I need to show why an interval has $\geq x$ balls suffices if $P(\exists \text{ interval with } \leq x \text{ balls}) \leq \frac{e^n}{n^x}$.

Fix an interval of size $n \cdot q$ and let

$$X = X_1 + \ldots + X_n$$

in which $X_i$ takes the value 1 if the $i$th ball falls into that interval.

$$P(X_i = 1) = \frac{q}{n} = \frac{1}{n}$$

$$E(X_i) = \frac{q}{n}$$

Hence $E(X) = n \cdot \frac{q}{n} = qn$.

We find the min $s$ such that

$$P(X \leq \sqrt{n (1-s)}) \leq \frac{1}{n^2}$$

Suffices if $s \geq \frac{\ln \frac{q}{n}}{n}$.

Hence $s > \frac{2 \sqrt{\ln \frac{q}{n}}}{n}$.

There are no more than $n$ such intervals (correct: $\frac{n}{n-\ln}$).

Hence by Boole's inequality

$$P(\exists \text{ interval with } \leq \sqrt{n (1-s) \text{ balls}}) \leq n \cdot \frac{1}{n^2} = \frac{1}{n}$$

Hence max value if $X = \sqrt{n (1- \frac{2 \sqrt{\ln \frac{q}{n}}}{n})}$.

II Fix any column, let $X = X_1 + \ldots + X_n$ in which

$X_i$ takes the value 1 if the $i$th position in the

column is nonempty.

$$P(X_i = 0) = (1 - \frac{1}{n})^{\text{only}}$$

Hence $P(X_i = 1) = 1 - (1 - \frac{1}{n})^{\text{only}}$. 
\[ E(X_i) = 1 - \left(1 - \frac{1}{n}\right)^{m/\lambda} \]

Hence \[ E(X) = n \left(1 - \left(1 - \frac{1}{n}\right)^{m/\lambda}\right) \]

\[ P(X \geq \mu + s) \leq e^{-\frac{bs^2}{3}} \]

We seek a \( s \) such that \[ e^{-\frac{bs^2}{3}} \leq \frac{1}{m^2} \quad \text{(since } \mu = cn) \]

i.e. \[ s \geq \sqrt{\frac{6 \ln m}{b \mu}} \]

Thus \[ \lambda = \mu \left(1 + \sqrt{\frac{6 \ln m}{b \mu}}\right) \]

where \( \mu = n \left(1 - \left(1 - \frac{1}{n}\right)^{m/\lambda}\right) \)