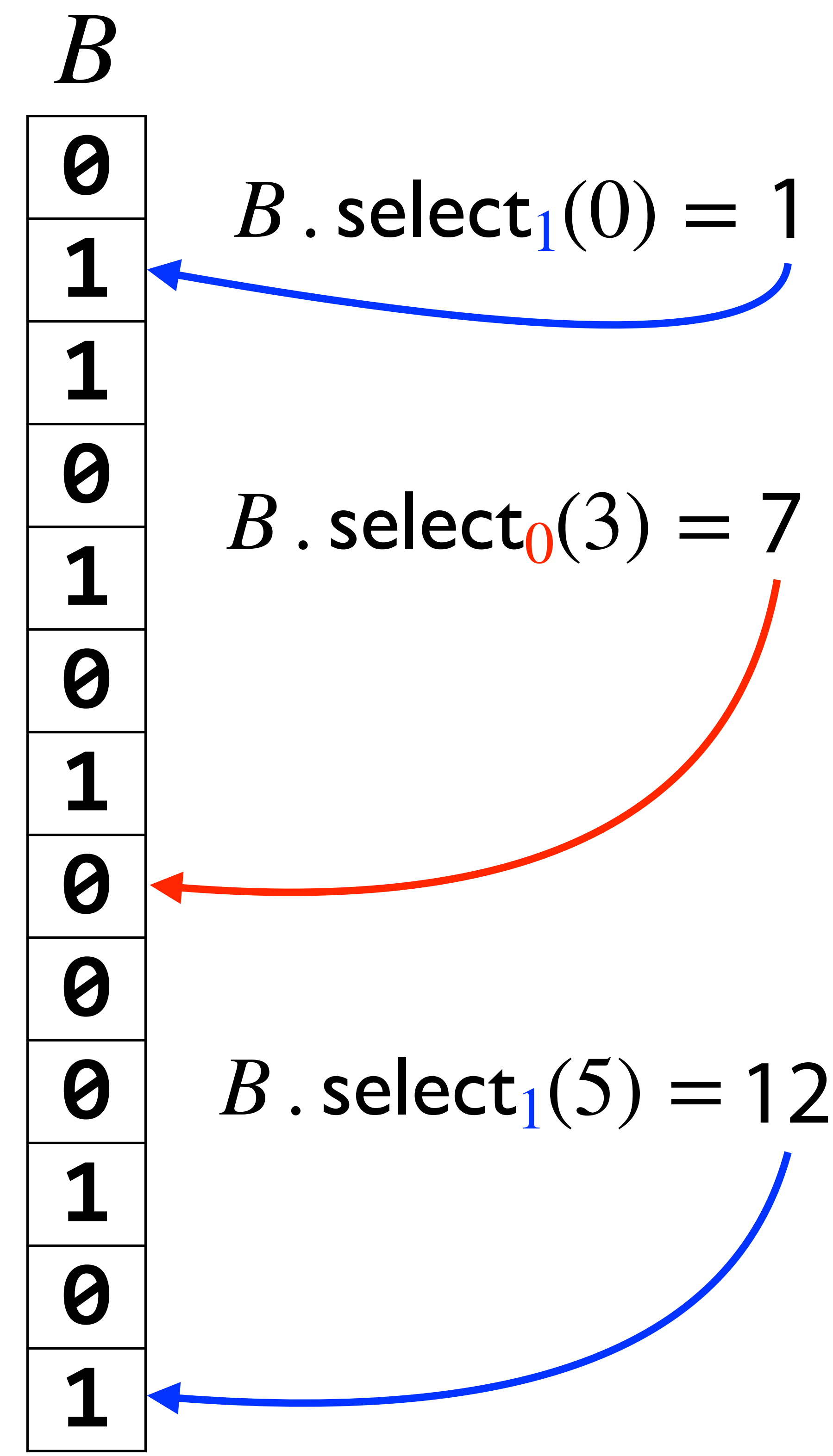
B
0
1
1
0
1
0
1
0
0
0
1
0
1

Bitvectors

$$B\,.\,\mathrm{select}_1(i) = \max\{\, j \mid B\,.\,\mathrm{rank}_1(j) = i \,\}$$

$$B\,.\,\mathrm{select}_0(i) = \max\{\, j \mid B\,.\,\mathrm{rank}_0(j) = i \,\}$$

(Don't forget: rank counts up to *but not including* given offset)



