First- and Second-order Expectation Semirings
with Applications to
Minimum-Risk Training on Translation Forests

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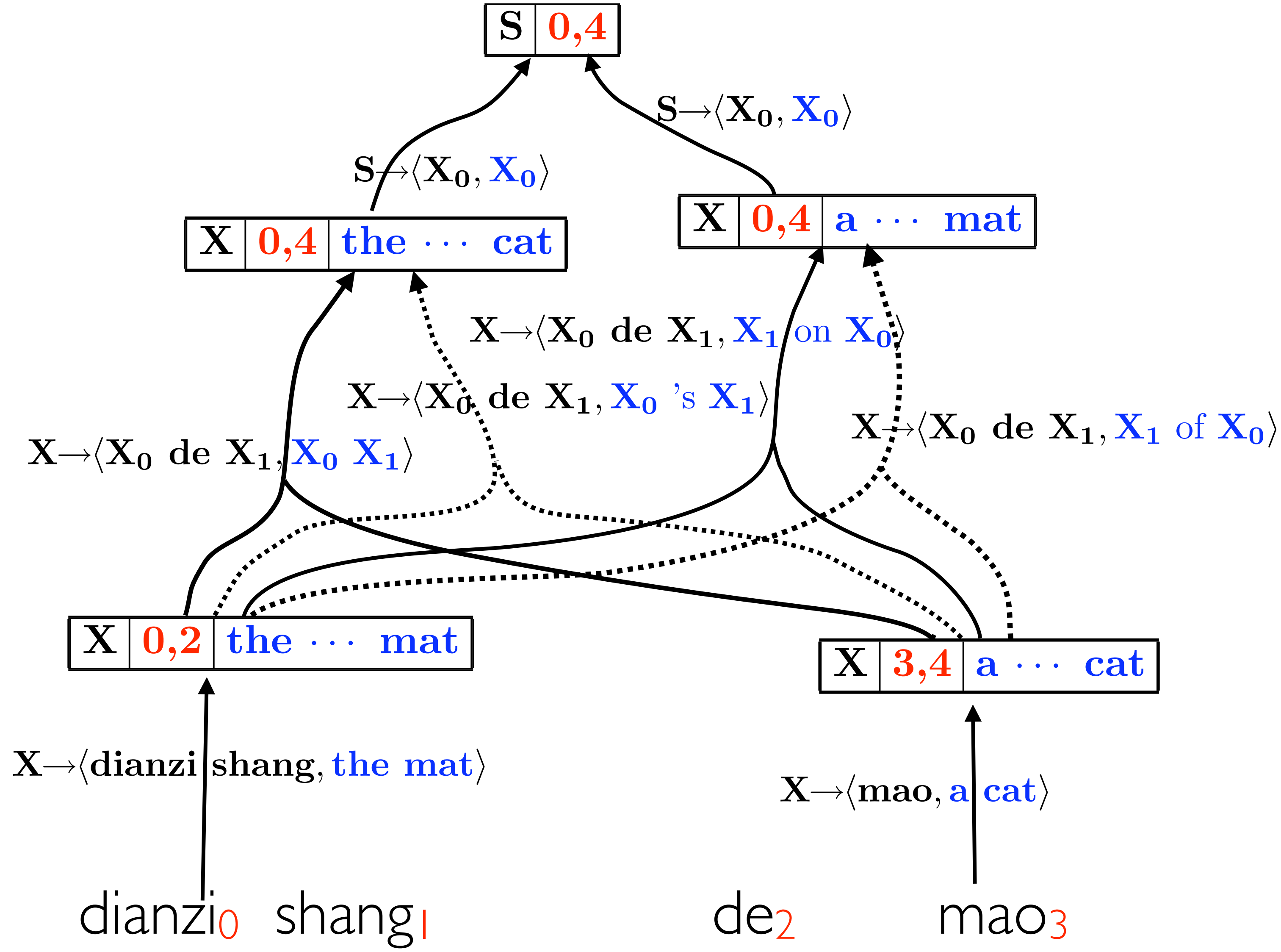
dianzi shang de mao

the cat on the mat

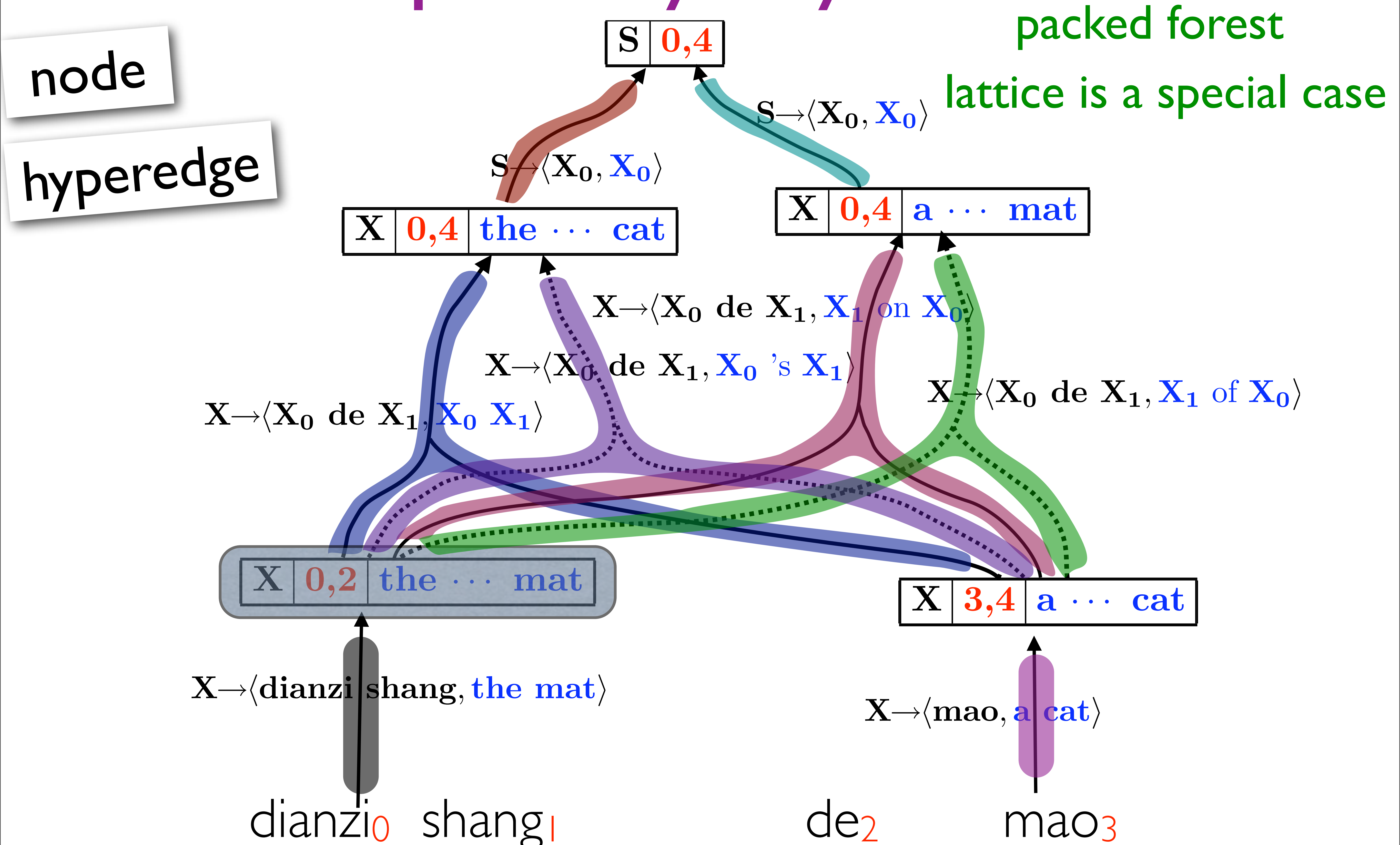
hypergraph



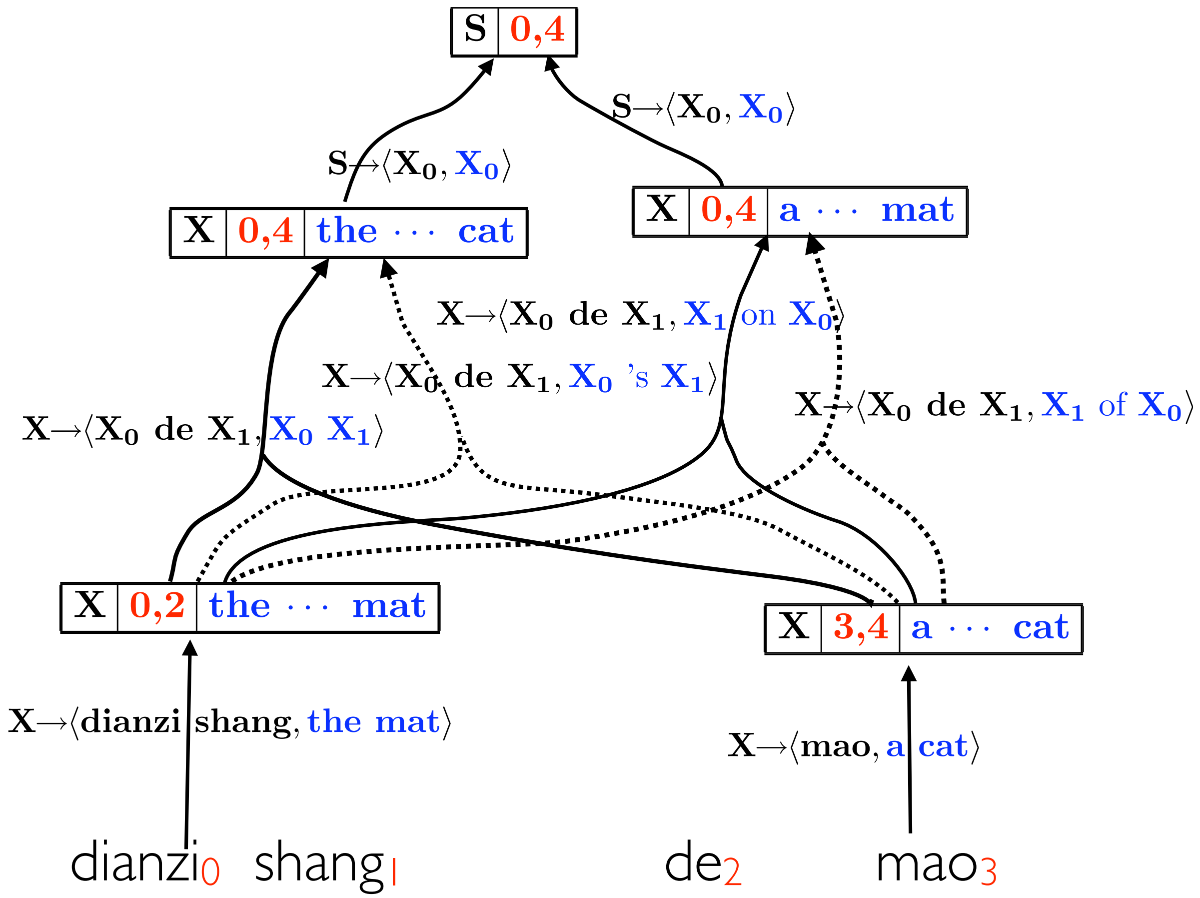
Joshua

dianzi shang de mao

A hypergraph is a compact data structure to encode exponentially many trees.



How many trees?



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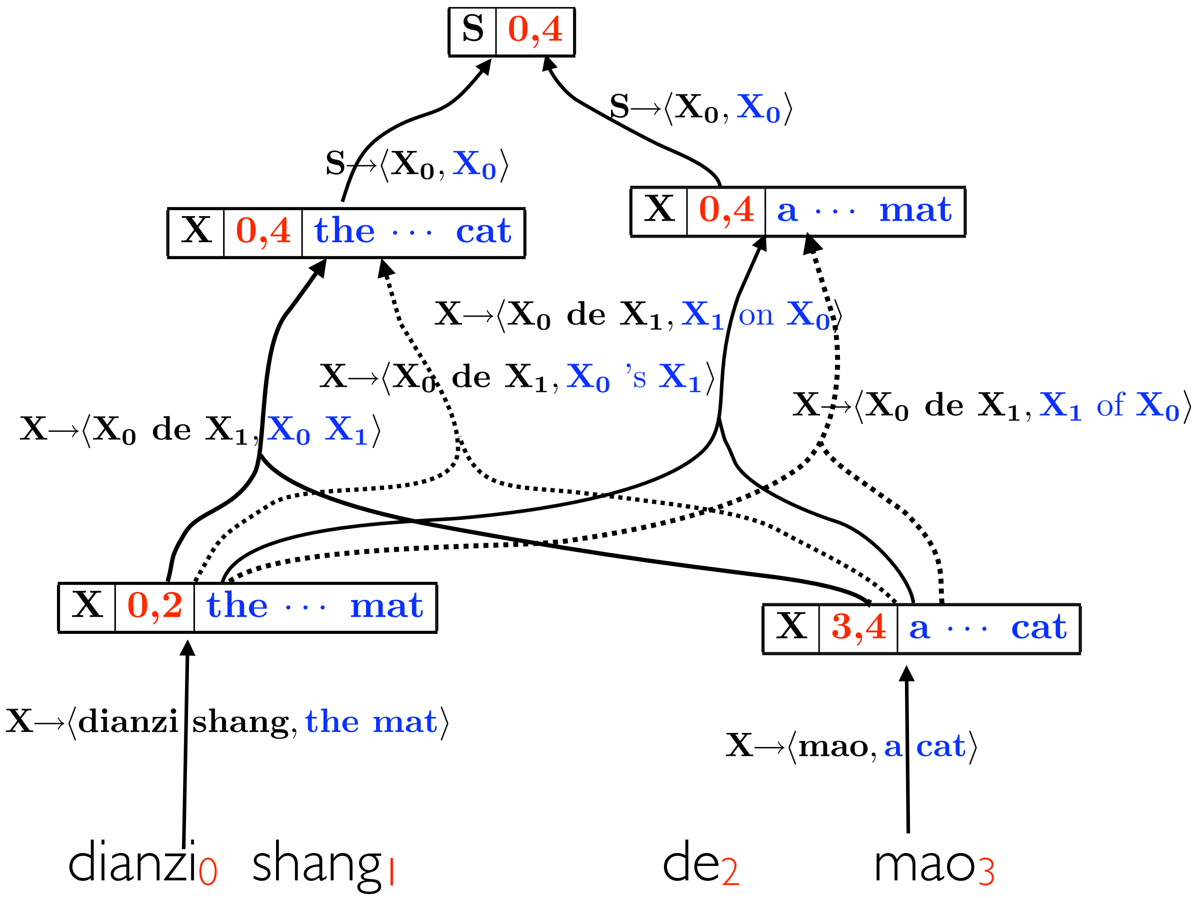
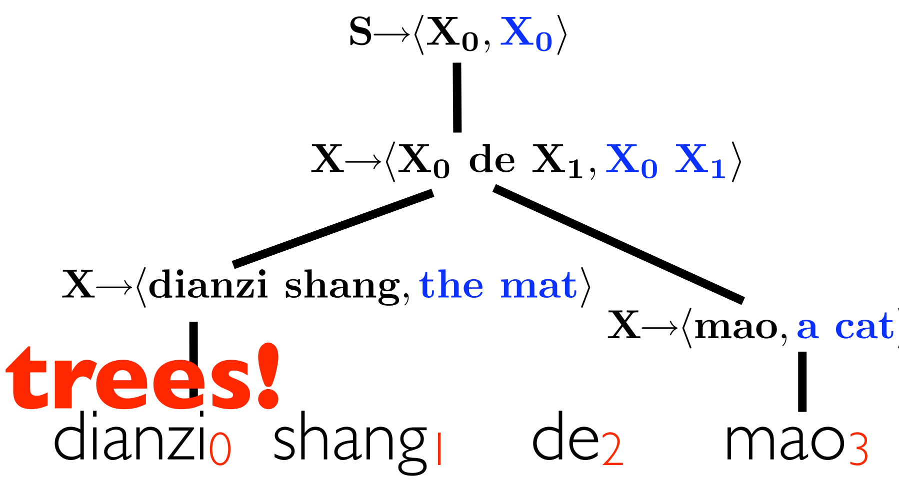
How many trees?

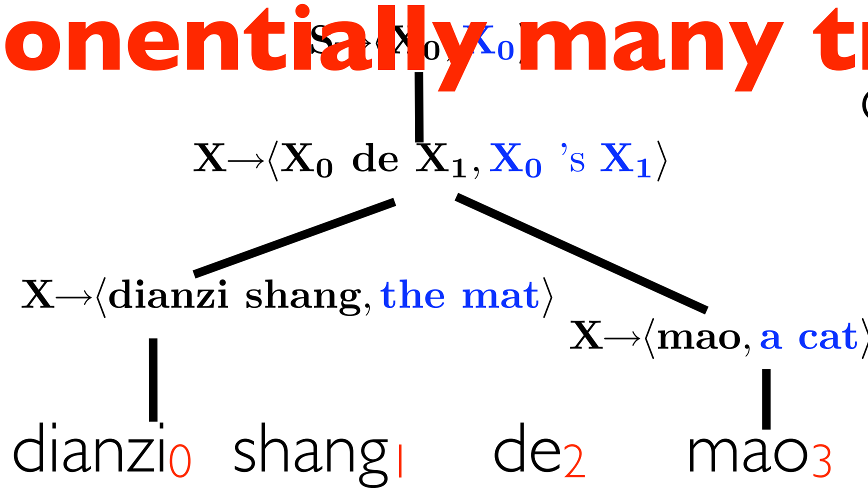
four ☺



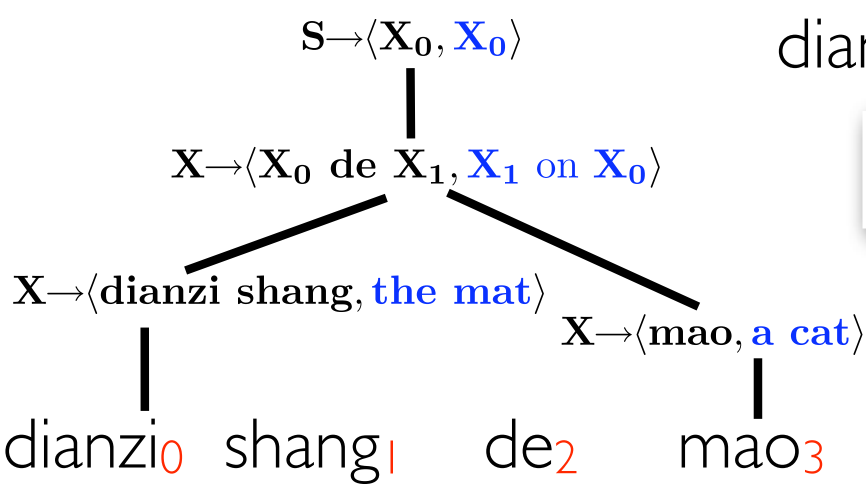
In general, exponentially many trees!



