First- and Second-order Expectation Semirings
with Applications to
Minimum-Risk Training on Translation Forests

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Center for Language and Speech Processing

Computer Science Department

Johns Hopkins University

dianzi shang de mao

dianzi shang de mao

the cat on the mat

dianzi shang de mao

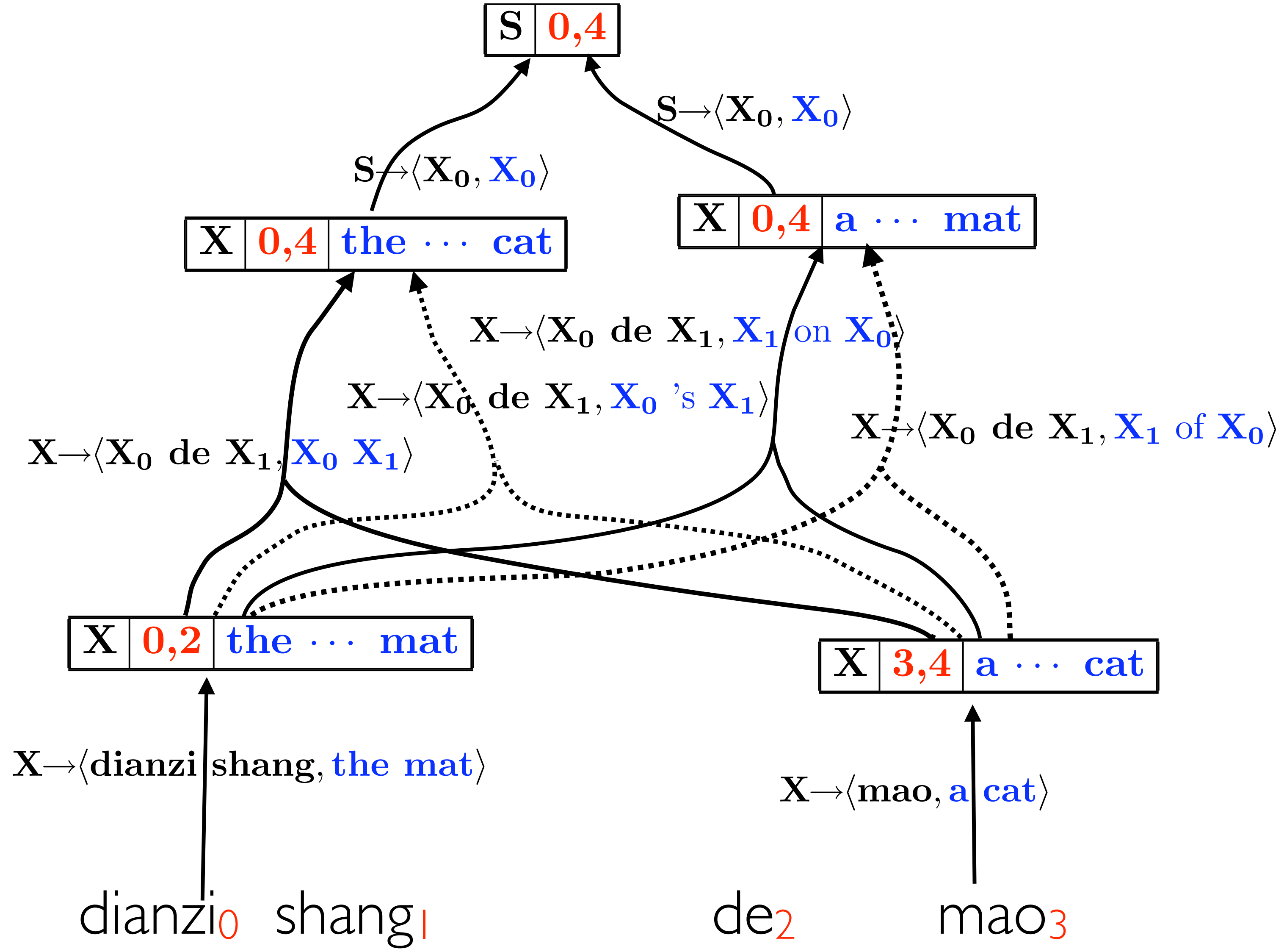
the cat on the mat

dianzi shang de mao

Joshua

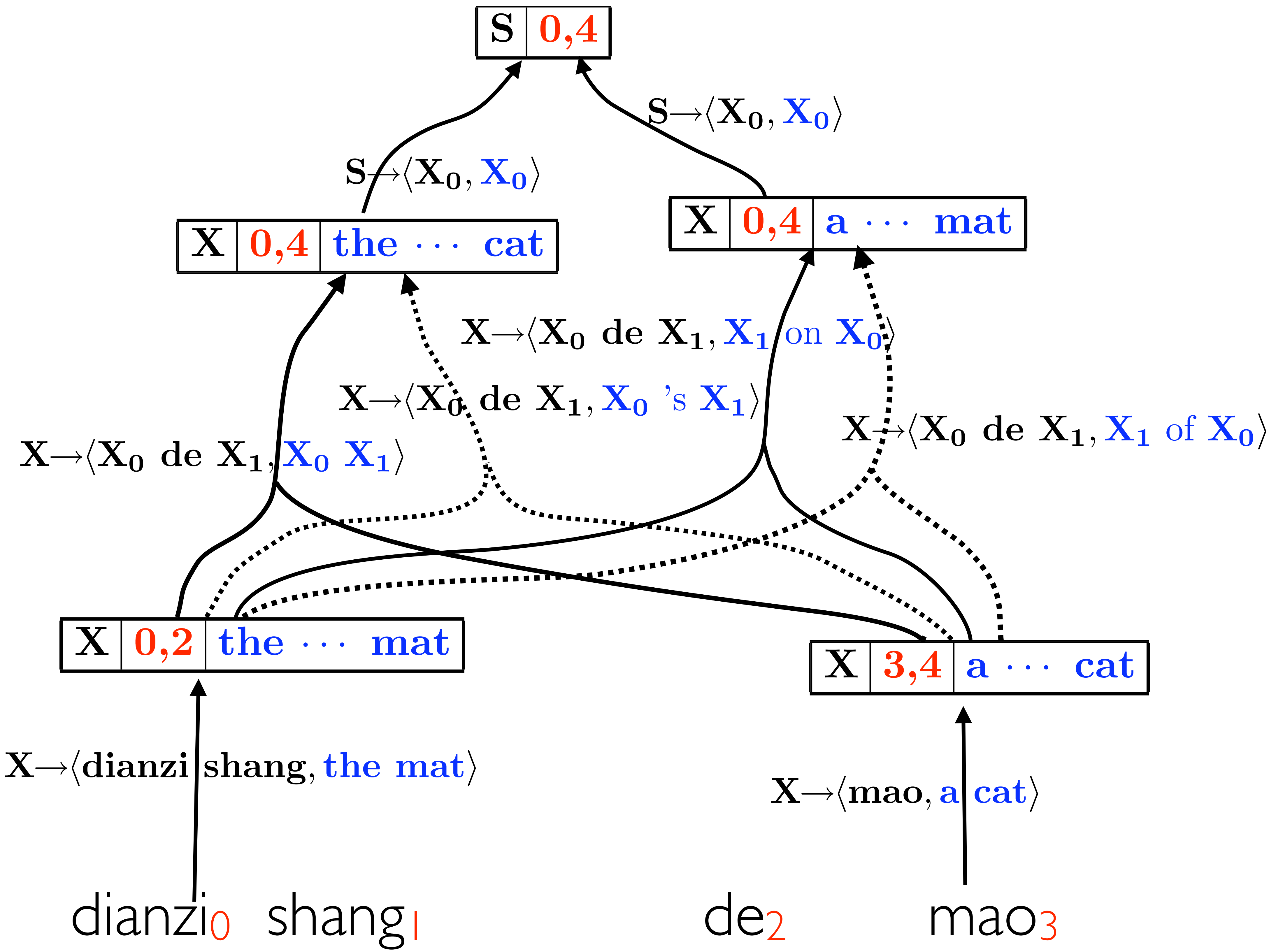