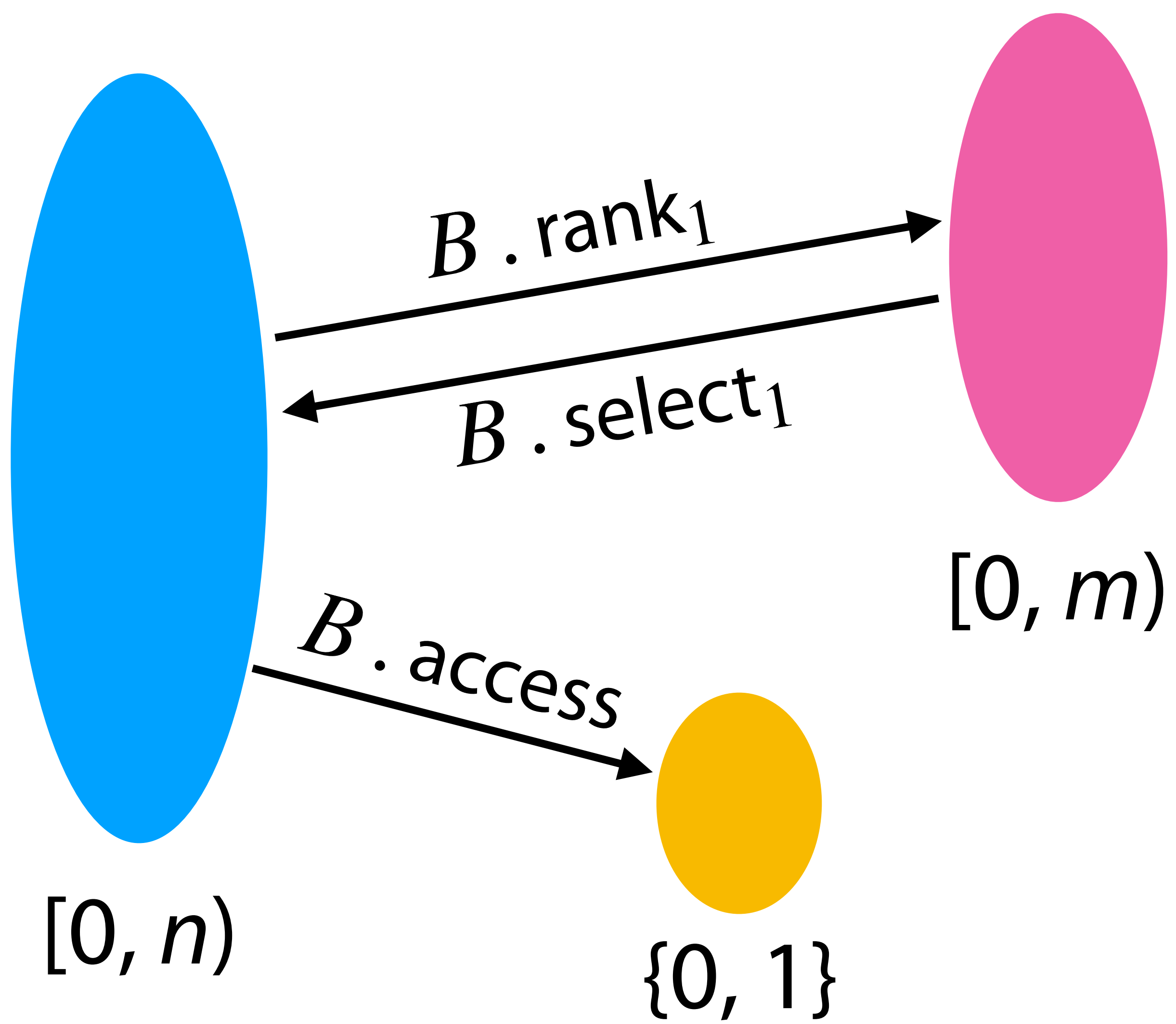
Bitvectors

$B\,.\,\text{access}(\,...\,)$

$B\,.\,\text{rank}(\,...\,)$

$B\,.\,\text{select}(\,...\,)$

Let $|\,B\,| = n$ and let m equal the number of set bits

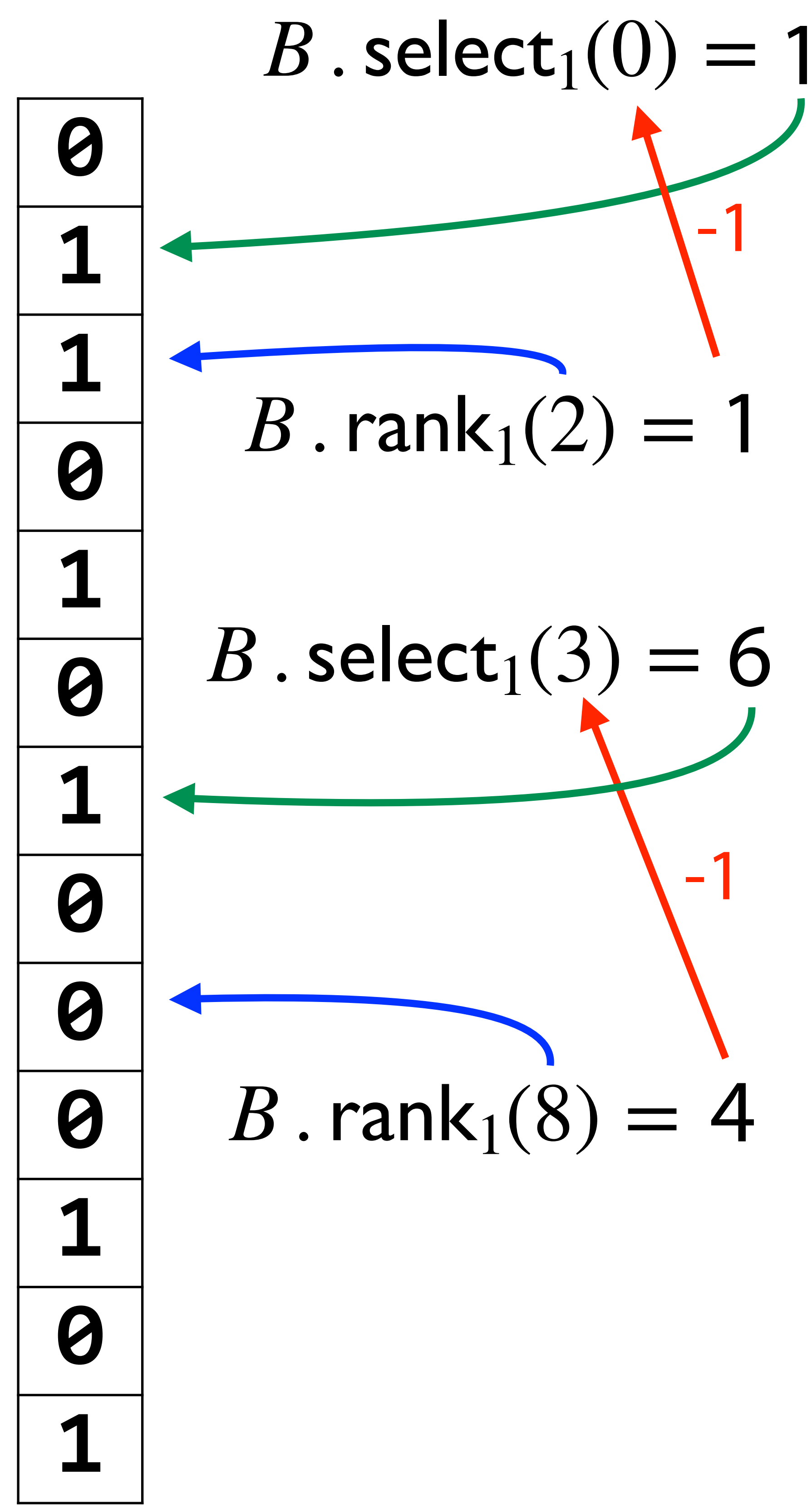


Bitvectors

What does this do?