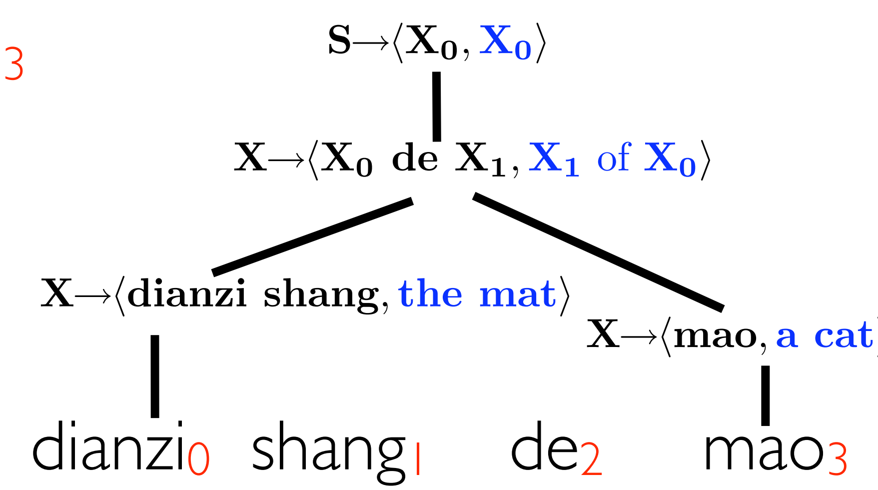
the mat a cat



the mat 's a cat

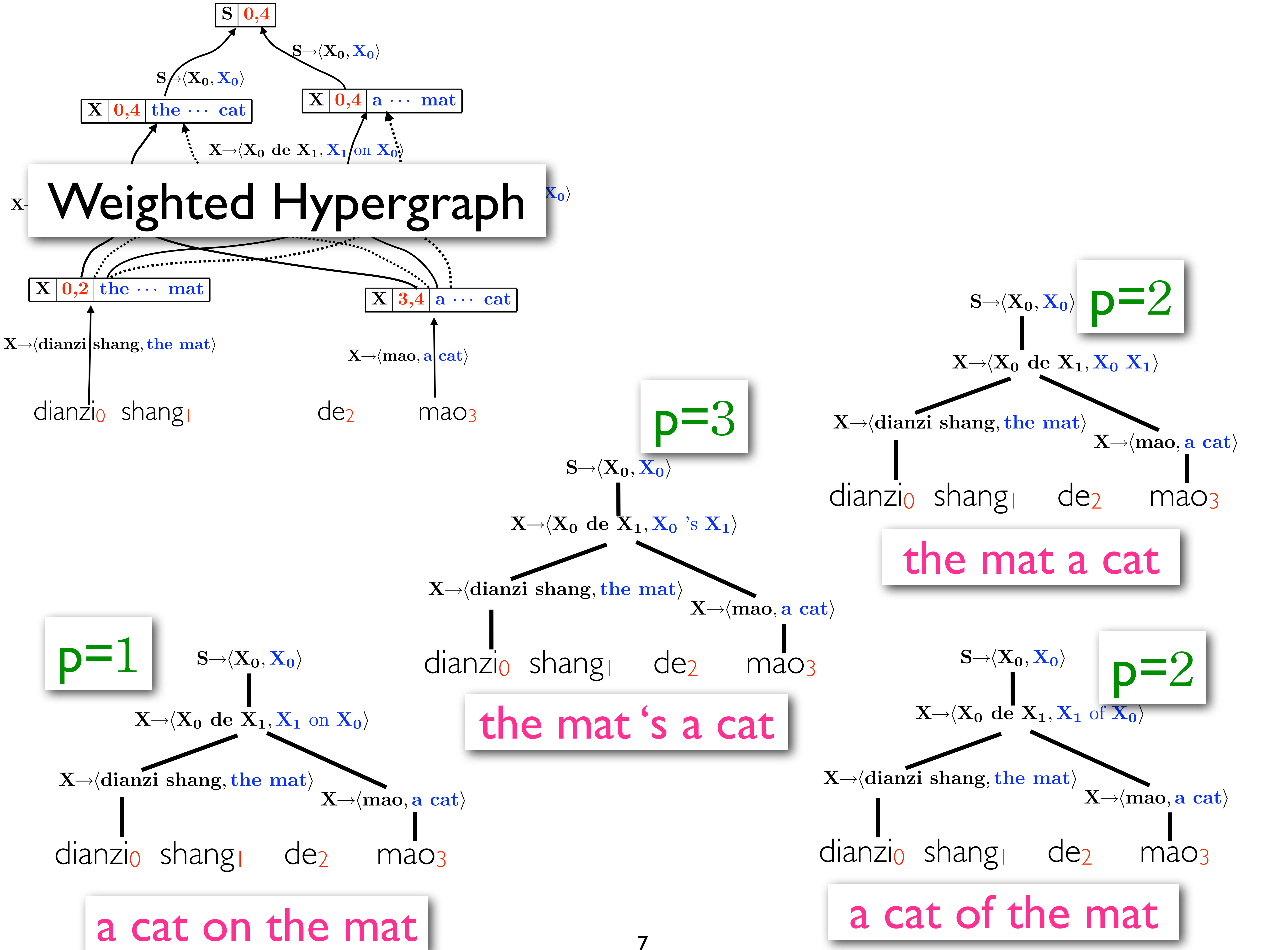


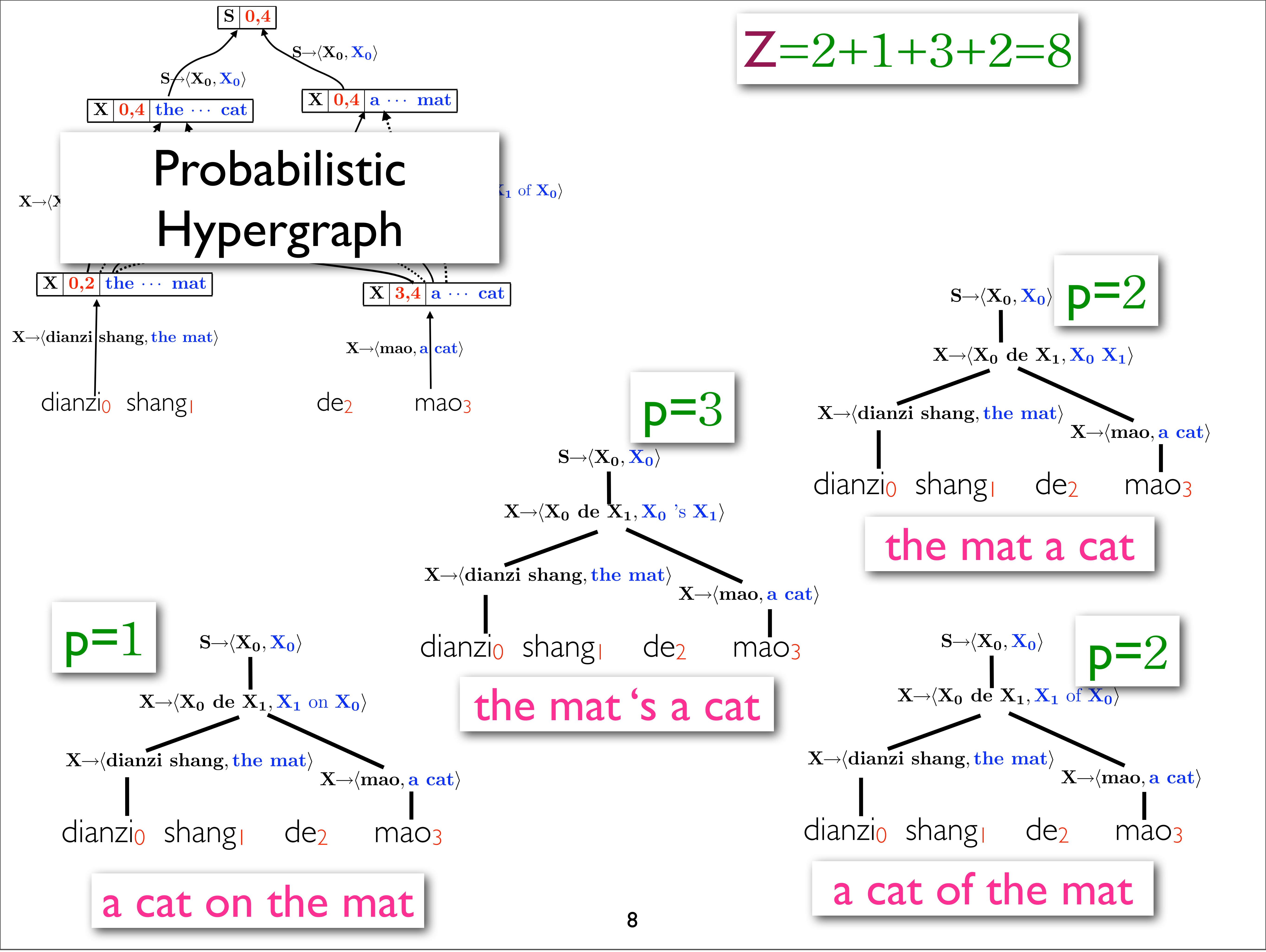
a cat on the mat



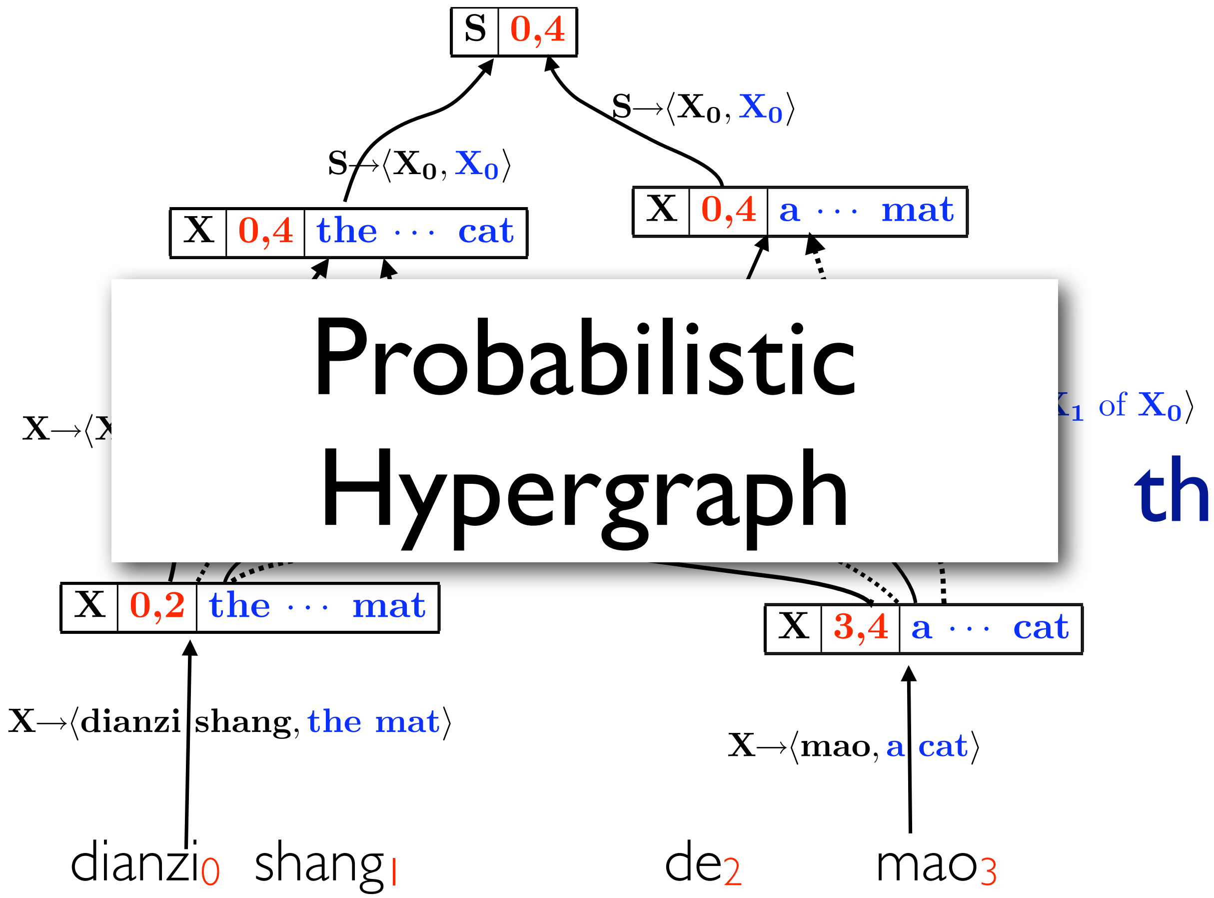
a cat of the mat

Weighted Hypergraph





Probabilistic Hypergraph
Z=2+1+3+2=8
S 0,4
S→⟨X0, X0⟩
X 0,4 the ··· cat
X 0,4 a ··· mat
X1 of X0⟩
X 0,2 the ··· mat
X 3,4 a ··· cat
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
p=1
X→⟨X0 de X1, X1 on X0⟩
a cat on the mat
p=3
X→⟨X0 de X1, X0 's X1⟩
the mat 's a cat
p=2
X→⟨X0 de X1, X0 X1⟩
the mat a cat
p=2
X→⟨X0 de X1, X1 of X0⟩
a cat of the mat
8

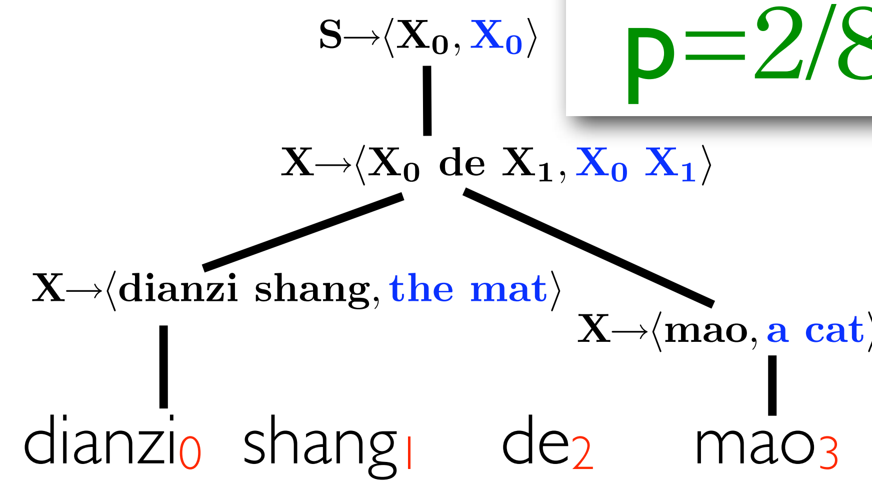


Probabilistic Hypergraph

Z=2+1+3+2=8

The hypergraph defines a probability distribution over **trees**!