dianzi shang de mao

hypergraph

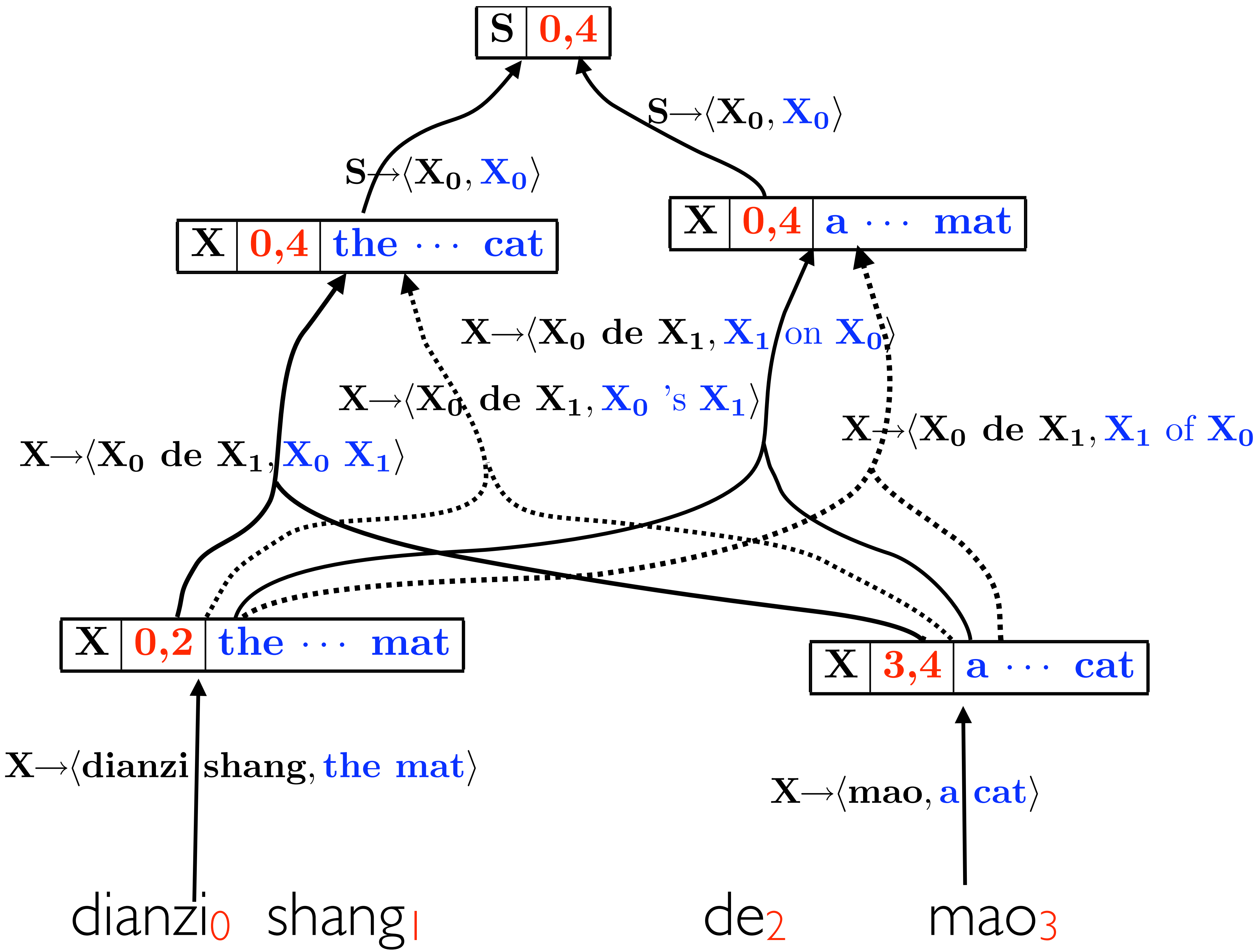


Joshua

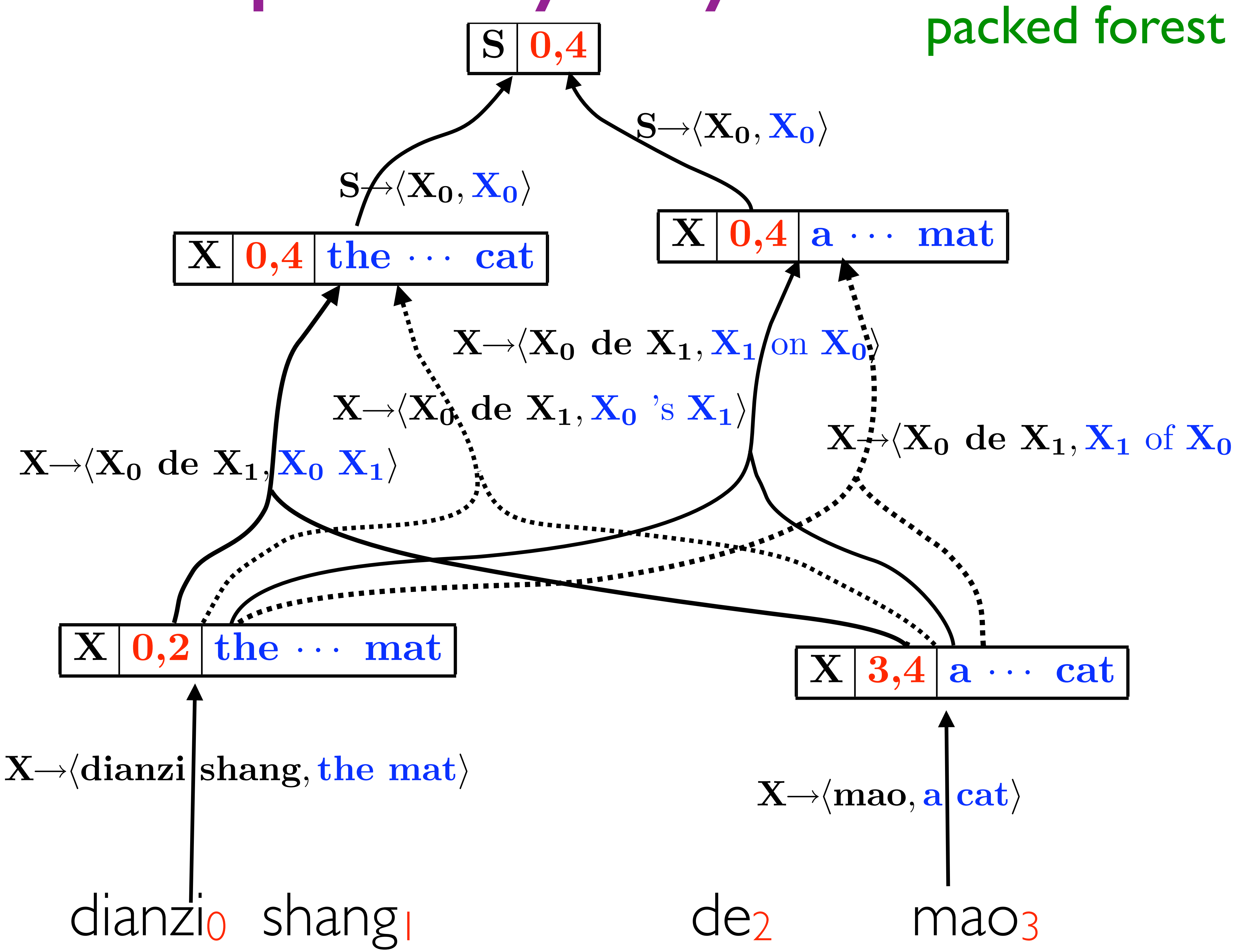
dianzi shang de mao


S | 0,4
S→⟨X0, X0⟩
S→⟨X0, X0⟩
X | 0,4 | the ··· cat
X | 0,4 | a ··· mat
X→⟨X0 de X1, X1 on X0⟩
X→⟨X0 de X1, X0 's X1⟩
X→⟨X0 de X1, X0 X1⟩
X→⟨X0 de X1, X1 of X0⟩
X | 0,2 | the ··· mat
X | 3,4 | a ··· cat
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3

A hypergraph is a compact data structure to encode exponentially many trees.