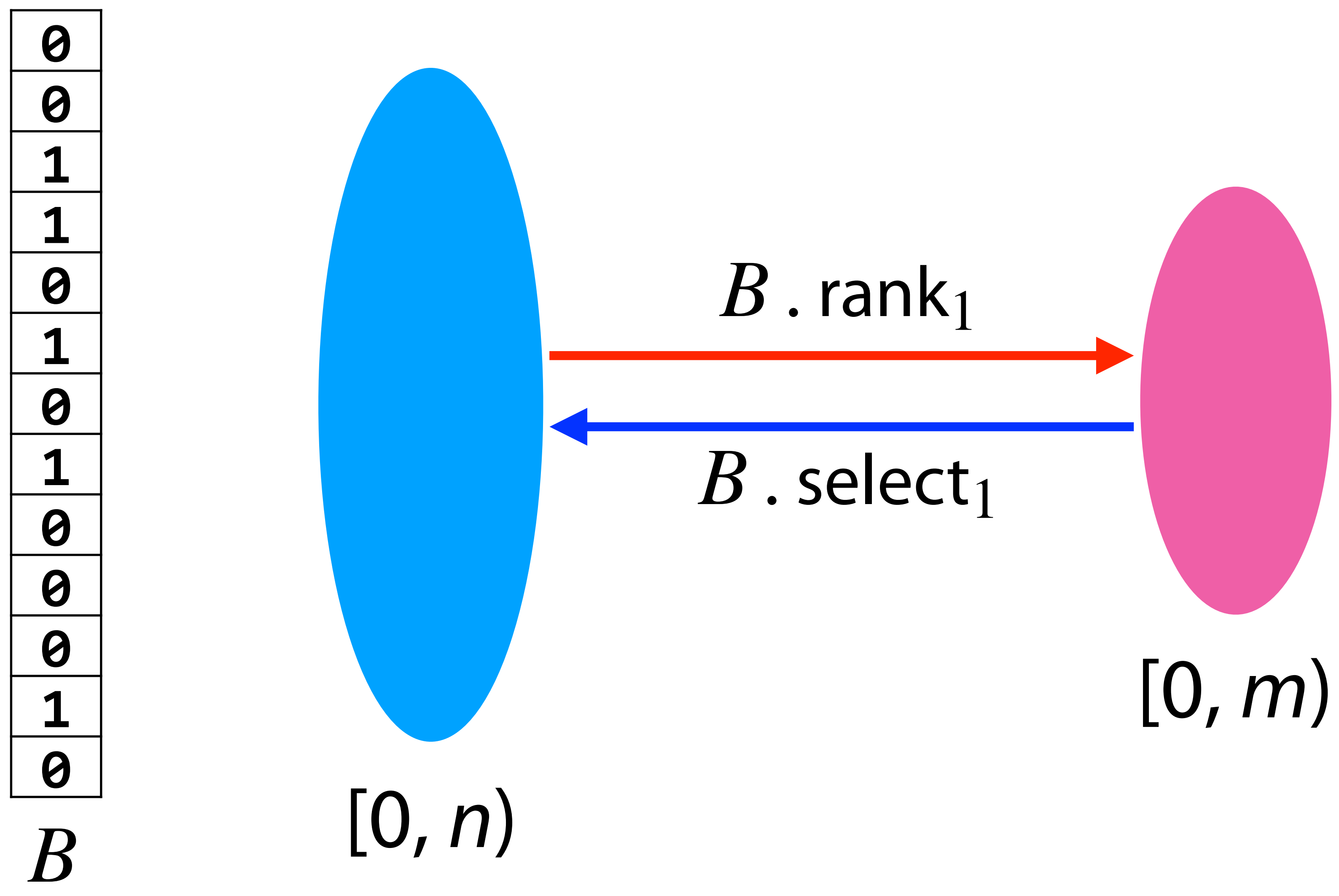
$B\,.\,\text{select}_1(B\,.\,\text{rank}_1(i)-1)$