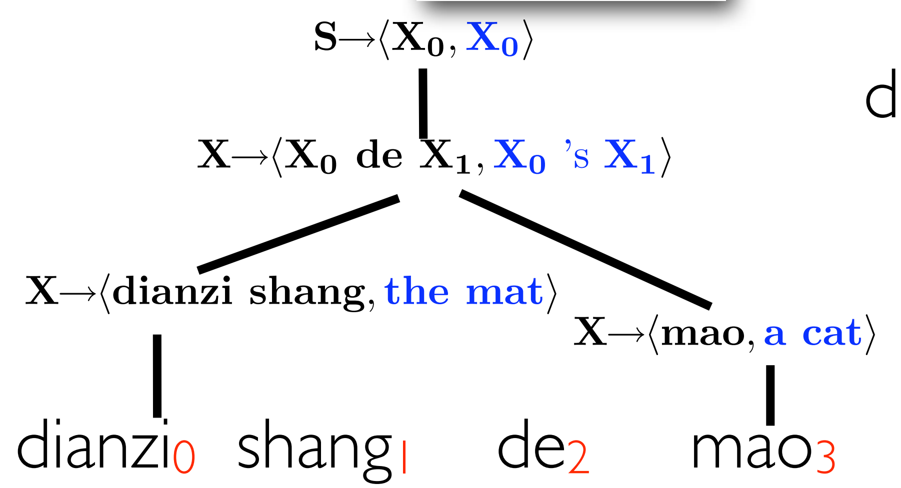
the distribution is parameterized by Θ



p=2/8

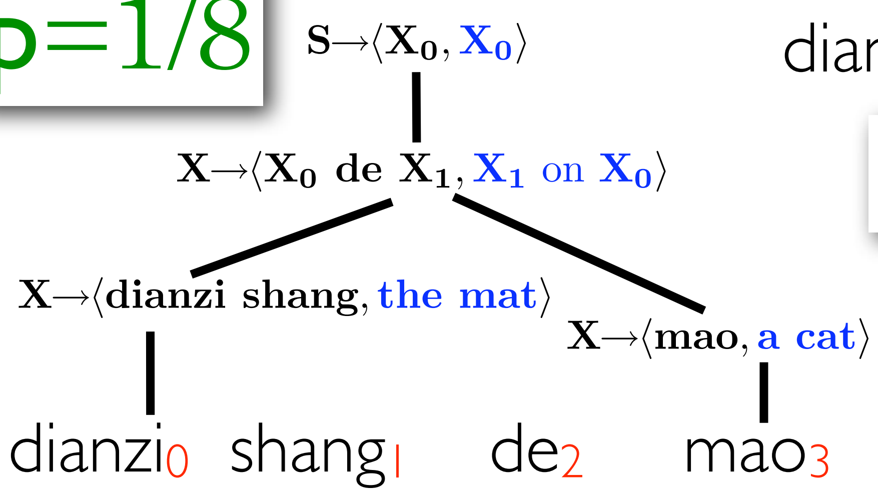
the mat a cat

p=3/8

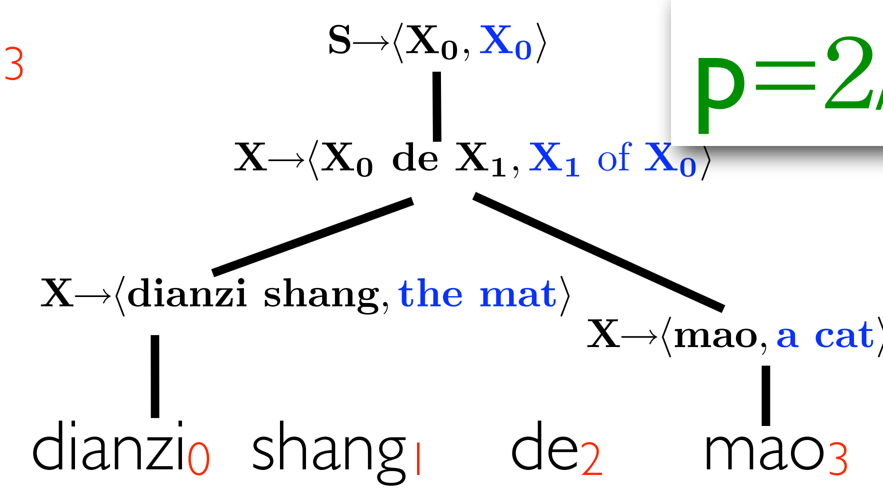


the mat 's a cat

p=1/8

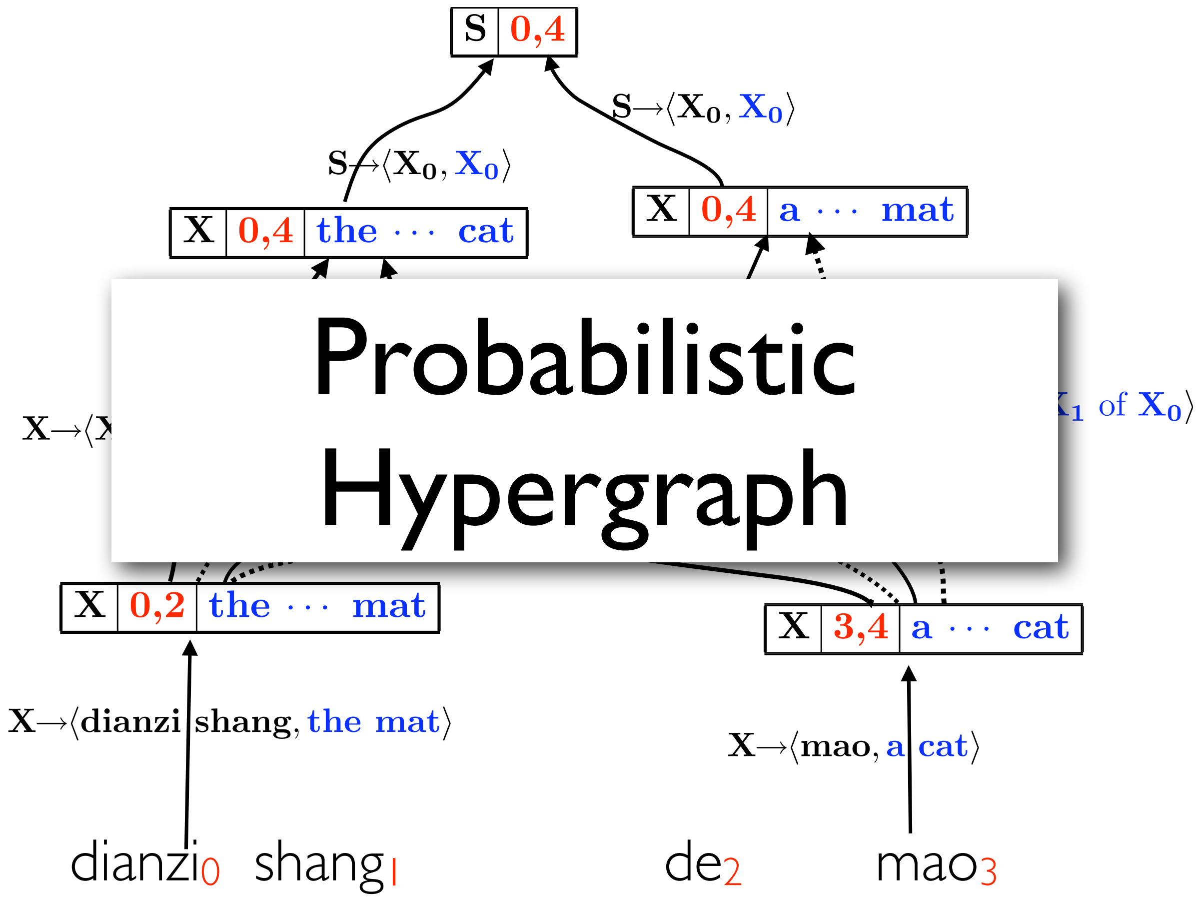


a cat on the mat



p=2/8

a cat of the mat



Probabilistic Hypergraph

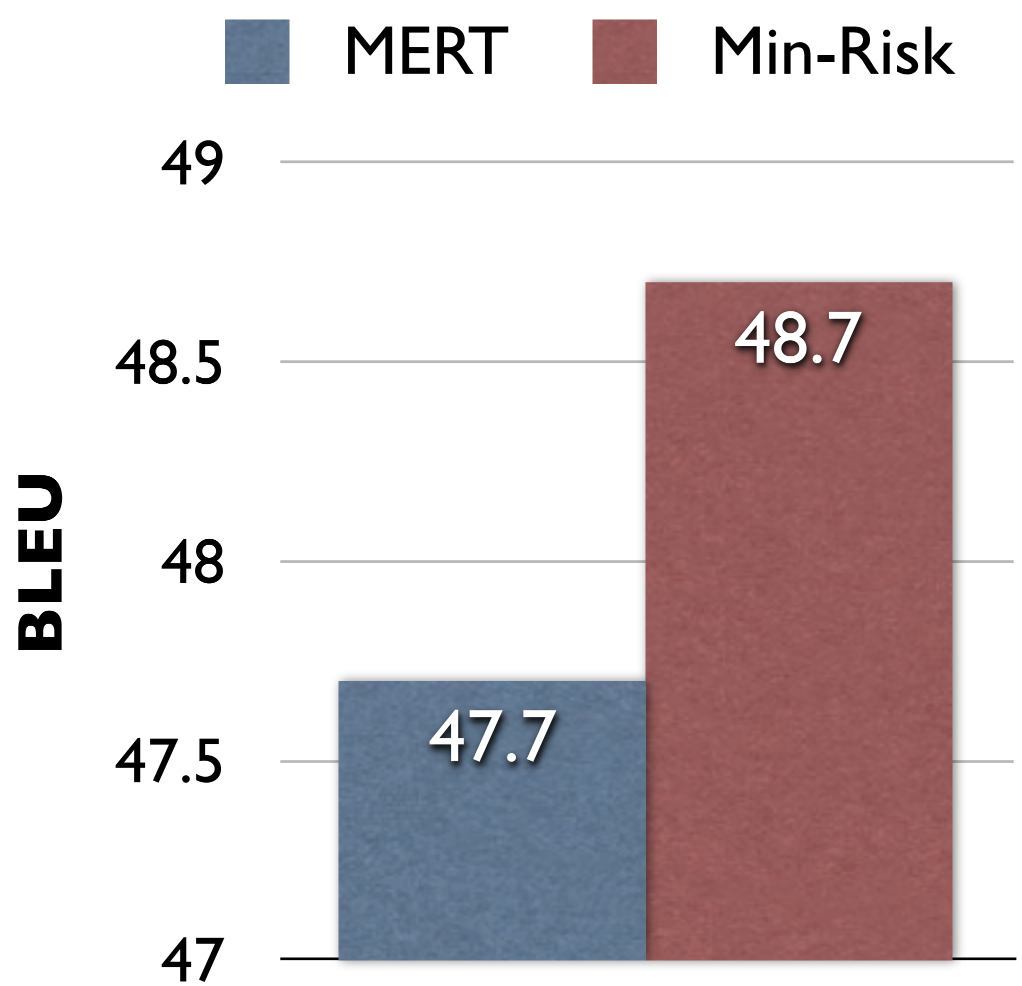
Entropy?

Bayes Risk?

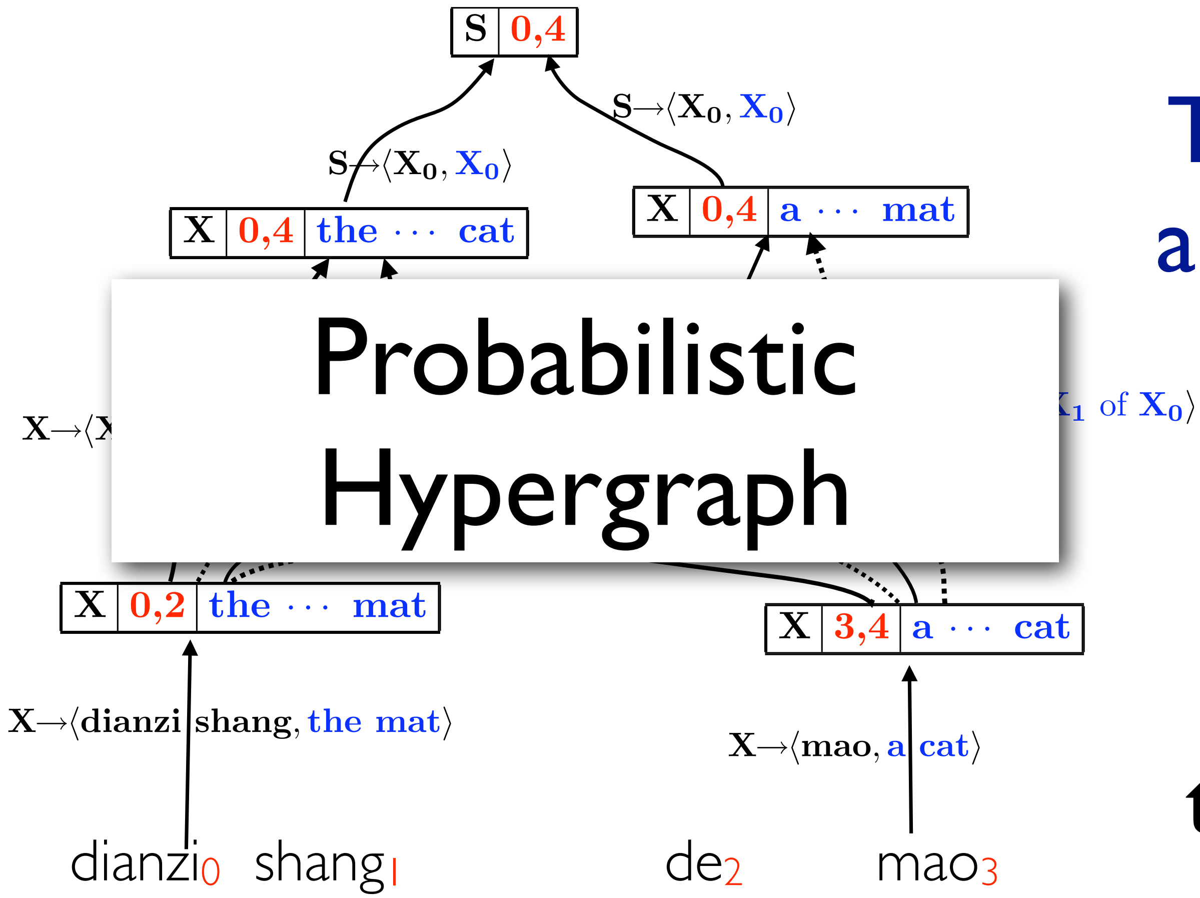
Minimum Risk Training on Translation Forests

Finding optimal parameters:

$$\theta^* = \mathit{argmin}\ \mathrm{Risk}(P_\theta) - \mathrm{Temperature} \times \mathrm{Entropy}(P_\theta)$$



BLEU scores on an IWSLT Chinese-English test set



This work provides a unified, elegant, and efficient framework in computing all of these!

useful in applying machine learning techniques into machine translation

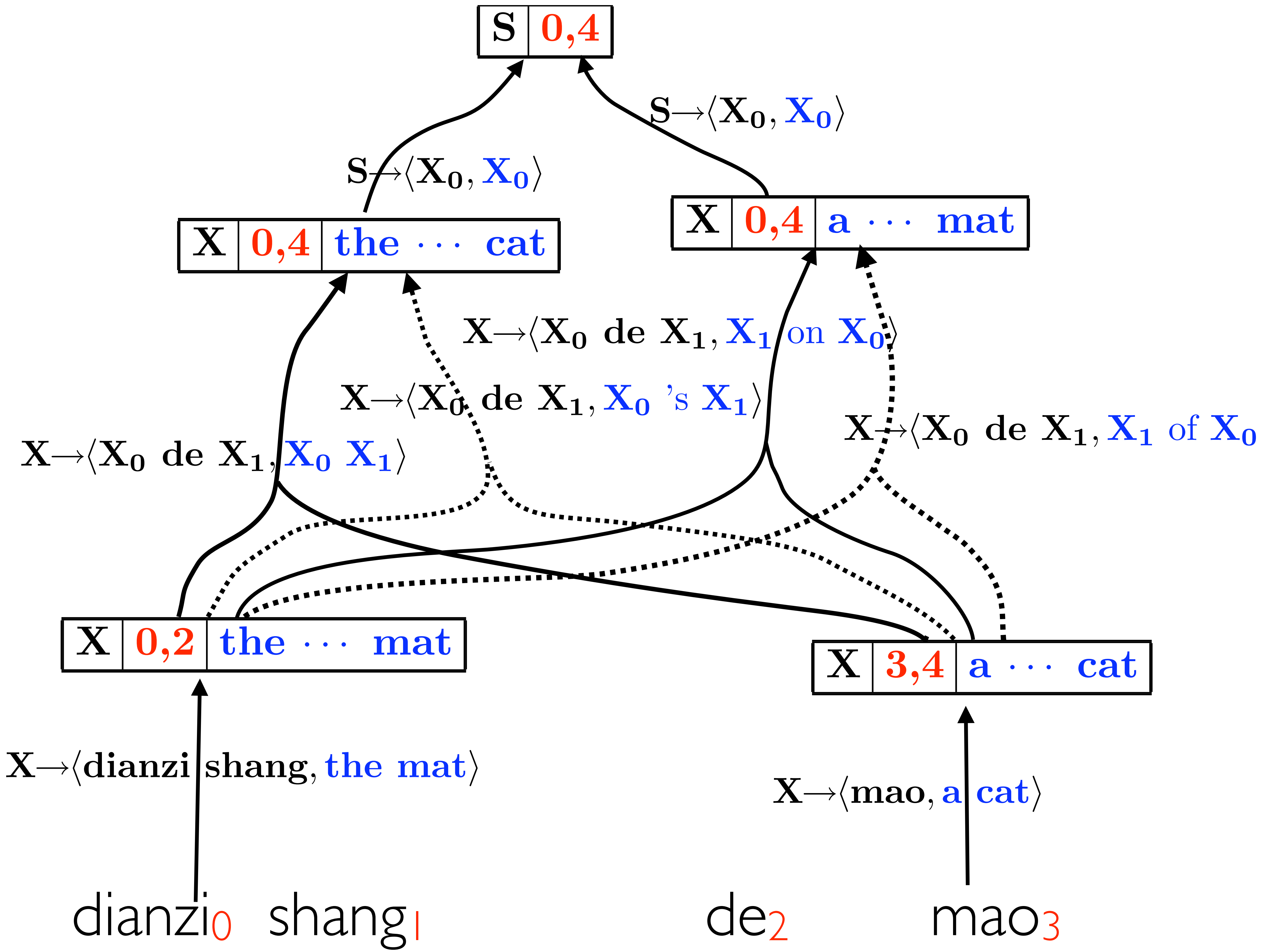
- First-order quantities:
  - expectation
    - entropy
    - Bayes risk
    - cross-entropy
    - KL divergence
    - feature expectations
  - first-order gradient of Z

- Second-order quantities:
  - Expectation over product
    - interaction between features
  - Hessian matrix of Z
    - second-order gradient descent
  - gradient of expectation
    - gradient of entropy or Bayes risk

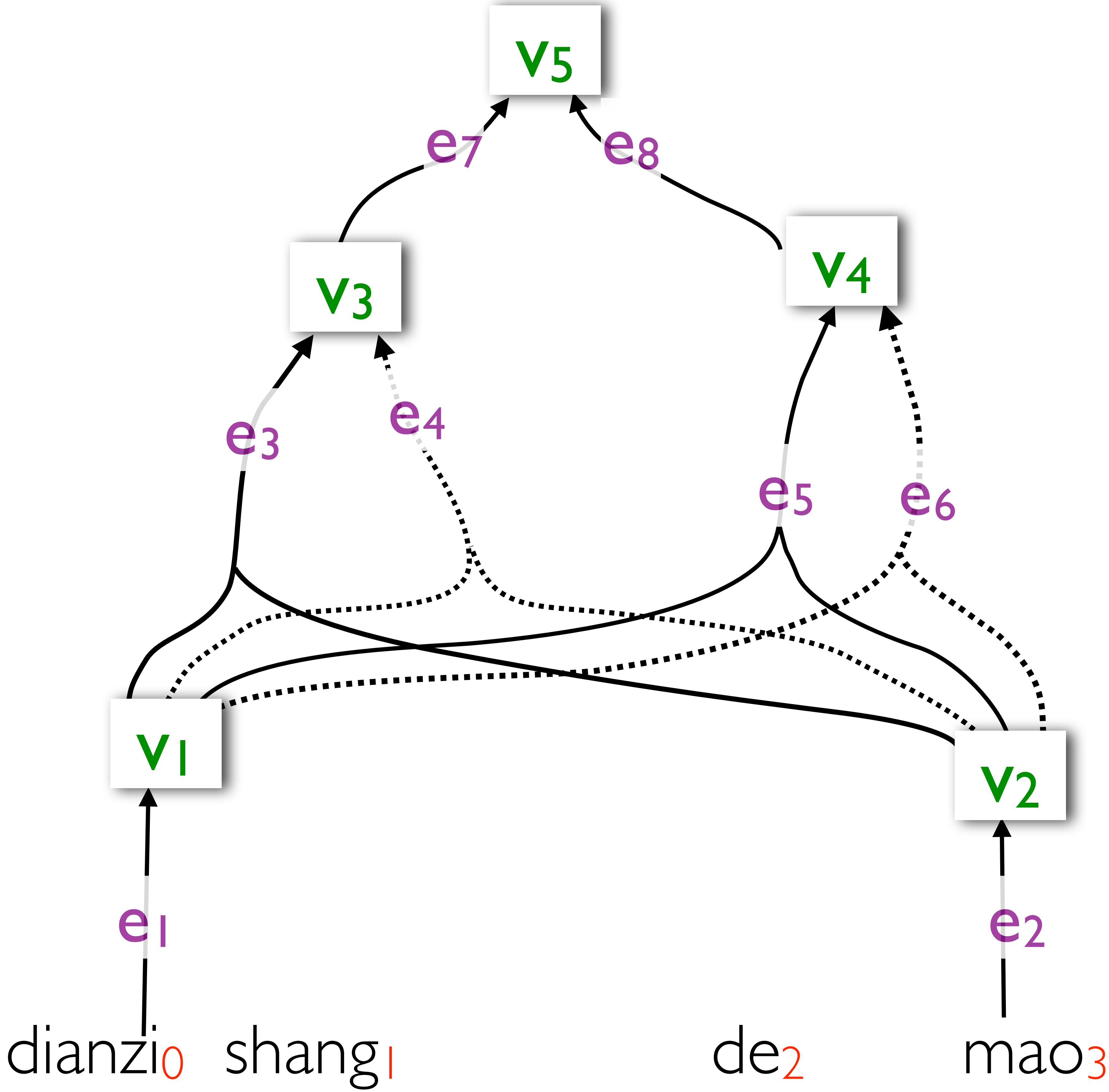
Outline

- Semiring-weighted Inside Algorithm
  - counting semiring (Goodman, 1999)
  - expectation semirings (Eisner, 2002)
  - second-order expectation semirings (new)
- Applications of the Semirings (new)
- Speed-up with Inside-outside (new)
- Minimum Risk Training over **Forests** (new)