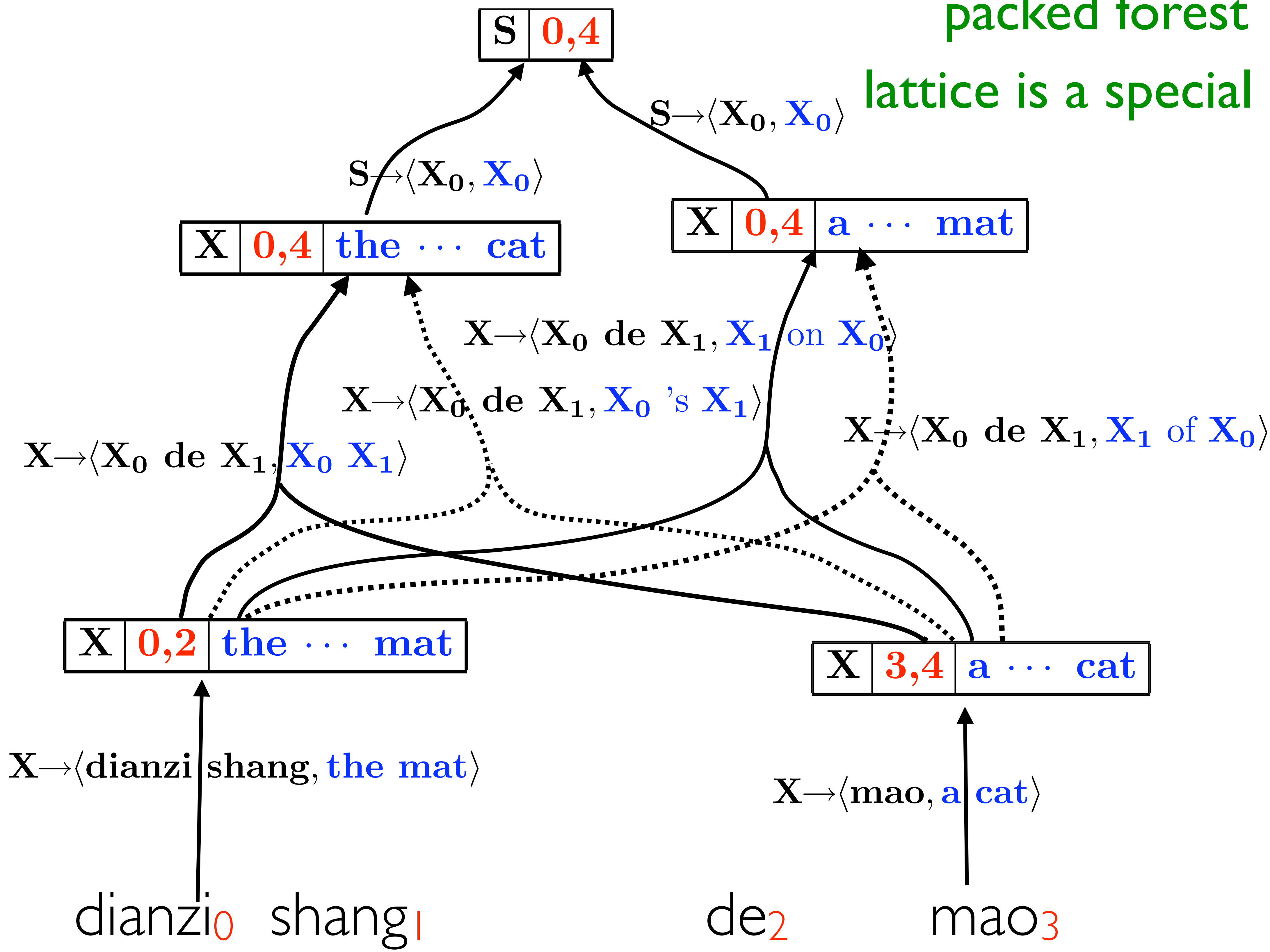
A hypergraph is a compact data structure to encode exponentially many trees.