Gives offset of next-earliest set bit -- *predecessor bit*



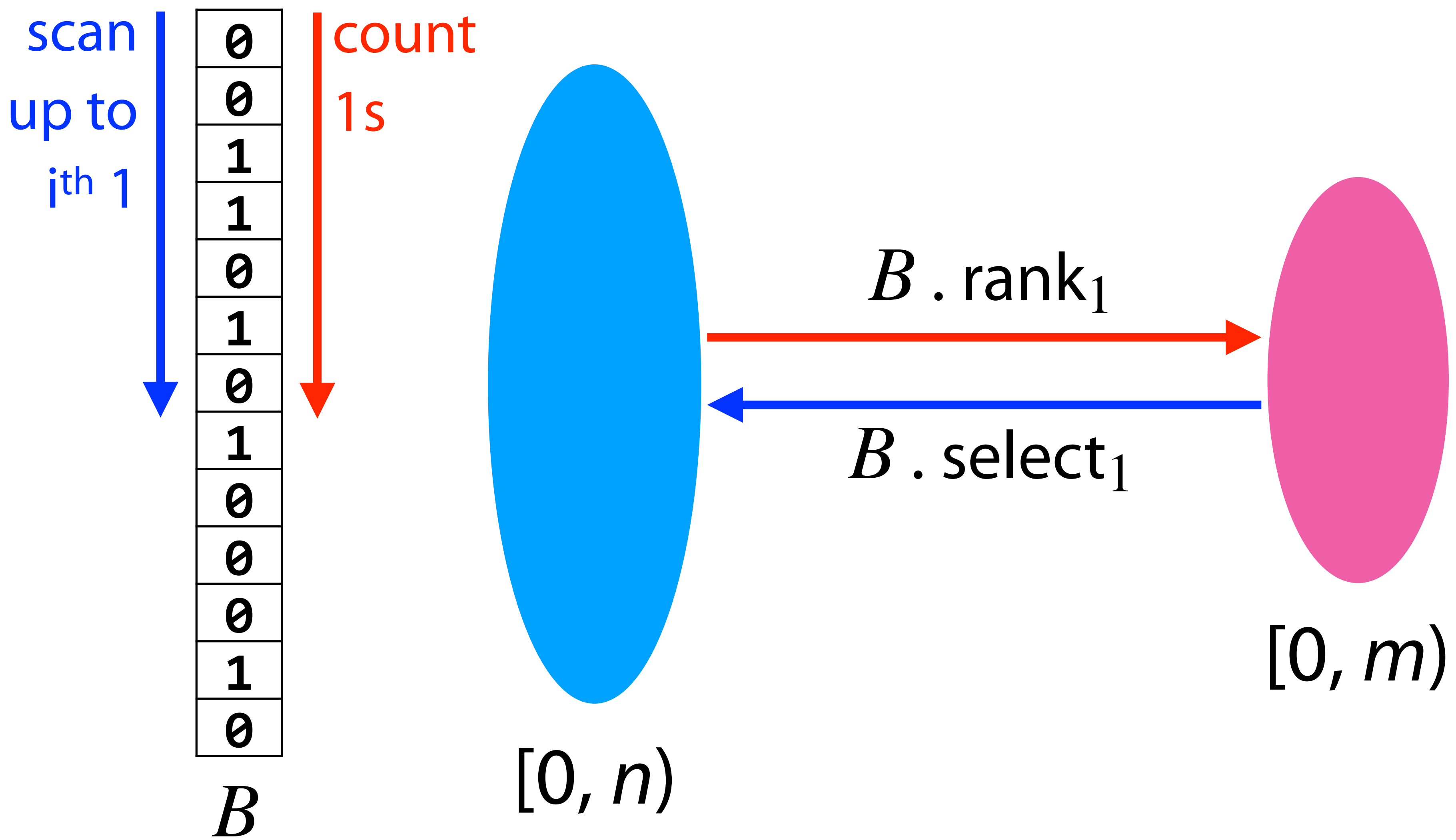
Bitvectors

How to implement $B\,.\,\mathrm{rank}_1$ & $B\,.\,\mathrm{select}_1$?