Semiring-weighted Inside Algorithm



S | 0,4
S→⟨X0, X0⟩
S→⟨X0, X0⟩
X | 0,4 | the ··· cat
X | 0,4 | a ··· mat
X→⟨X0 de X1, X1 on X0⟩
X→⟨X0 de X1, X0 's X1⟩
X→⟨X0 de X1, X0 X1⟩
X→⟨X0 de X1, X1 of X0⟩
X | 0,2 | the ··· mat
X | 3,4 | a ··· cat
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3



v5
e7
e8
v3
v4
e3
e4
e5
e6
v1
v2
e1
e2
dianzi0 shang1 de2 mao3

What is a Semiring?

$$\langle K, \oplus, \otimes \rangle$$

a **set** with **plus** and **times** operations

Compute the Number of Derivation Trees

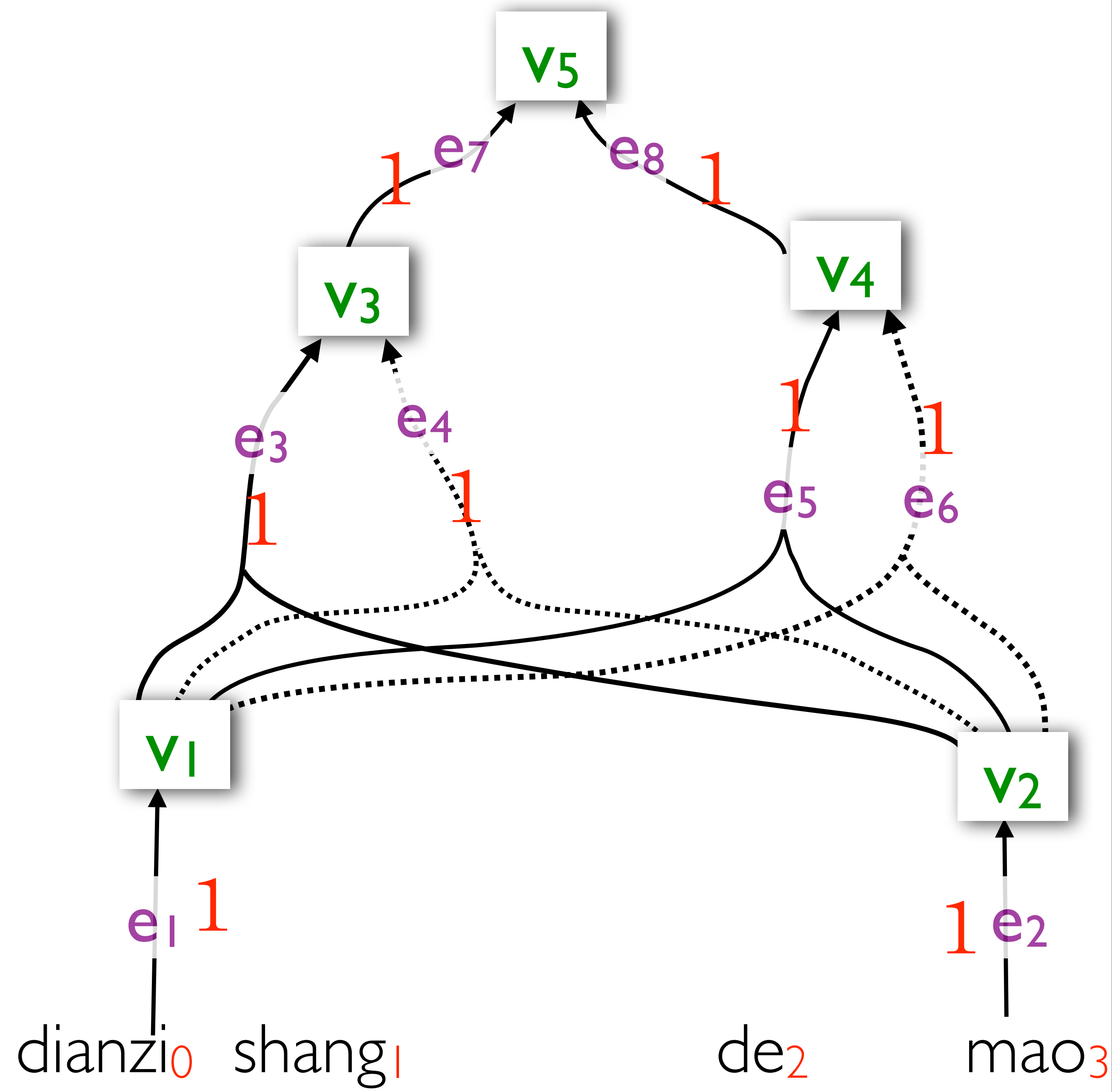
Counting semiring: ordinary integers

Inside algorithm:

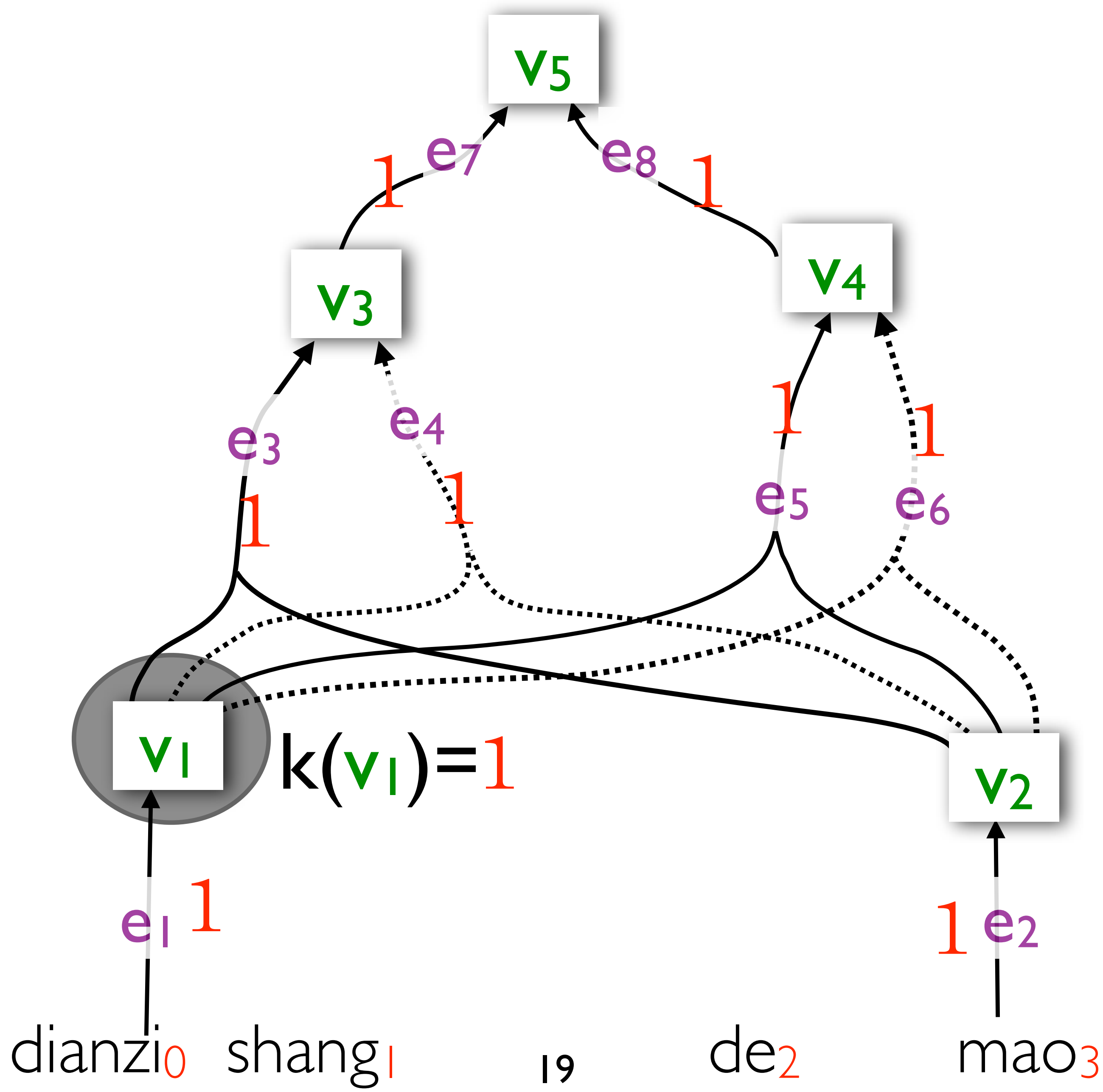
Inputs:
- a hypergraph
- a weight for each hyperedge

Output: k(root)

Complexity:
O(size of the hypergraph)



$k(v_1) = k(e_1)$



Bottom-up process in computing the number of trees

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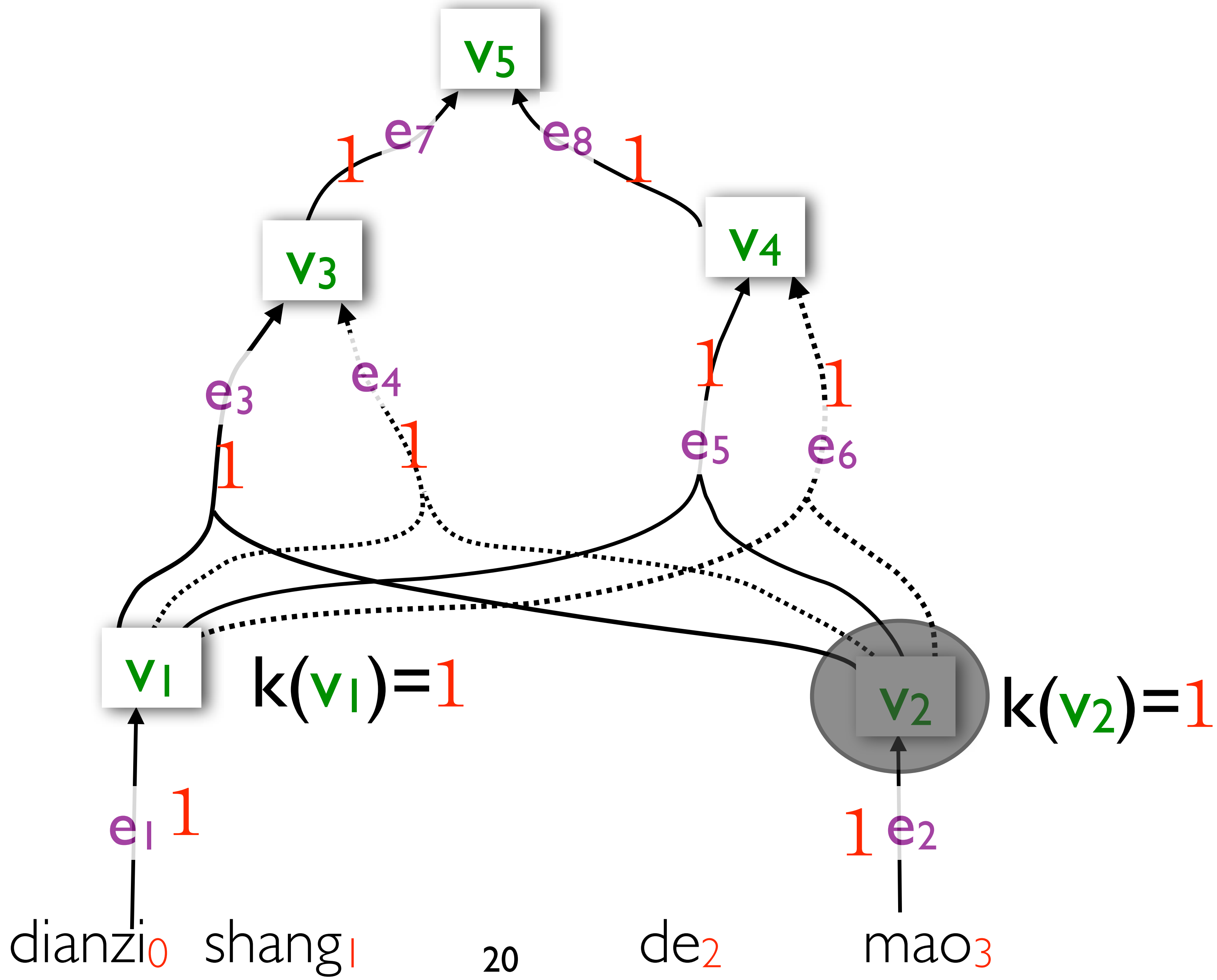
$k(v_1) = k(e_1)$ $k(v_2) = k(e_2)$

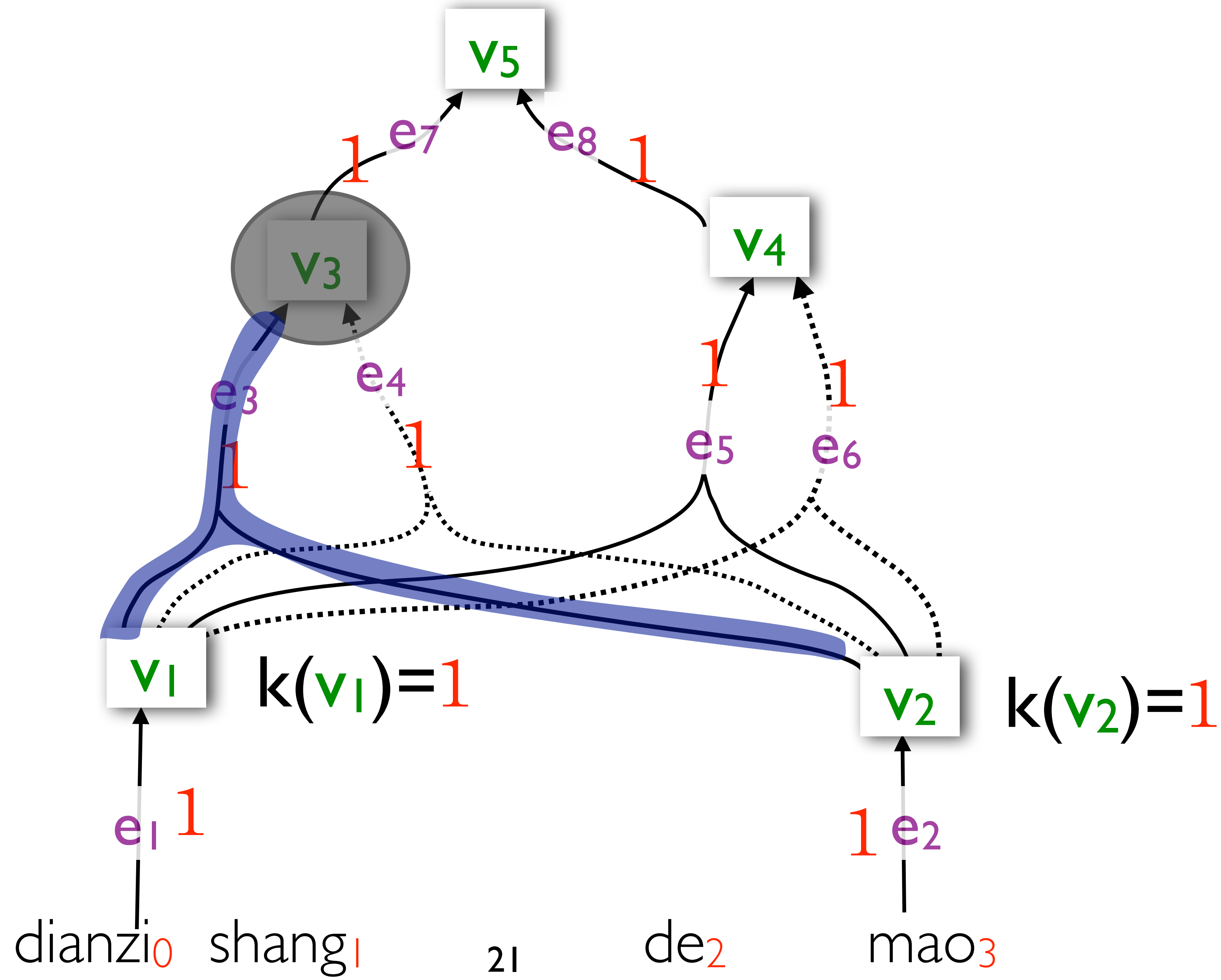


Bottom-up process in computing the number of trees

Compute $k(v_3)$: the weight at node v_3

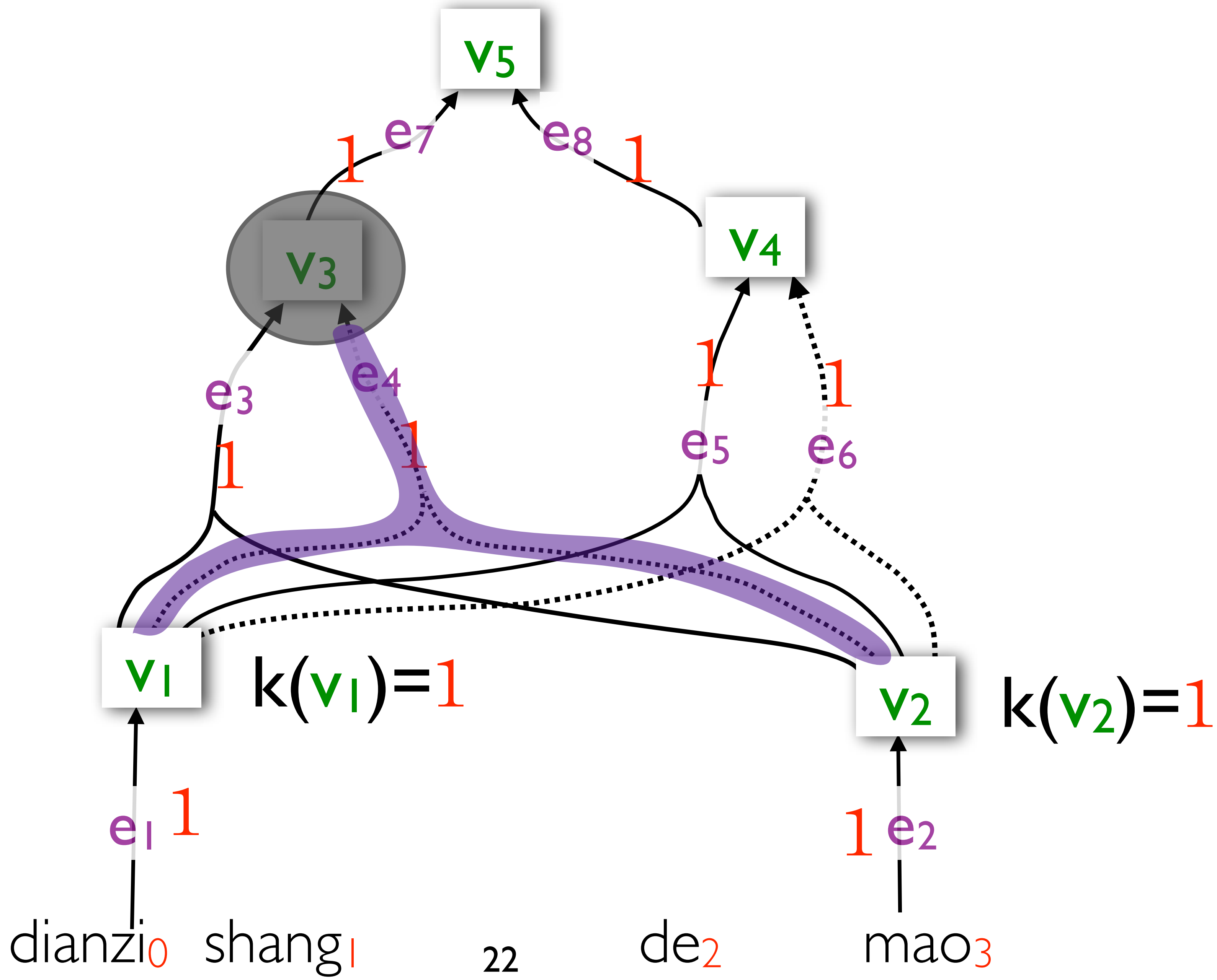
Hyperedge e_3: $k(e_3) \bigotimes k(v_1) \bigotimes k(v_2) = 1 \otimes 1 \otimes 1 = 1$