Bitvectors

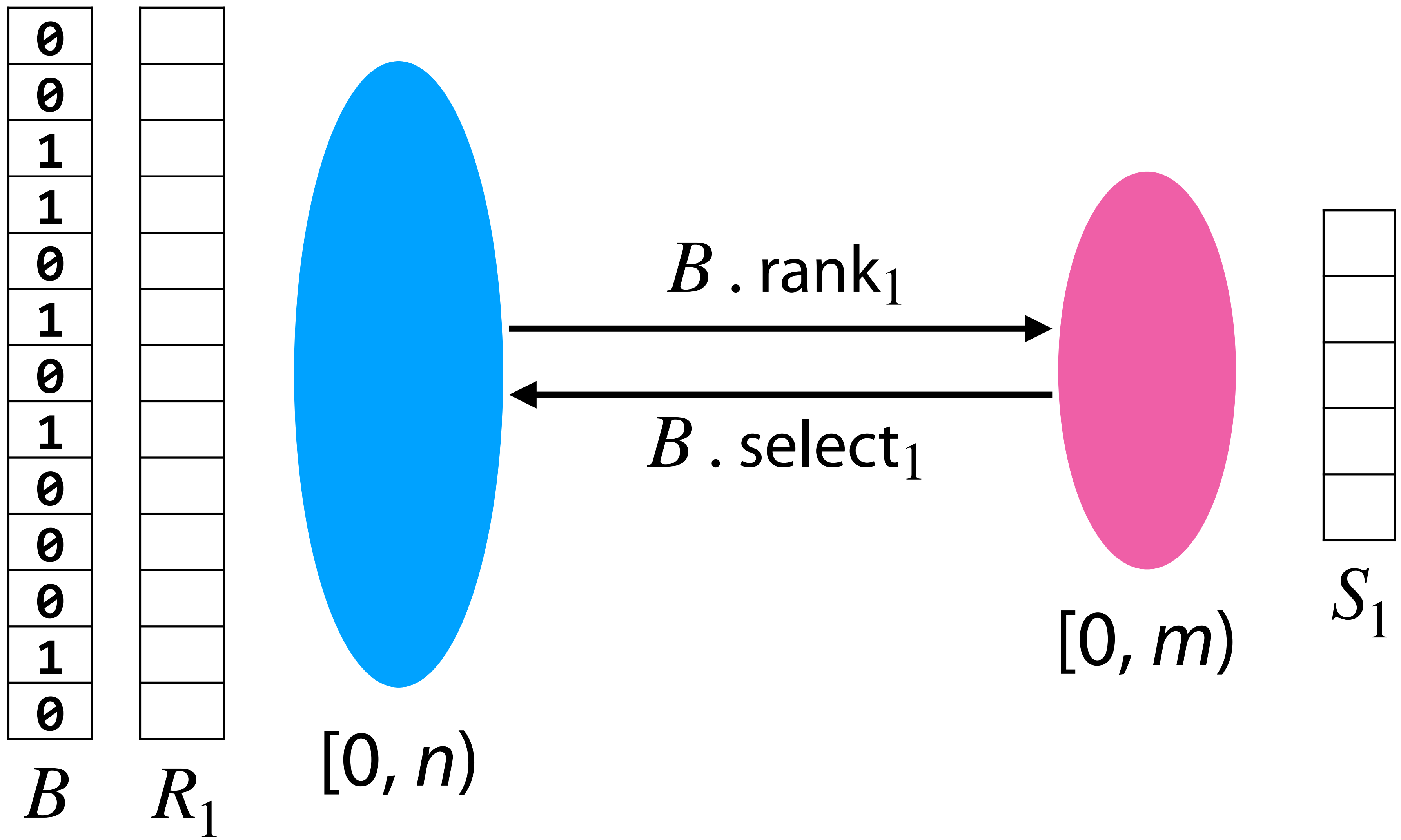
Idea 0: linear scans over B



Can we be more efficient?

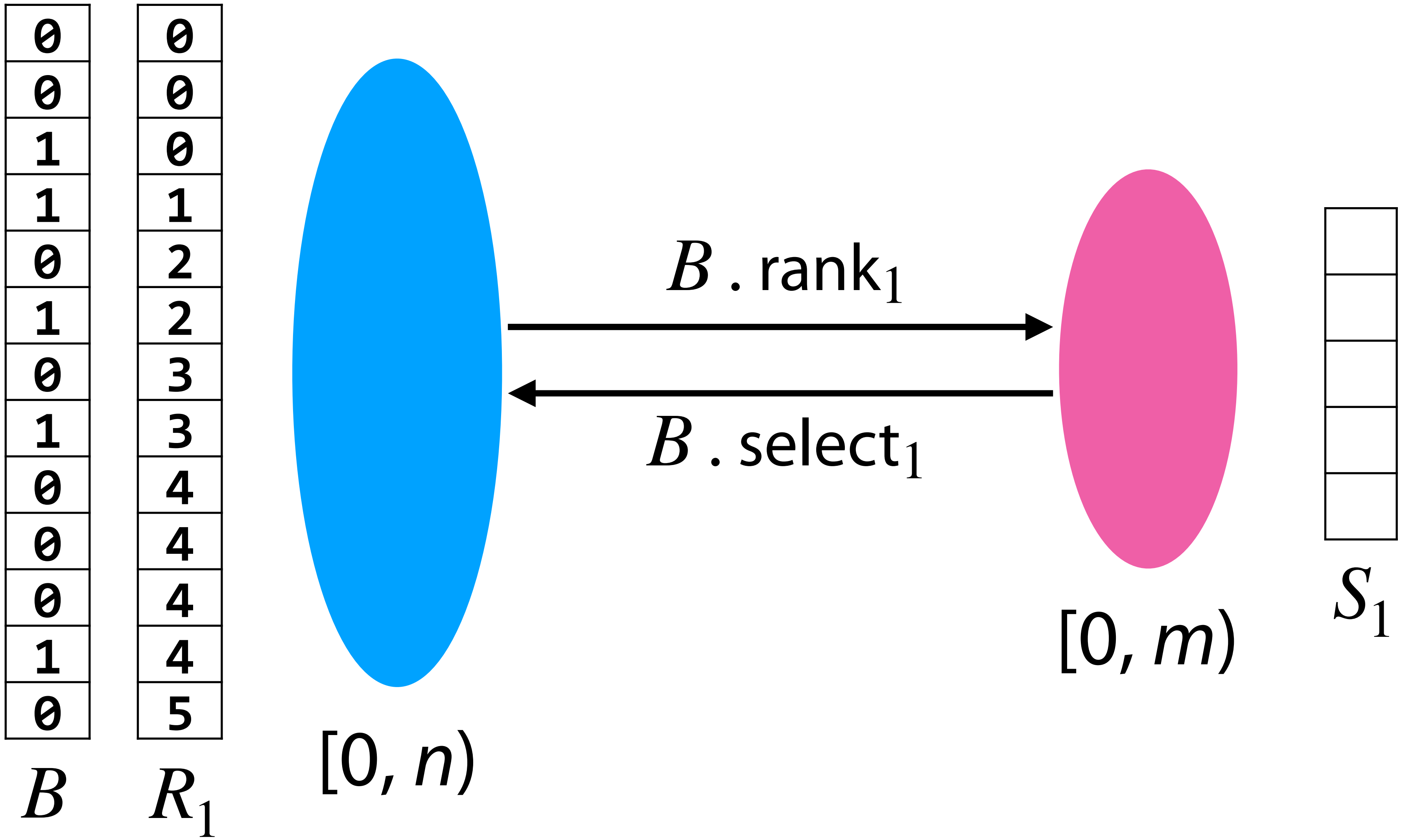
Bitvectors

Idea 1: Pre-calculate all answers



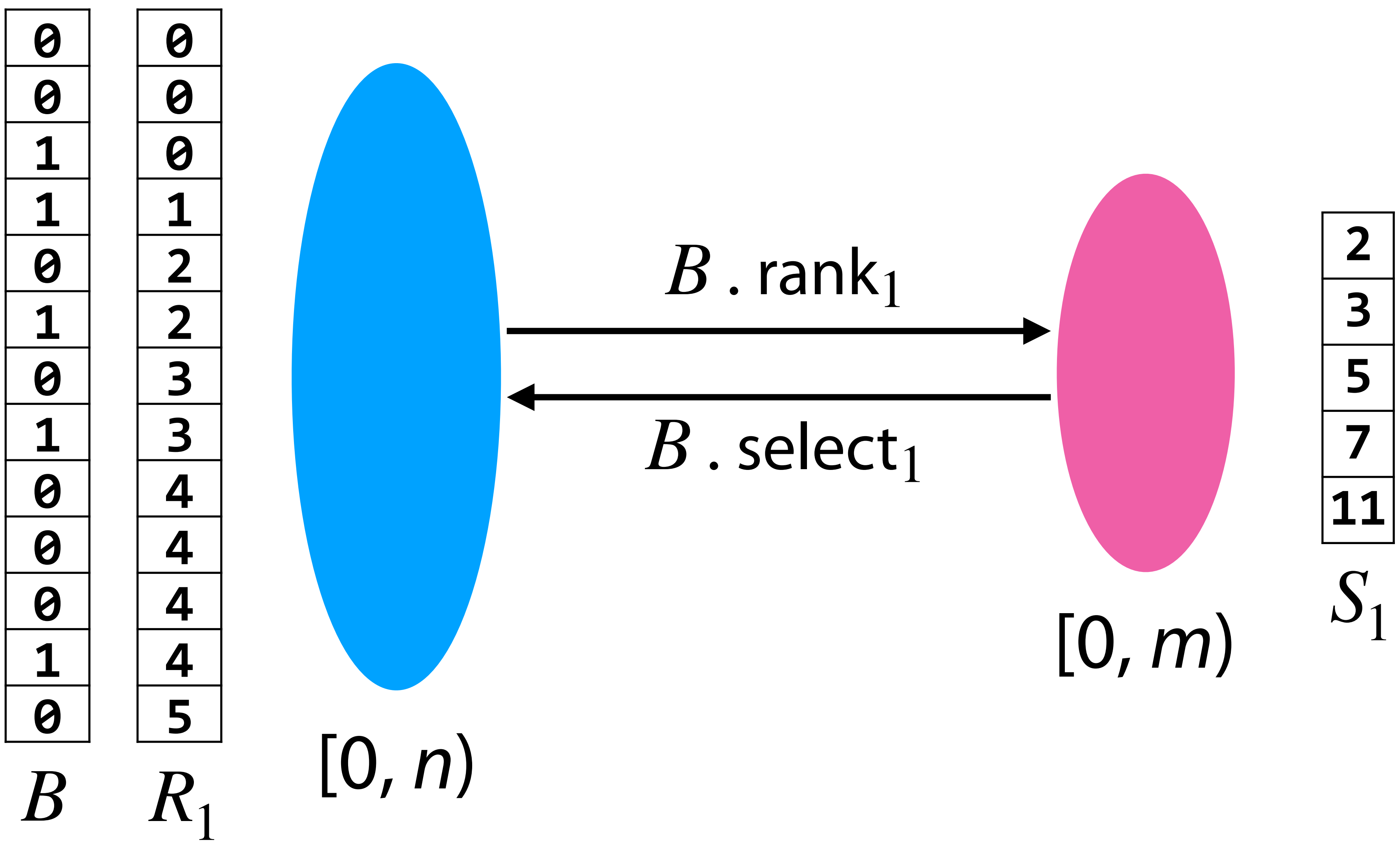
Bitvectors

Idea 1: Pre-calculate all answers



Bitvectors

Idea 1: Pre-calculate all answers



$O(m \log n + n \log m)$ bits required

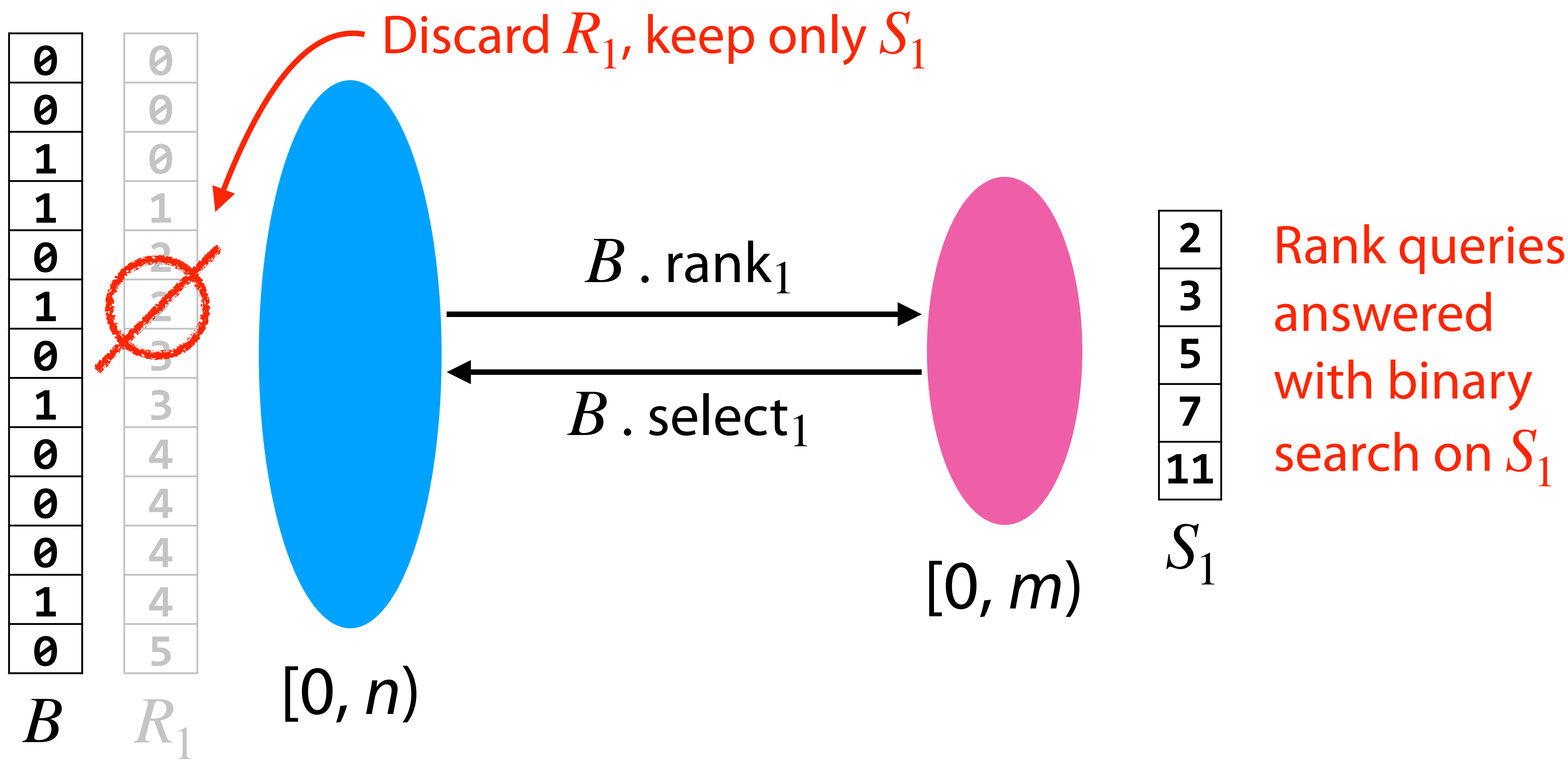
Bitvectors

Idea 1: Pre-calculate all answers

	Time	Space (bits)	Note
B . access	$O(1)$	n	Lookup
B . rank$_1$	$O(1)$	$O(n \log m)$	Pre-calculate R_1
B . select$_1$	$O(1)$	$O(m \log n)$	Pre-calculate S_1

Bitvectors

Idea 2: Pre-calculate all answers for $B\,.\,\mathrm{select}_1$



$O(m \log n)$ bits. $B\,.\,\mathrm{rank}_1$ is $O(\log m)$ time.

Bitvectors

Idea 2: Pre-calculate all answers for $B\,.\,\text{select}_1$

	Time	Space (bits)	Note
$B\,.\,\text{access}$	$O(1)$	n	Lookup
$B\,.\,\text{rank}_1$	$O(\log m)$	$O(m \log n)$	Binary search on S_1
$B\,.\,\text{select}_1$	$O(1)$	$O(m \log n)$	Pre-calculate S_1

Bitvectors

Is this possible?

	Time	Space (bits)	Note
B . access	$O(1)$	n	Lookup
B . rank$_1$	$O(1)$	$\check{o}(n)$	??🧚‍♂️🦄🧚‍♀️??
B . select$_1$	$O(1)$	$\check{o}(n)$	??🧚‍♂️🦄🧚‍♀️??