Bottom-up process in computing the number of trees



Compute $k(v_3)$: the weight at node v_3

Hyperedge e_4: $k(e_4) \bigotimes k(v_1) \bigotimes k(v_2) = 1 \otimes 1 \otimes 1 = 1$



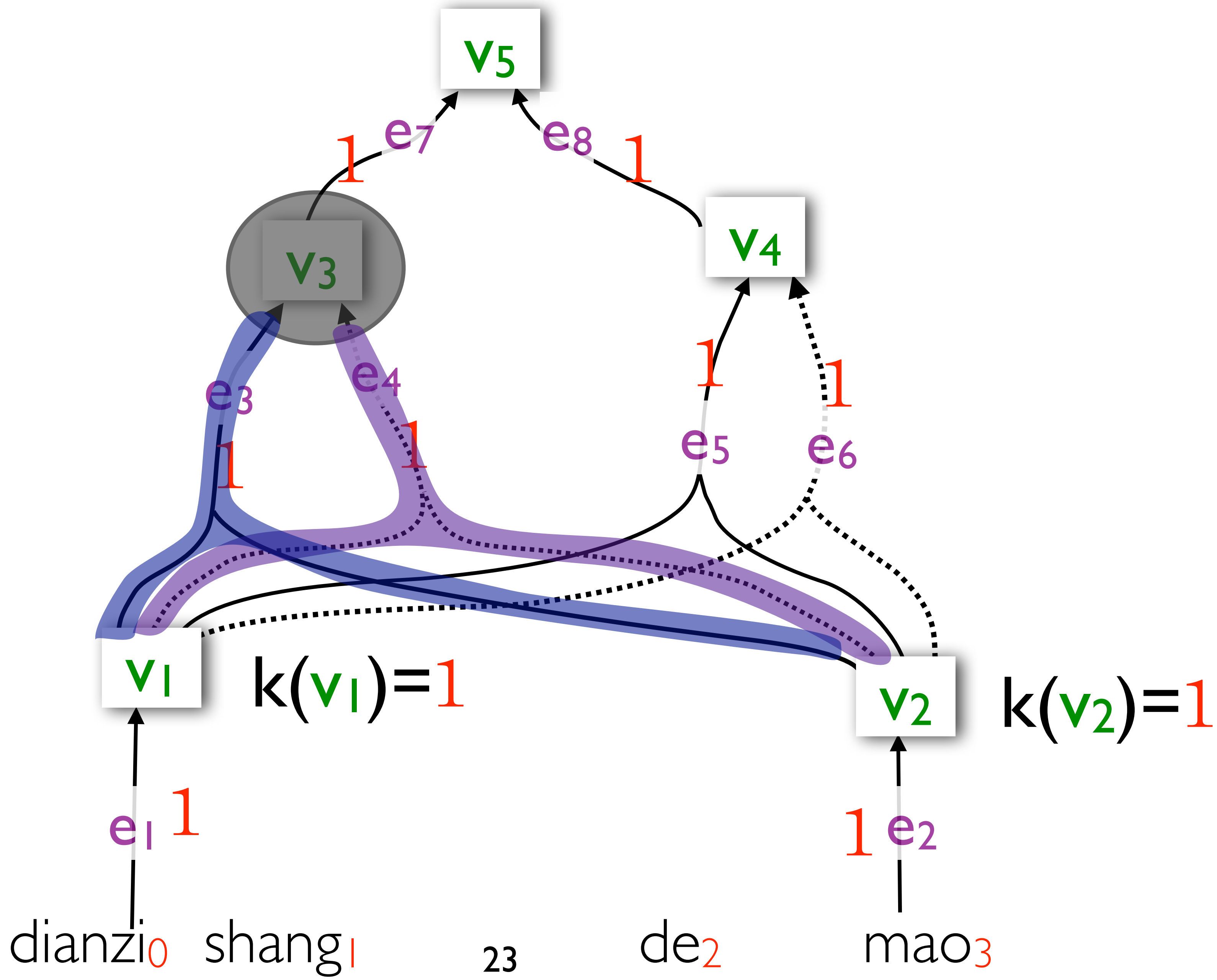
Bottom-up process in computing the number of trees

Compute $k(v_3)$: the weight at node v_3

Hyperedge e_3: $k(e_3) \otimes k(v_1) \otimes k(v_2) = 1$

Hyperedge e_4: $k(e_4) \otimes k(v_1) \otimes k(v_2) = 1$

Bottom-up process in computing the number of trees

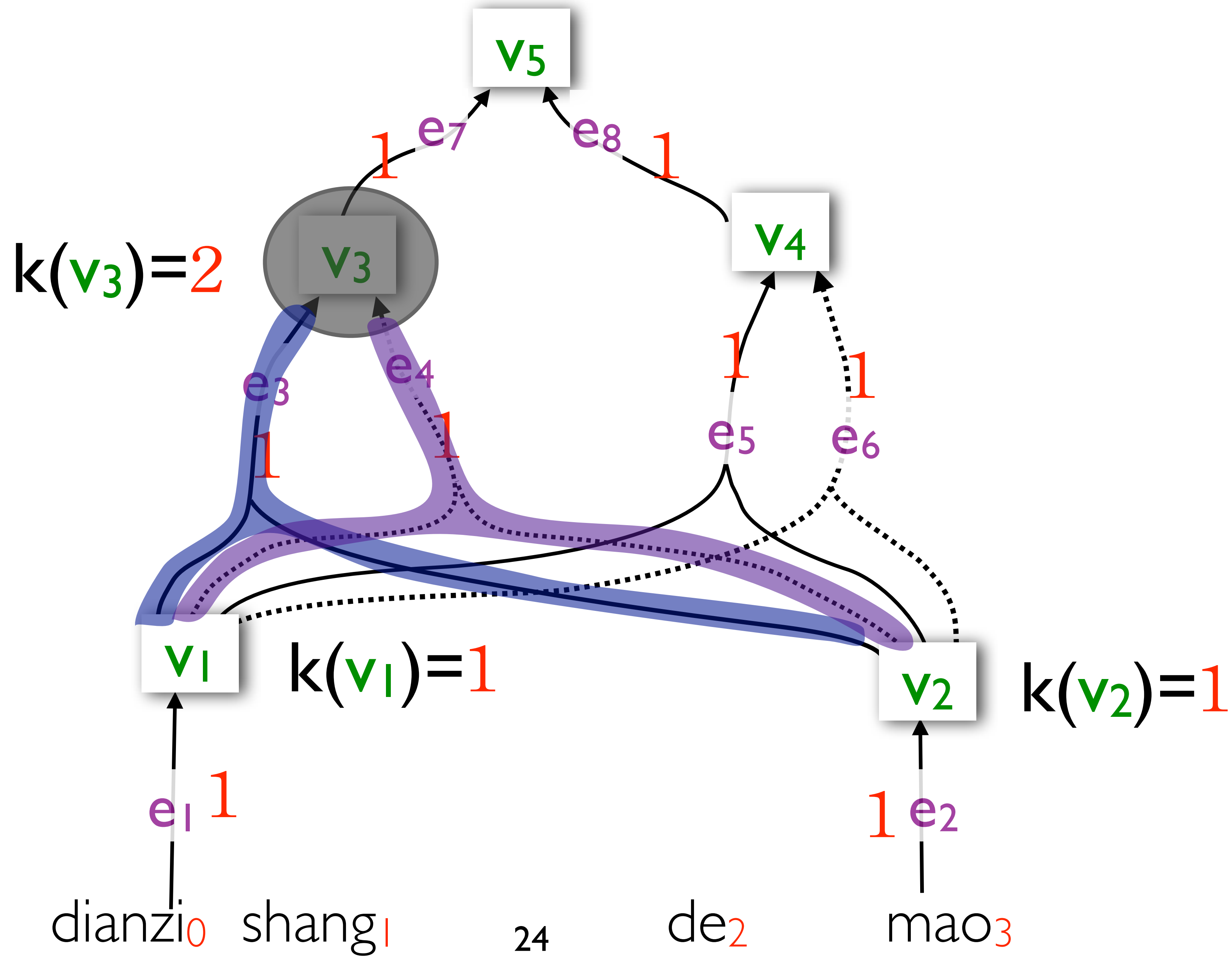


Compute $k(v_3)$: the weight at node v_3

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

$$1 \oplus 1 = 2$$

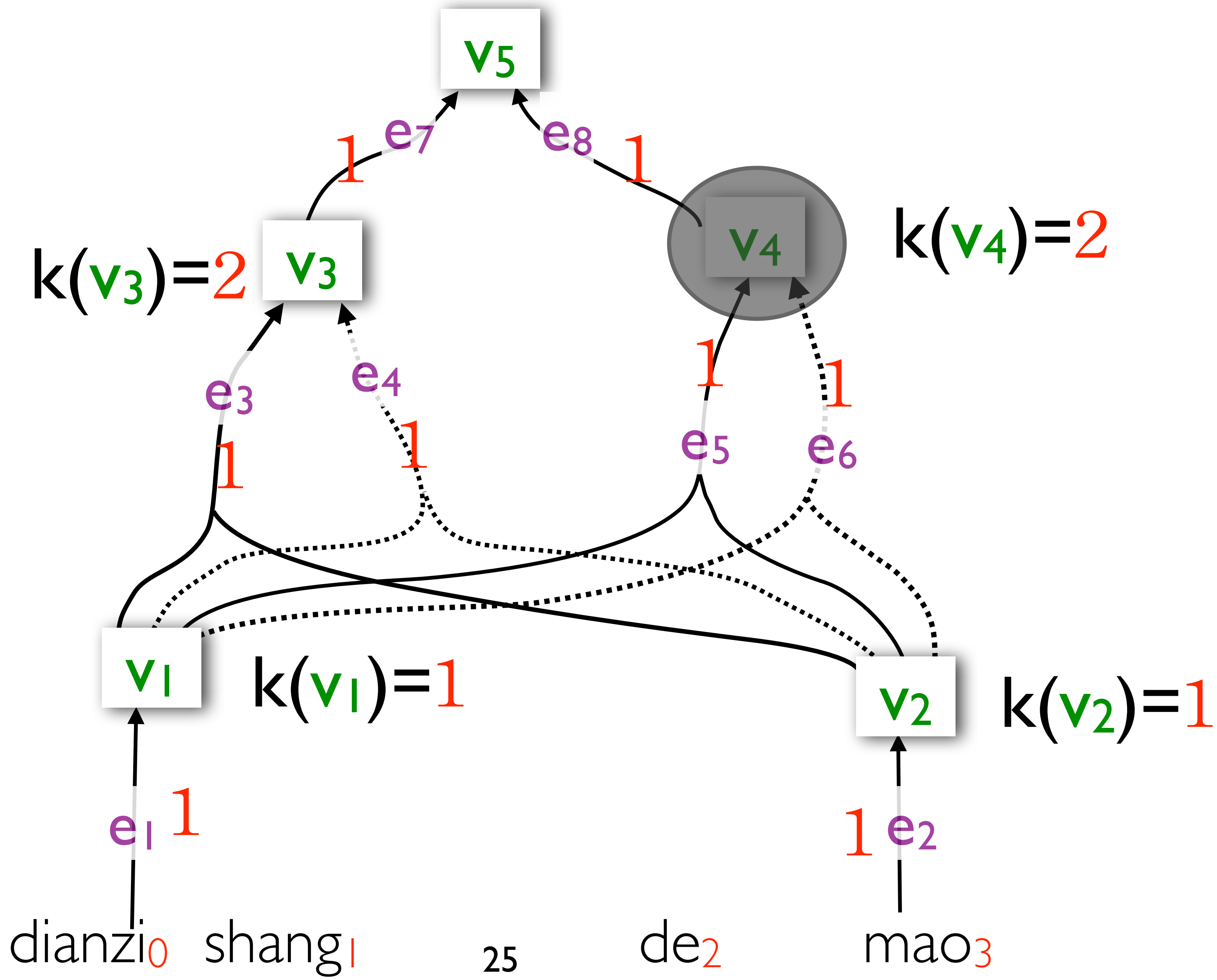
Bottom-up process in computing the number of trees



$k(v_1) = k(e_1)$ $\quad$ $k(v_2) = k(e_2)$

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$$



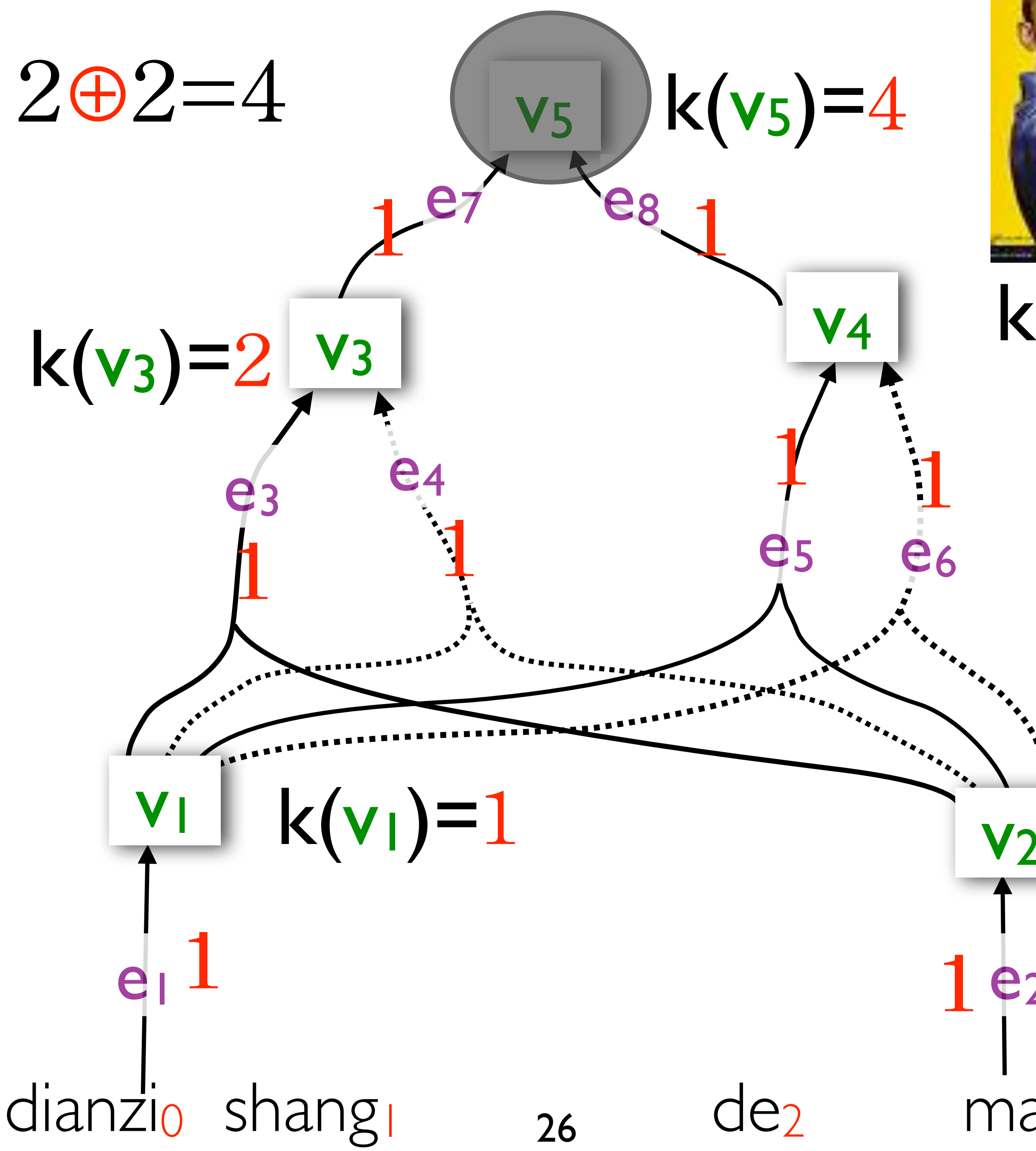
Bottom-up process in computing the number of trees

$k(v_1) = k(e_1)$ $\quad$ $k(v_2) = k(e_2)$

$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$

$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$

$k(v_5) = k(e_7) \otimes k(v_3) \oplus k(e_8) \otimes k(v_4)$



Bottom-up

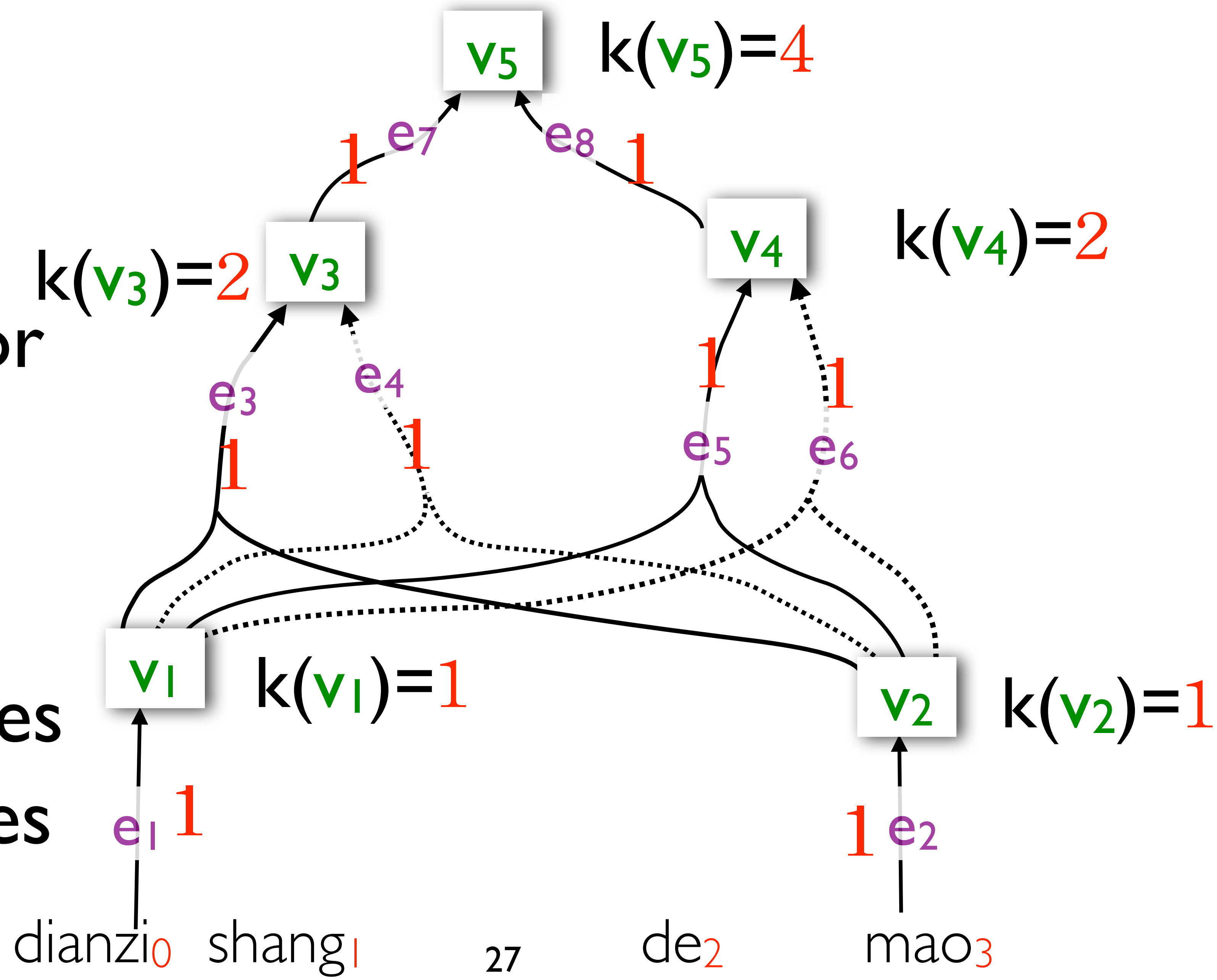
process in computing the number of trees

$k(v_1) = k(e_1)$ $\quad$ $k(v_2) = k(e_2)$

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_5) = k(e_7) \otimes k(v_3) \oplus k(e_8) \otimes k(v_4)$$



Summary:

- **input**: a weight for each edge
- **output**: a weight for each node
- $\oplus$ is used at nodes
- $\otimes$ is used at edges

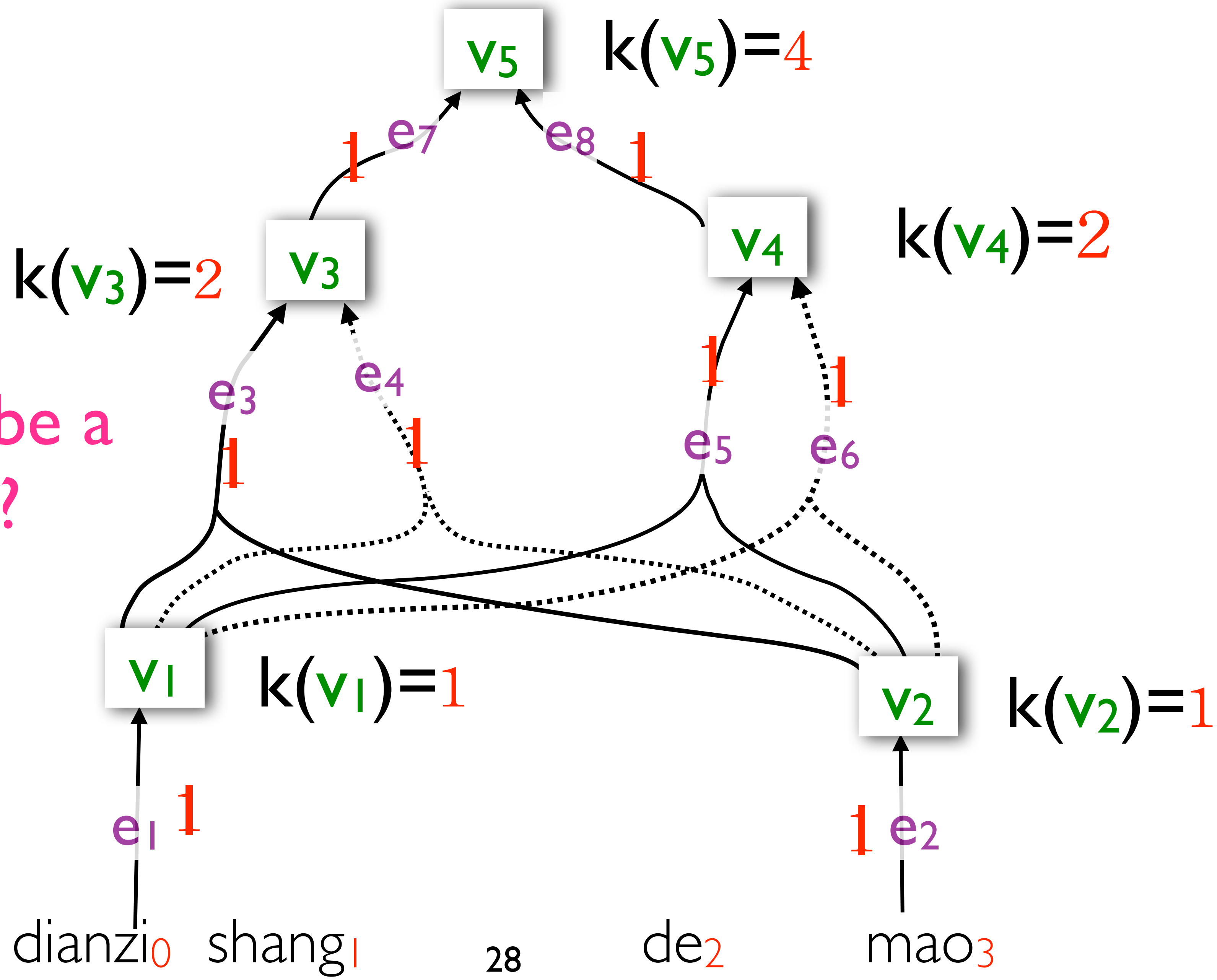
$k(v_1) = k(e_1)$ $\qquad$ $k(v_2) = k(e_2)$

$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$

$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$

$k(v_5) = k(e_7) \otimes k(v_3) \oplus k(e_8) \otimes k(v_4)$

Does it have to be a single integer?



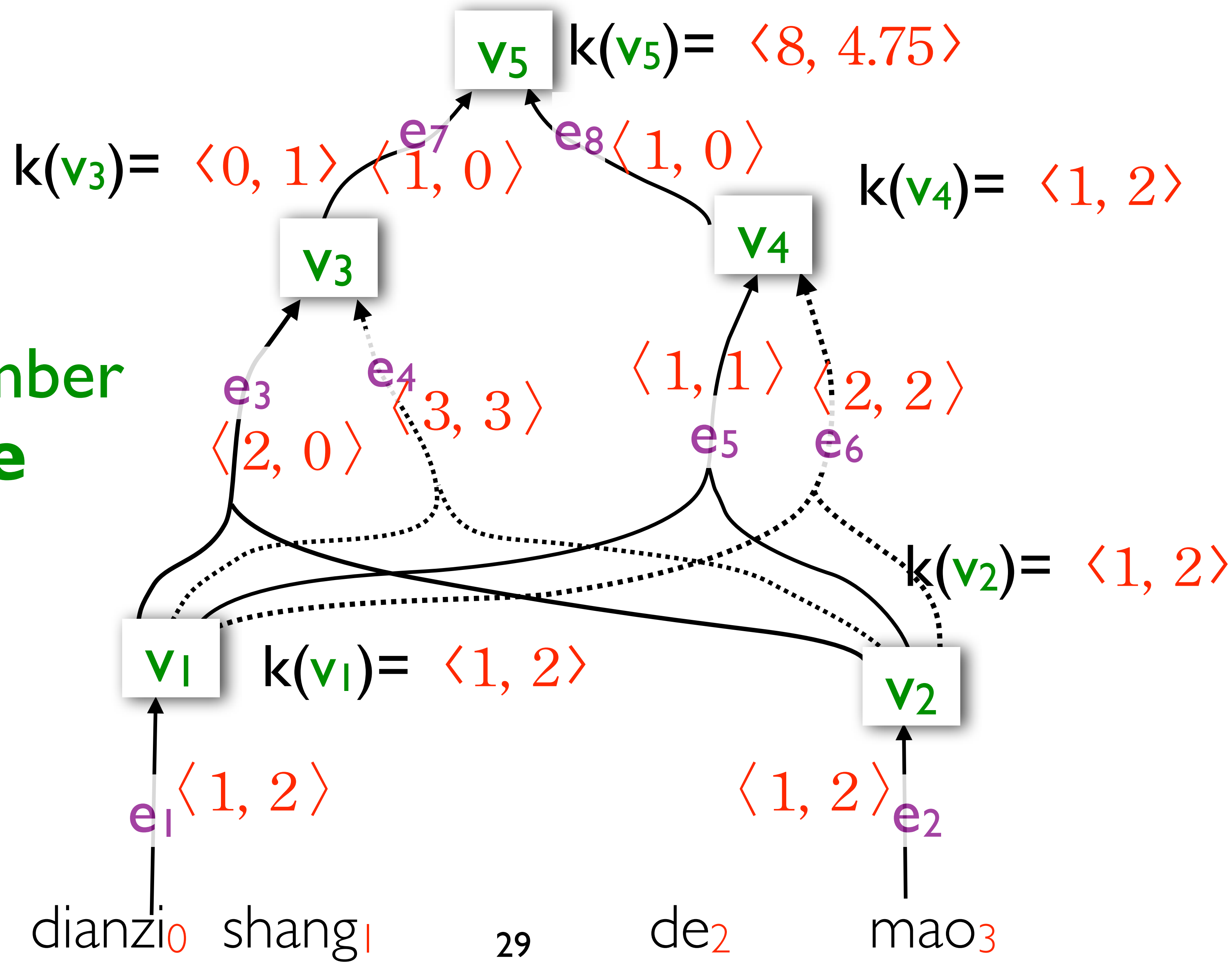
$$k(v_1) = k(e_1) \qquad k(v_2) = k(e_2)$$

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_5) = k(e_7) \otimes k(v_3) \oplus k(e_8) \otimes k(v_4)$$

A semiring member is a **2-tuple**

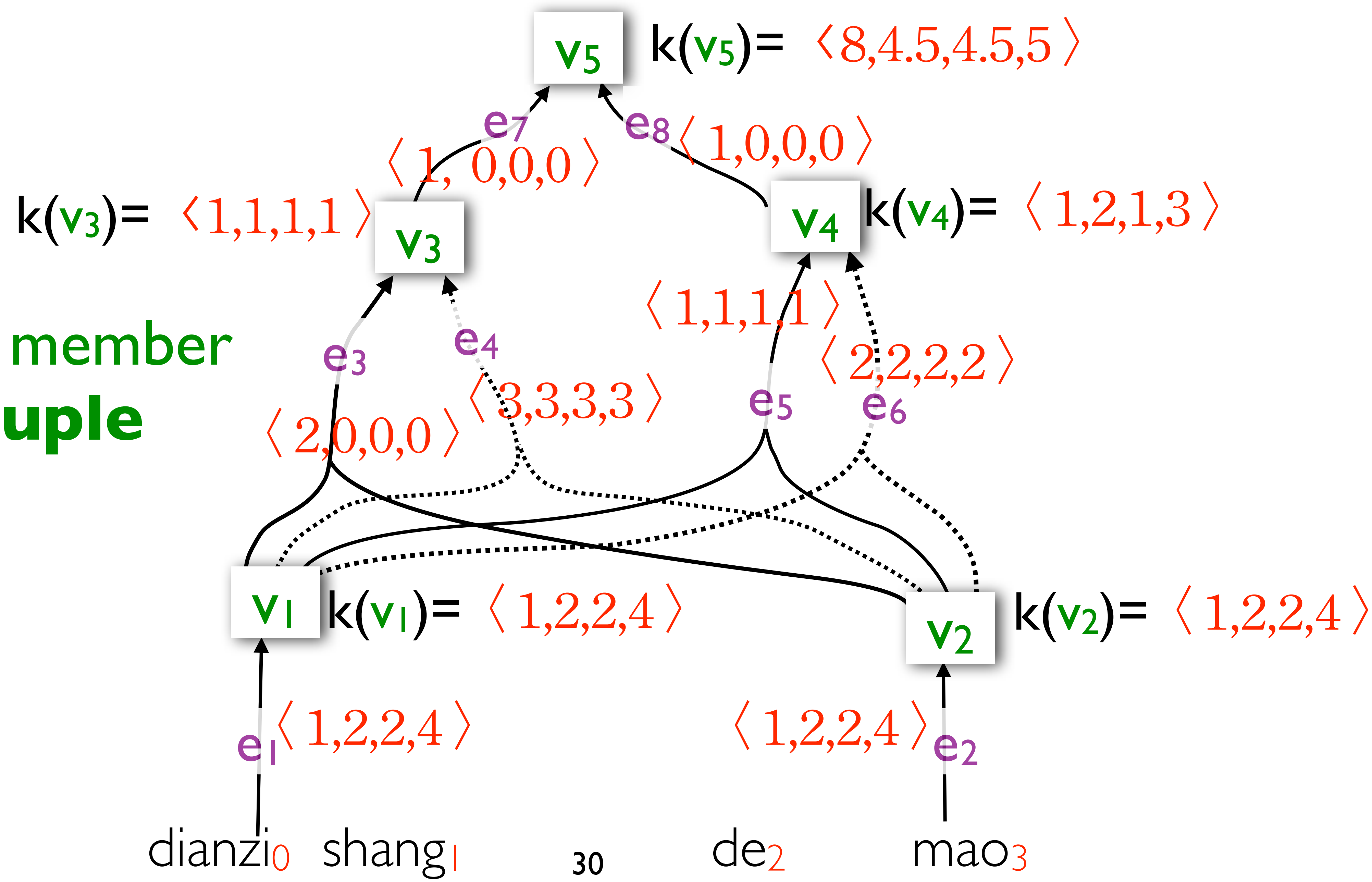


$$k(v_1) = k(e_1) \qquad\qquad k(v_2) = k(e_2)$$

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_5) = k(e_7) \otimes k(v_3) \oplus k(e_8) \otimes k(v_4)$$



A semiring member is a 4-tuple

Why do we want to work on **tuples**?

expectation semiring

second-order expectation semiring

useful for
parameter estimation
at **training** time

Goodman (1999) defines many other semirings useful at "**testing**" time

First- and Second-order Expectation Semirings

First-order:

(Eisner, 2002)

- each member is a 2-tuple: $\langle p, r \rangle$

$\langle p_1, r_1 \rangle \otimes \langle p_2, r_2 \rangle$	$\langle p_1 p_2,\ p_1 r_2 + p_2 r_1 \rangle$
$\langle p_1, r_1 \rangle \oplus \langle p_2, r_2 \rangle$	$\langle p_1 + p_2,\ r_1 + r_2 \rangle$

Second-order:

- each member is a 4-tuple: $\langle p, r, s, t \rangle$

$\langle p_1, r_1, s_1, t_1 \rangle \otimes \langle p_2, r_2, s_2, t_2 \rangle$	$\langle p_1 p_2,\ p_1 r_2 + p_2 r_1,\ p_1 s_2 + p_2 s_1,$ $p_1 t_2 + p_2 t_1 + r_1 s_2 + r_2 s_1 \rangle$
$\langle p_1, r_1, s_1, t_1 \rangle \oplus \langle p_2, r_2, s_2, t_2 \rangle$	$\langle p_1 + p_2,\ r_1 + r_2,\ s_1 + s_2,\ t_1 + t_2 \rangle$

First- and Second-order Expectation Semirings

First-order:

- each member is a 2-tuple: $\langle p, r \rangle$

$\langle p_1, r_1 \rangle \otimes \langle p_2, r_2 \rangle$	$\langle p_1 p_2,\ p_1 r_2 + p_2 r_1 \rangle$
$\langle p_1, r_1 \rangle \oplus \langle p_2, r_2 \rangle$	$\langle p_1 + p_2,\ r_1 + r_2 \rangle$

What does p, r, s, or t mean?

Second-order:

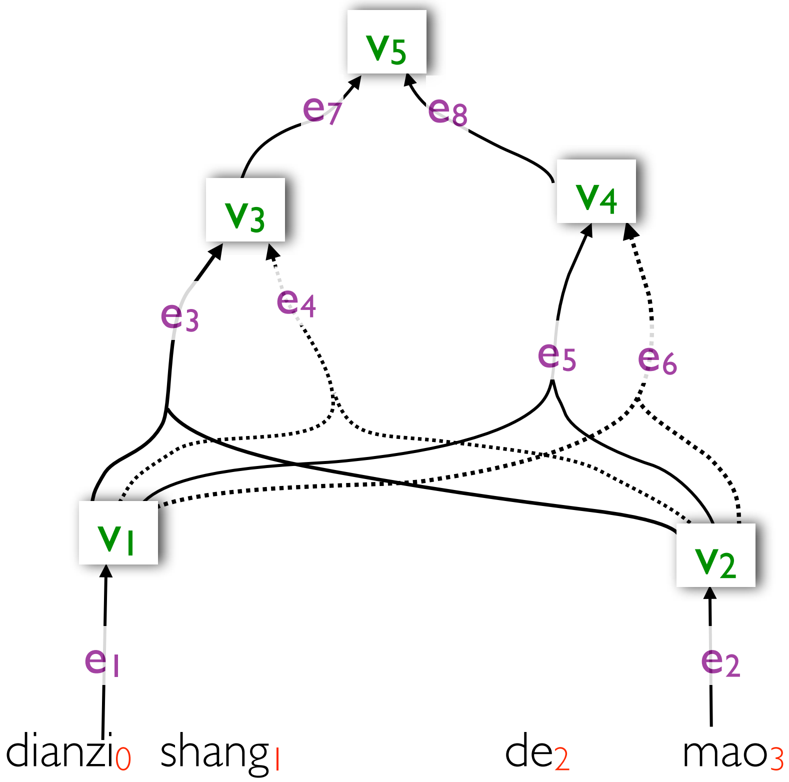
- ea

application-dependent

$\langle p_1, r_1, s_1, t_1 \rangle \otimes \langle p_2, r_2, s_2, t_2 \rangle$	$\langle p_1 p_2,\ p_1 r_2 + p_2 r_1,\ p_1 s_2 + p_2 s_1,$ $p_1 t_2 + p_2 t_1 + r_1 s_2 + r_2 s_1 \rangle$
$\langle p_1, r_1, s_1, t_1 \rangle \oplus \langle p_2, r_2, s_2, t_2 \rangle$	$\langle p_1 + p_2,\ r_1 + r_2,\ s_1 + s_2,\ t_1 + t_2 \rangle$

Applications

Compute Quantities on a Hypergraph: a Recipe



Three steps:

- choose a semiring

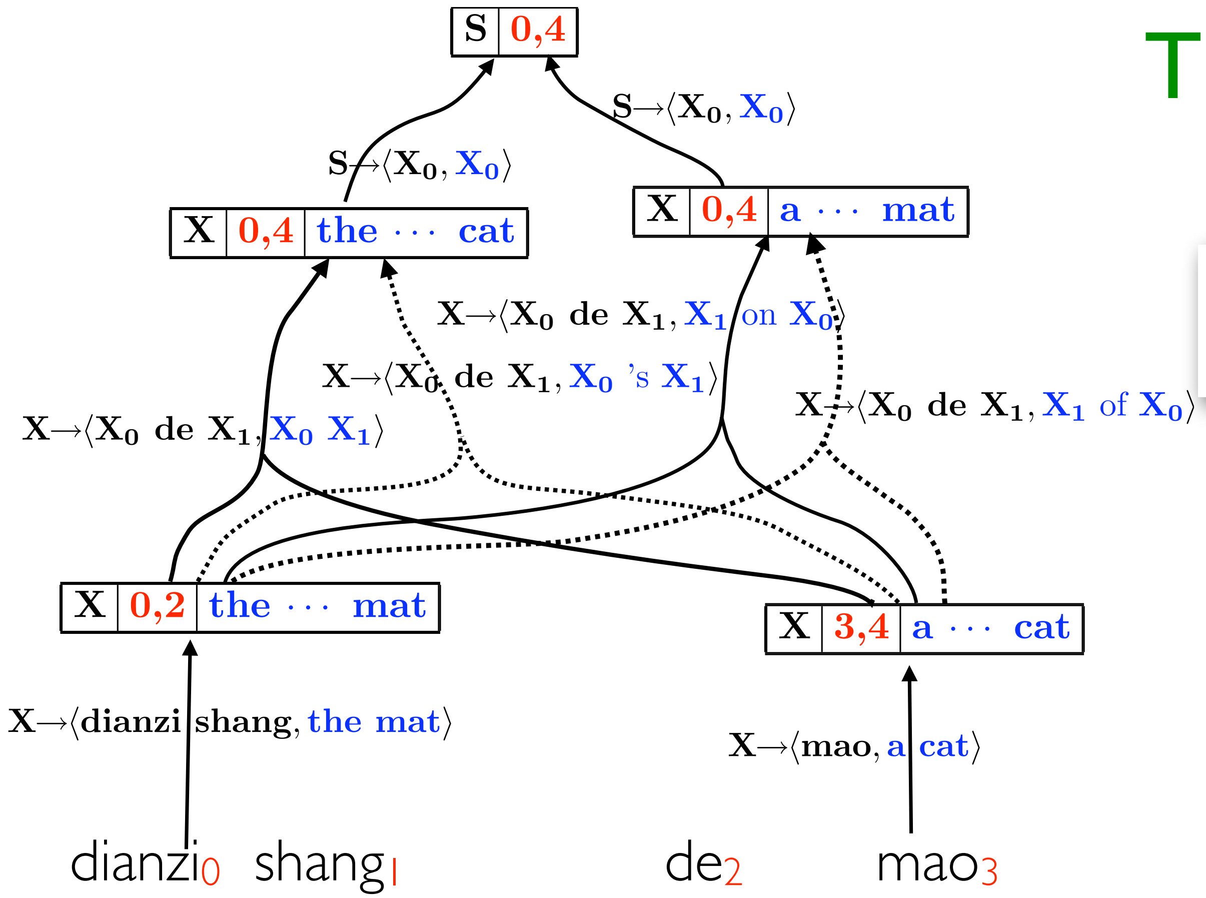
  first- or second-order expectation semiring

- specify a weight for each edge

- run the inside algorithm

Compute First-order Expectations

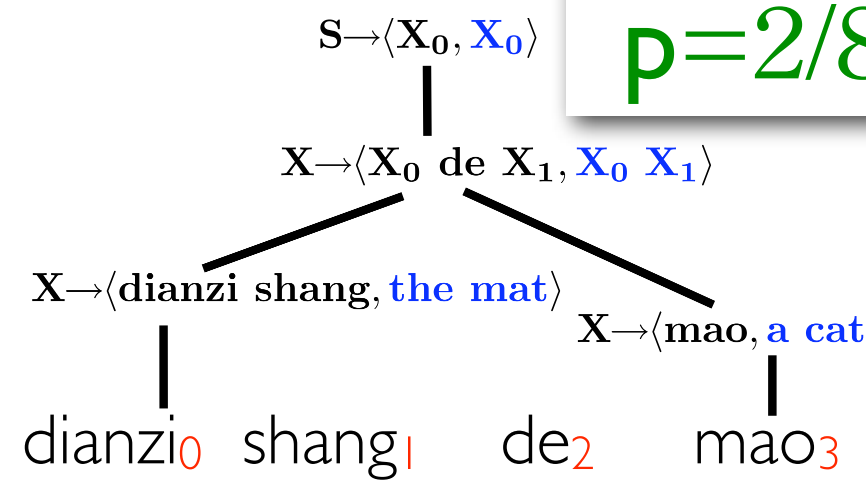
The hypergraph defines a probability distribution over trees!



expected translation length?

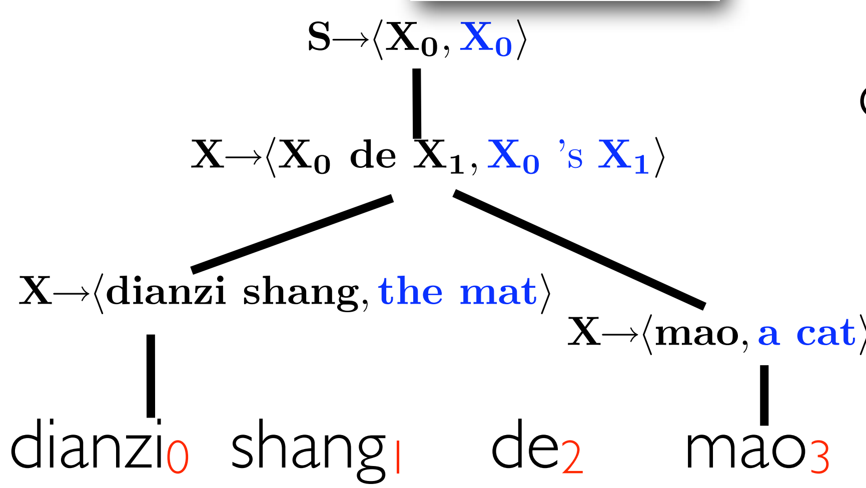
$4*2/8+5*6/8=4.75$

p=2/8



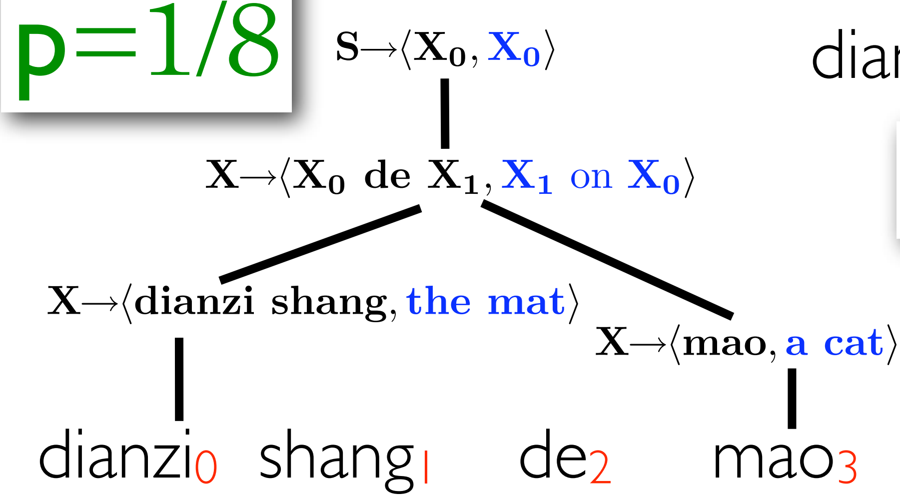
the mat a cat 4

p=3/8



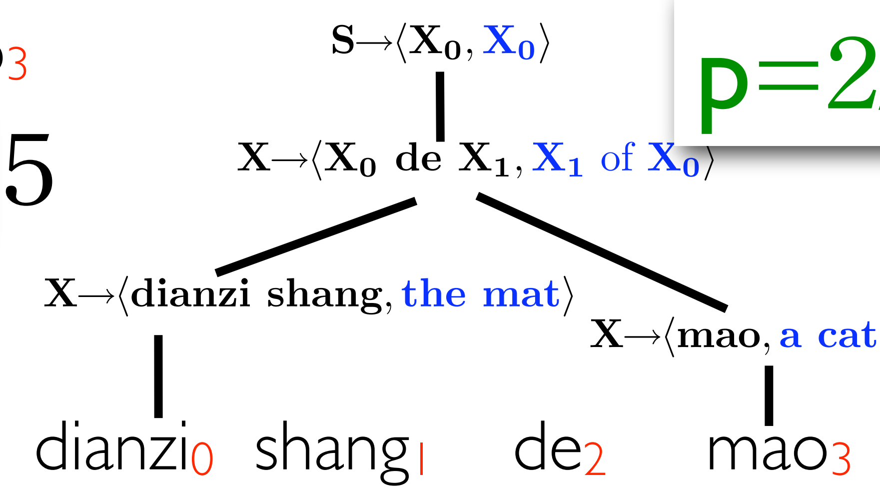
the mat 's a cat 5

p=1/8



a cat on the mat 5

p=2/8



a cat of the mat 5

Compute the expected translation length

using an expectation semiring

Compute Expected Translation Length

- choose a semiring

  expectation semiring

- specify a weight for each edge

$$k_e \stackrel{\text{def}}{=} \langle p_e, p_e r_e \rangle$$

p_e: transition probability or log-linear score at edge e

r_e: number of English words generated at edge e

- run the inside algorithm

Expectations on Hypergraphs

- Expectation over a hypergraph

$$\bar{r} \stackrel{\text{def}}{=} \mathbb{E}_p[r] = \sum_{d \in \text{HG}} p(d)r(d)$$

  - the distribution *p*(*d*) is defined by the hypergraph
  - *r*(*d*) is a function over a derivation *d*

    e.g., the length of the translation yielded by *d*

  - *r*(*d*) is additively decomposed

$$r(d) \stackrel{\text{def}}{=} \sum_{e \in d} r_e$$

e.g., translation length is additively decomposed!

Compute expectation using an expectation semiring:

$$k_e \stackrel{\text{def}}{=} \langle p_e, p_e r_e \rangle$$

p_e : transition probability or log-linear score at edge e

r_e ?

Entropy: $r_e \stackrel{\text{def}}{=} \log p_e$

Cross-entropy: $r_e \stackrel{\text{def}}{=} \log q_e$

Bayes risk: $r_e \stackrel{\text{def}}{=} \text{loss at edge } e$ (Tromble et al. 2008)

Why?

entropy is an **expectation**

$$\mathrm{H}(p) = \mathbb{E}_p[-\log p] = -\sum_{d \in \mathrm{HG}} p(d) \log p(d)$$

log *p*(*d*) is additively decomposed!

Compute expectation using an expectation semiring:

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Bayes risk: $r_e \stackrel{\text{def}}{=} \text{loss at edge } e$ (Tromble et al. 2008)

Why?

cross-entropy is an **expectation**

$$\mathrm{H}(p, q) = \mathbb{E}_p(-\log q) = -\sum_{d \in \mathrm{HG}} p(d) \log q(d)$$

log $q(d)$ is additively decomposed!

Compute expectation using an expectation semiring:

$$k_e \stackrel{\text{def}}{=} \langle p_e, p_e r_e \rangle$$

p_e : transition probability or log-linear score at edge e

r_e ?

Entropy:

$$r_e \stackrel{\text{def}}{=} \log p_e$$

Cross-entropy:

$$r_e \stackrel{\text{def}}{=} \log q_e$$

Bayes risk:

$$r_e \stackrel{\text{def}}{=} \text{loss at edge } e$$

(Tromble et al. 2008)

Why?

Bayes risk is an **expectation**

$$\mathrm{H}(p,q) = \mathbb{E}_p(L) = -\sum_{d \in \mathrm{HG}} p(d) \cdot L(Y(d))$$

L(*Y*(*d*)) is additively decomposed!

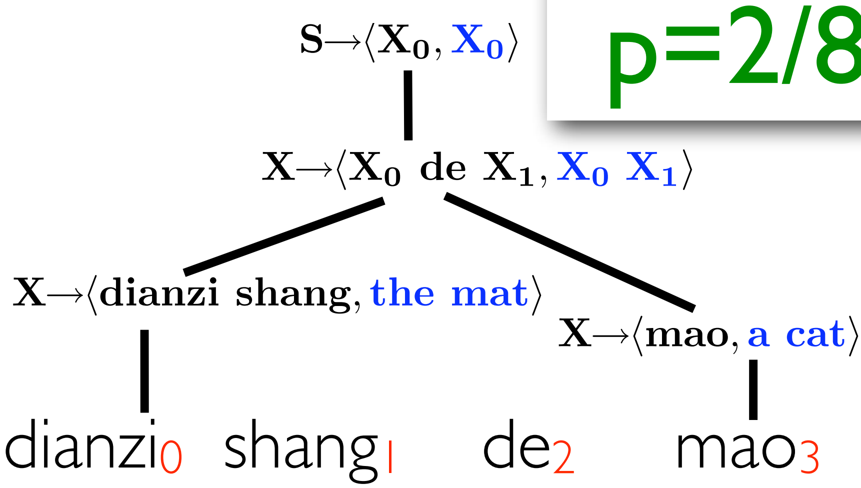
Compute Expectations over Products

expected translation length: 4.75

variance of translation length?

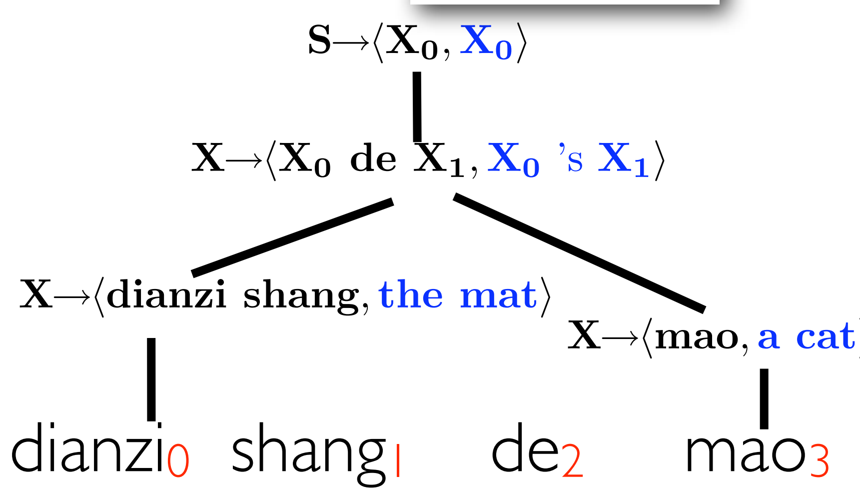
$2/8 \times (4-4.75)^2 + 6/8 \times (5-5.75)^2 \approx 0.56$

p=2/8



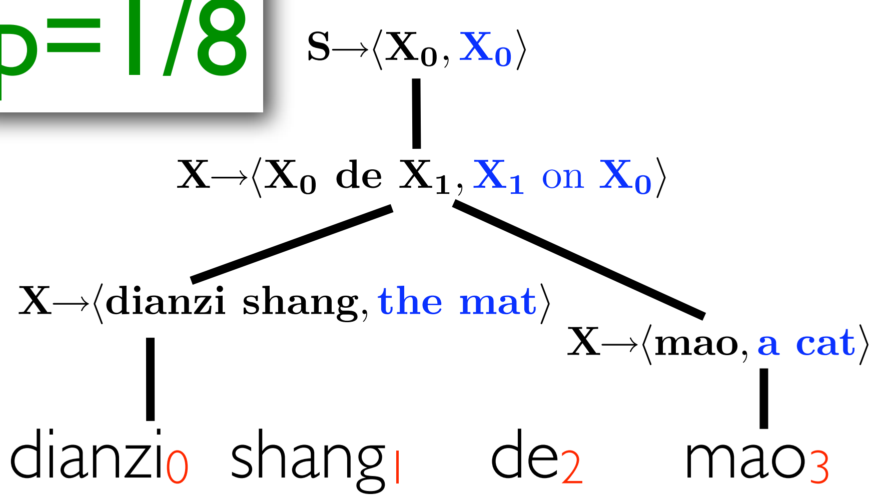
the mat a cat

p=3/8



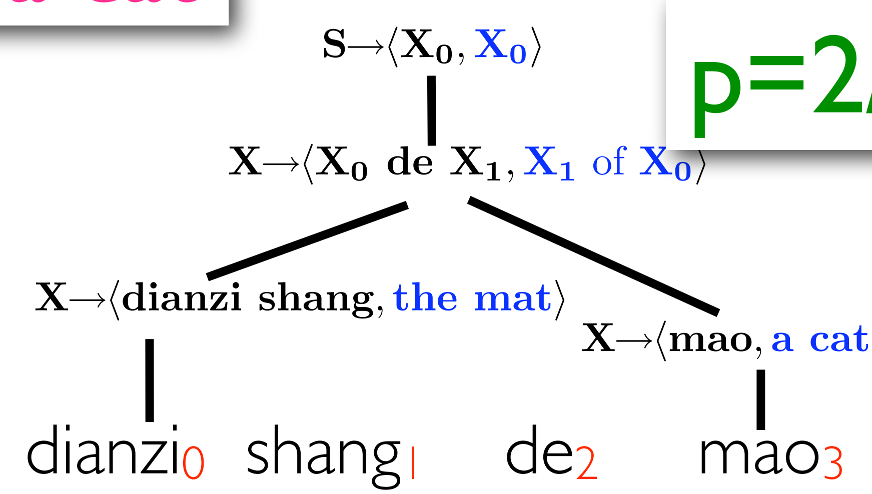
the mat 's a cat

p=1/8



a cat on the mat

p=2/8



a cat of the mat

Compute the Variance in Translation Length

- choose a semiring

  second-order expectation semiring

- specify a weight for each edge

$$k_e \stackrel{\text{def}}{=} \langle p_e, p_e r_e, p_e s_e, p_e r_e s_e \rangle$$

p_e: transition probability or log-linear score at edge e

$r_e = s_e$: number of English words generated at edge e

- run the inside algorithm

Second-order Expectations on Hypergraphs

- Expectation of **products** over a hypergraph

$$\bar{t} \stackrel{\mathrm{def}}{=} \mathbb{E}_p[r \cdot s] = \sum_{d \in \mathrm{HG}} p(d) r(d) s(d)$$

- r and s are additively decomposed

$$r(d) \stackrel{\mathrm{def}}{=} \sum_{e \in d} r_e$$

$$s(d) \stackrel{\mathrm{def}}{=} \sum_{e \in d} s_e$$

r and s can be identical or different functions.

Applications of expectation semirings: a summary

First-order:

Quantity	Weight for edge e	Value at root
Expectation	$\langle p_e, p_e r_e \rangle$	$\langle Z, \overline{r} \rangle$
First-order gradient	$\langle p_e, \nabla p_e \rangle$	$\langle Z, \nabla Z \rangle$

Second-order:

Covariance matrix	$\langle p_e, p_e r_e, p_e s_e, p_e r_e s_e \rangle$	$\langle Z, \overline{r}, \overline{s}, \overline{t} \rangle$
Hessian matrix	$\langle p_e, \nabla p_e, \nabla p_e, \nabla^2 p_e \rangle$	$\langle Z, \nabla Z, \nabla Z, \nabla^2 Z \rangle$
Gradient of expectation	$\langle p_e, p_e r_e, \nabla p_e, (\nabla p_e) r_e + p_e (\nabla r_e) \rangle$	$\langle Z, \overline{r}, \nabla Z, \nabla \overline{r} \rangle$

$$p_e = \exp(\Phi_e \cdot \theta) \qquad \nabla p_e = p_e \Phi_e$$

- choose a semiring
- define a weight for each edge
- run inside algorithm (or inside-outside for speedup)

Applications of expectation semirings: a summary

First-order:

Quantity	Weight for edge e	Value at root
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Second-order:

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Entropy is an expectation!

- choose a semiring
- define a weight for each edge
- run inside algorithm (or inside-outside for speedup)

Applications of expectation semirings: a summary

First-order:

Quantity	Weight for edge e	Value at root
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Second-order:

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Bayes risk is an expectation!

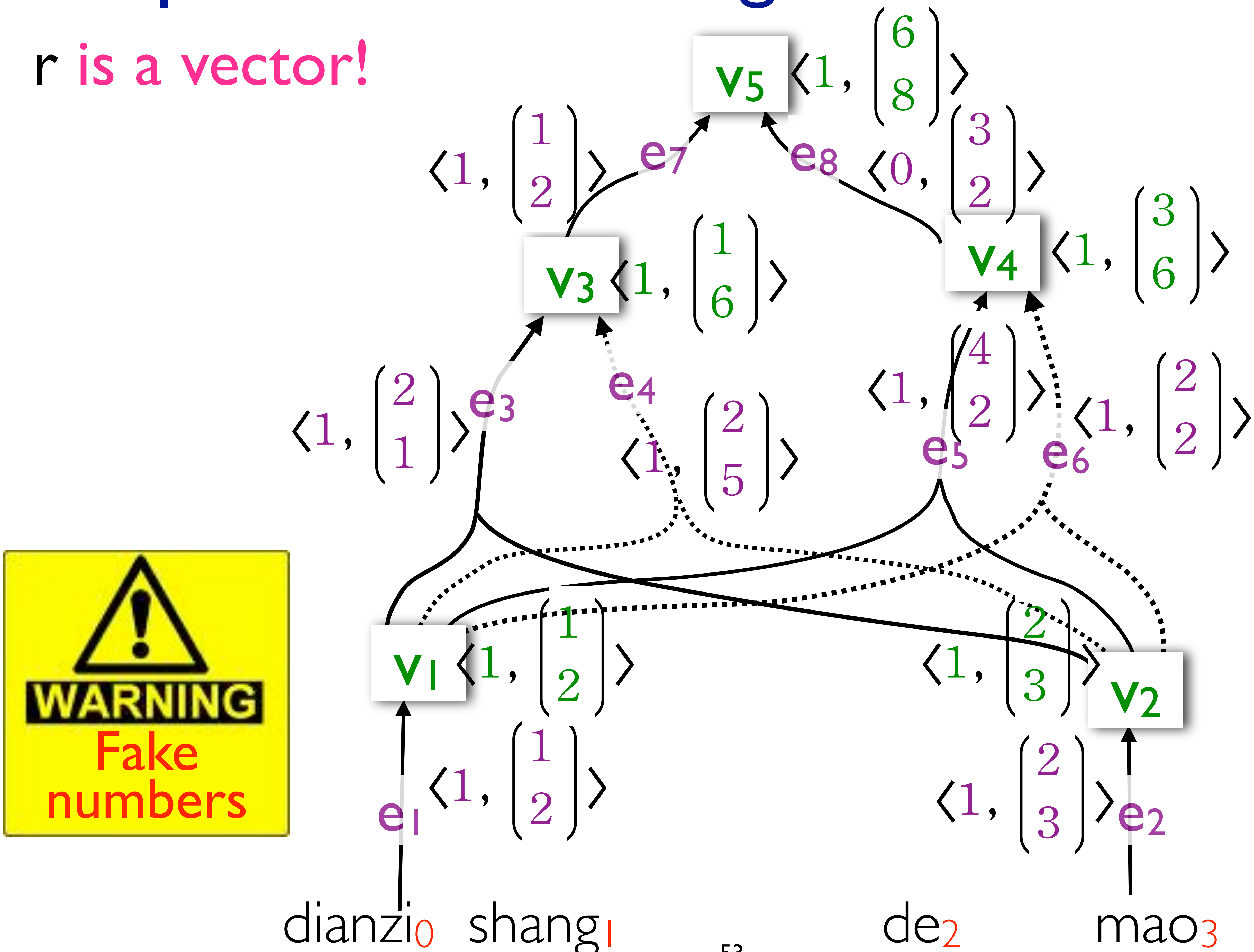
- choose a semiring
- define a weight for each edge
- run inside algorithm (or inside-outside for speedup)

Inside-Outside Speedup

Why is it slow?

Expectation Semirings with Vectors

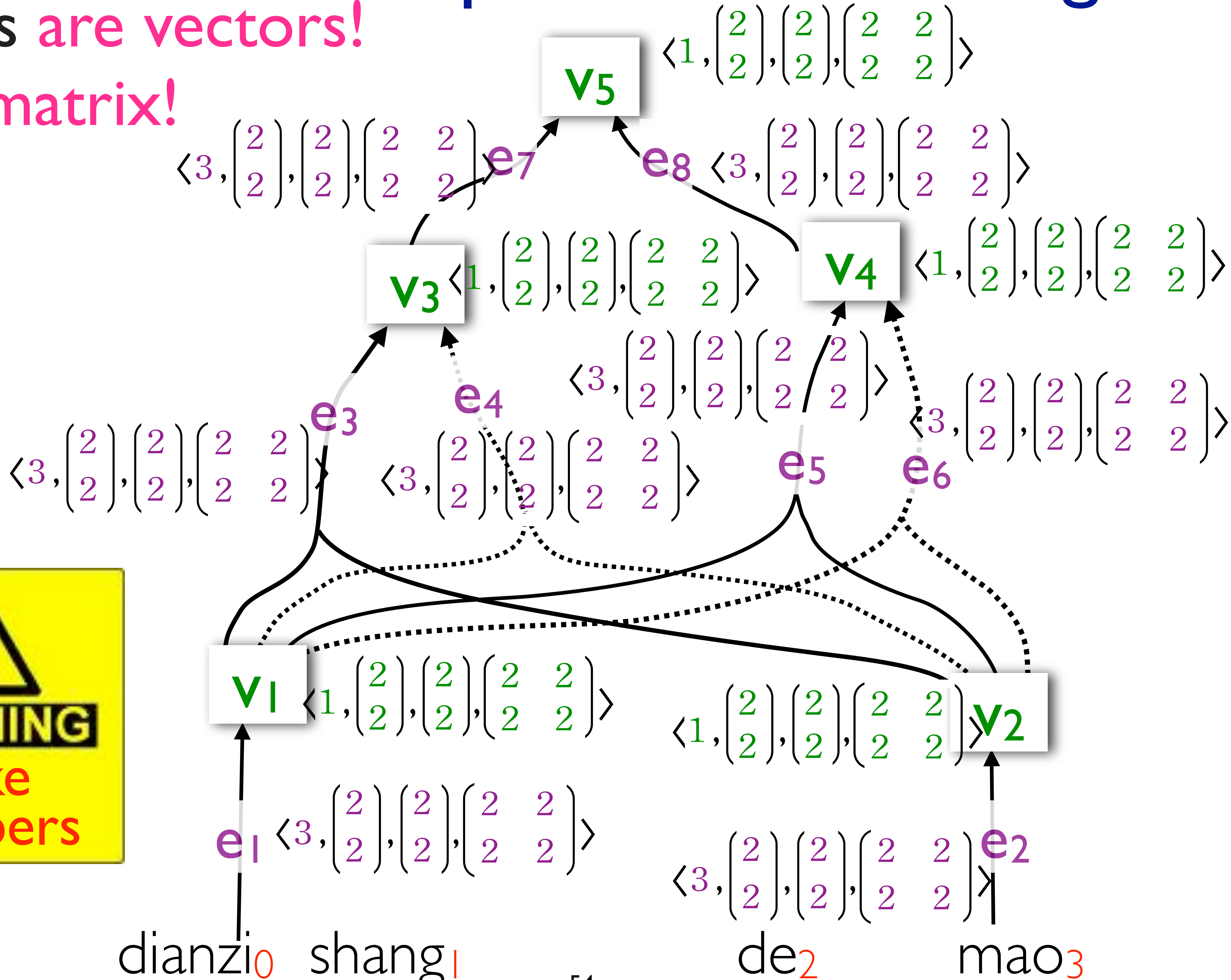
r is a vector!



Second-order Expectation Semirings

r and s are vectors!

t is a matrix!



Inside-Outside Speedup

See the paper for details!

Minimum Risk Training over Translation Forests

Minimum Risk Training

Finding optimal parameters:

$$\theta^* = \mathit{argmin}\ \text{Risk}(P_\theta) - \text{Temperature} \times \text{Entropy}(P_\theta)$$

- P_θ is a probability distribution over trees, parameterized by the parameters θ

Why Minimum Risk Training?

	Min-Risk	MERT	CRF	Perceptron	MIRA
Gradient descent	☺	☹	☺	?	?
BLEU	☺	☺	☹	☹	☺
Latent variable	☺	☹	☺	☹	☹
Oracle translation	☺	☺	☹	☹	☹
Model regularization	☺	☹	☺	☺	☺

Experimental Results

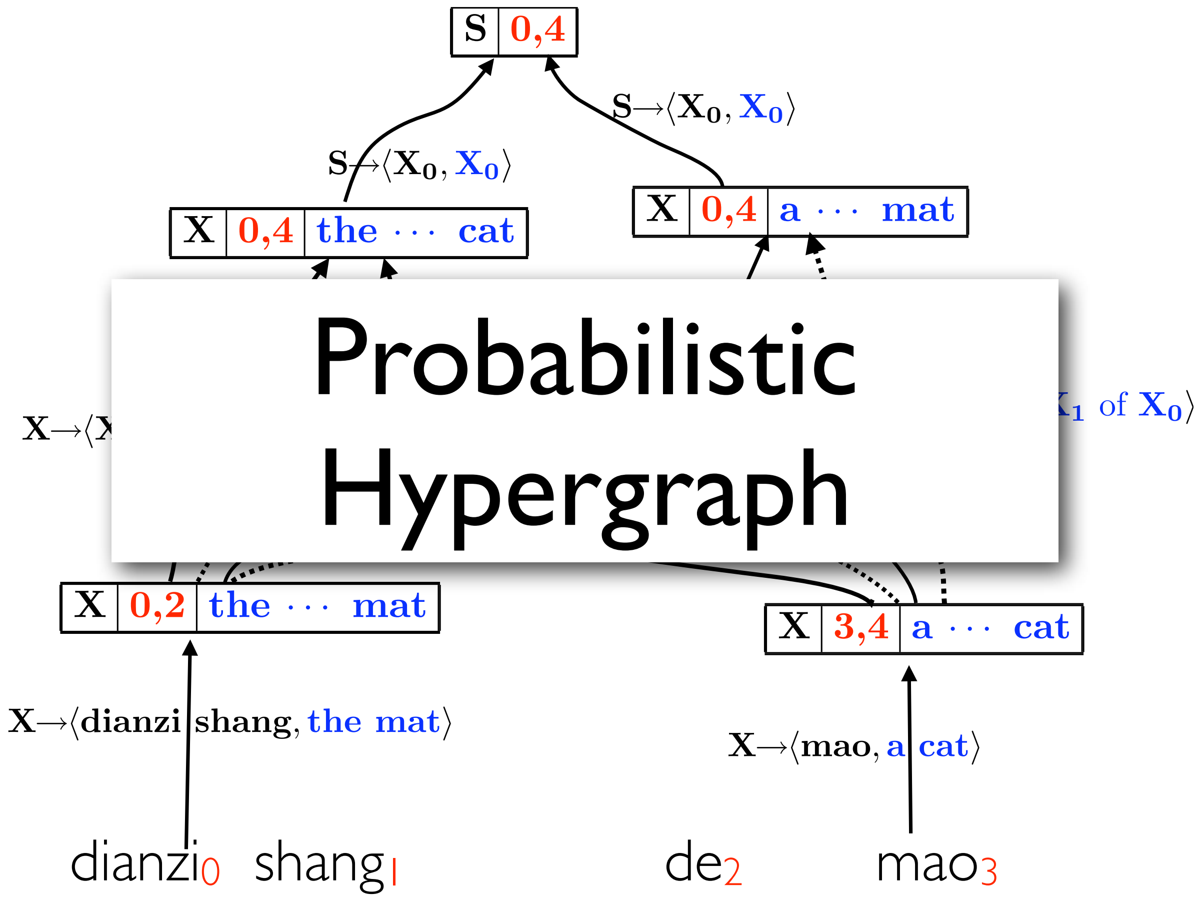
- Data set: IWSLT CN-EN 2005

	Training scheme	test
(Och, 2002)	MERT (Nbest, small)	47.7
(Smith and Eisner, 2006)	MR (Nbest, small)	47.7
NEW!	MR (hypergraph, small)	48.4
NEW!	MR (hypergraph, large)	**48.7**

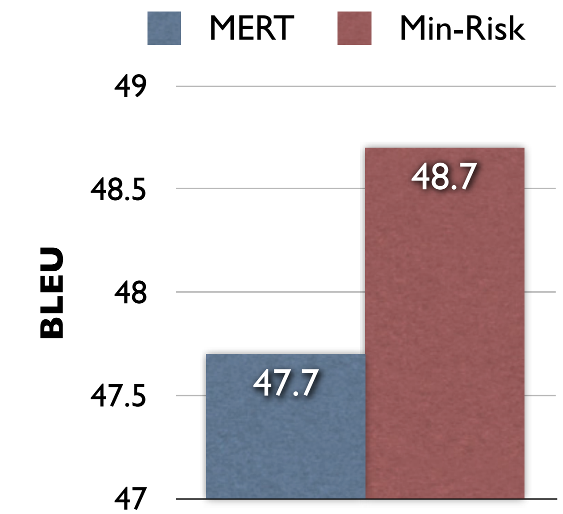
Small: five features as in regular MERT

Large: two-stage forest-reranking with **21k** additional target-side ngram features

Summary



Improved BLEU score!



This work provides a unified, elegant, and efficient framework in computing all of these!

- First-order quantities:
  - expectation
    - entropy
    - cross-entropy
    - KL divergence
    - Bayes risk
    - feature expectations
  - first-order gradient of Z

- Second-order quantities:
  - Covariance matrix
    - feature interaction
  - Hessian matrix of Z
    - second-order gradient descent
  - gradient of expectation
    - gradient of entropy or Bayes risk

Future: machine learning for MT

feature interaction

second-order gradient descent

....

minimum risk

deterministic annealing

active learning

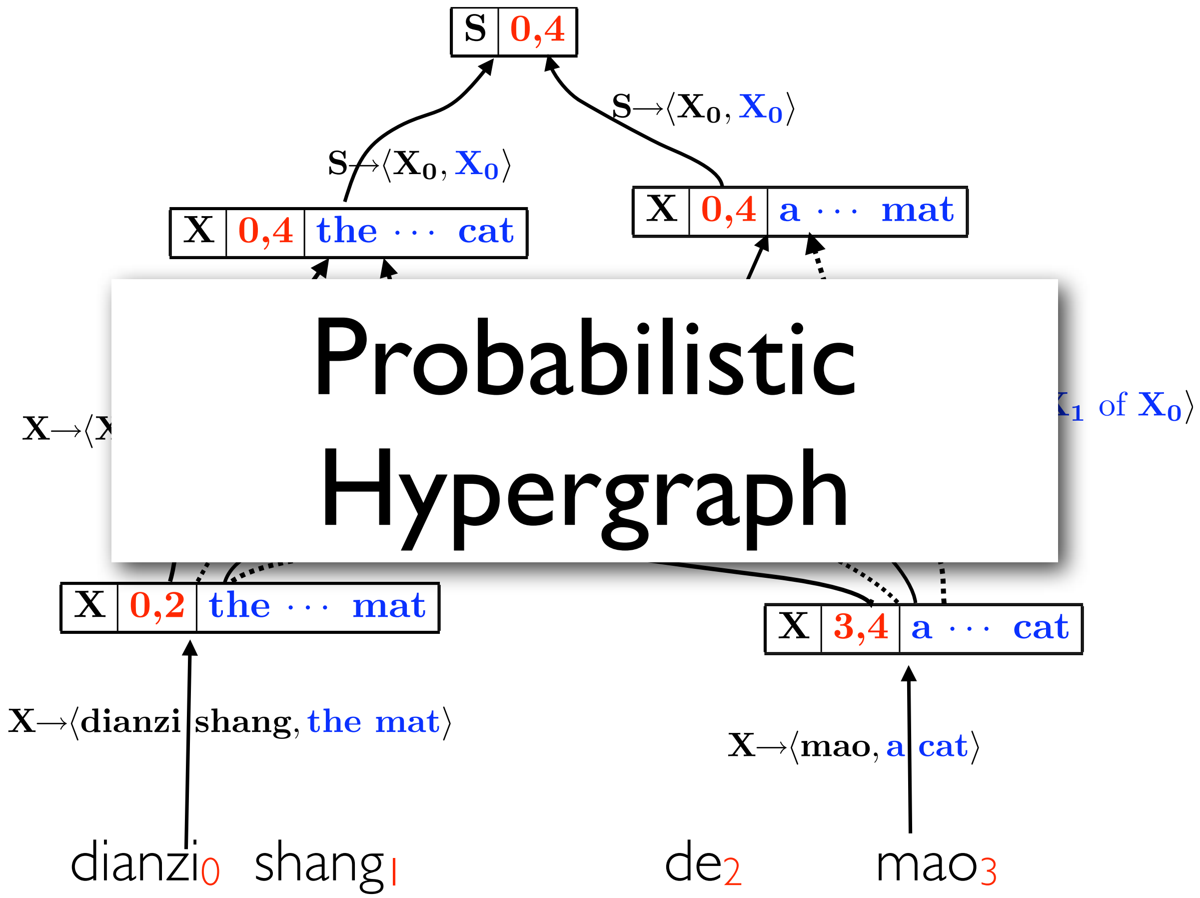
semi-supervised learning

semirings for parameter estimation

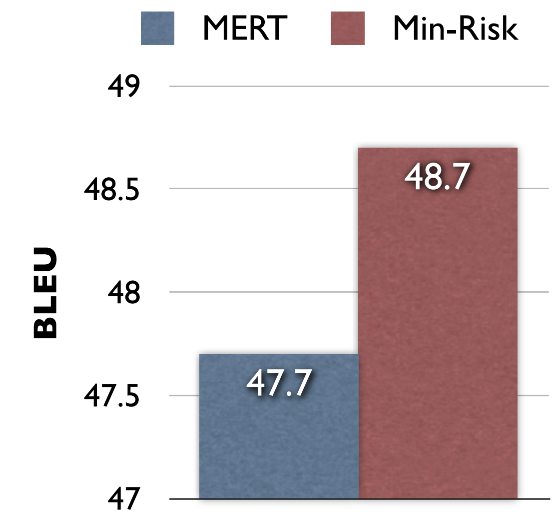

Joshua
semiring

Thank you!

谢谢！



Improved BLEU score!



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