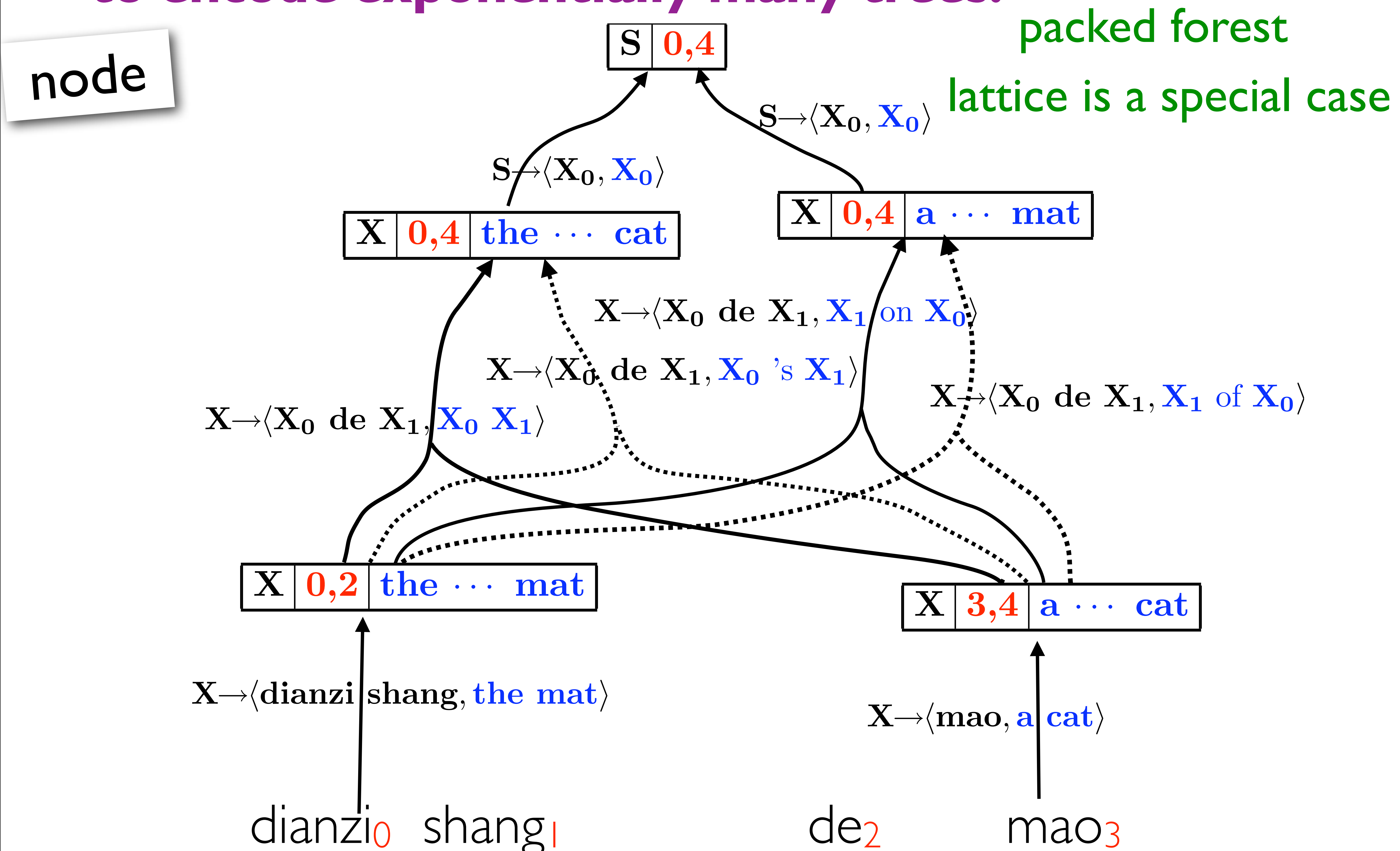
A hypergraph is a compact data structure to encode exponentially many trees.

packed forest

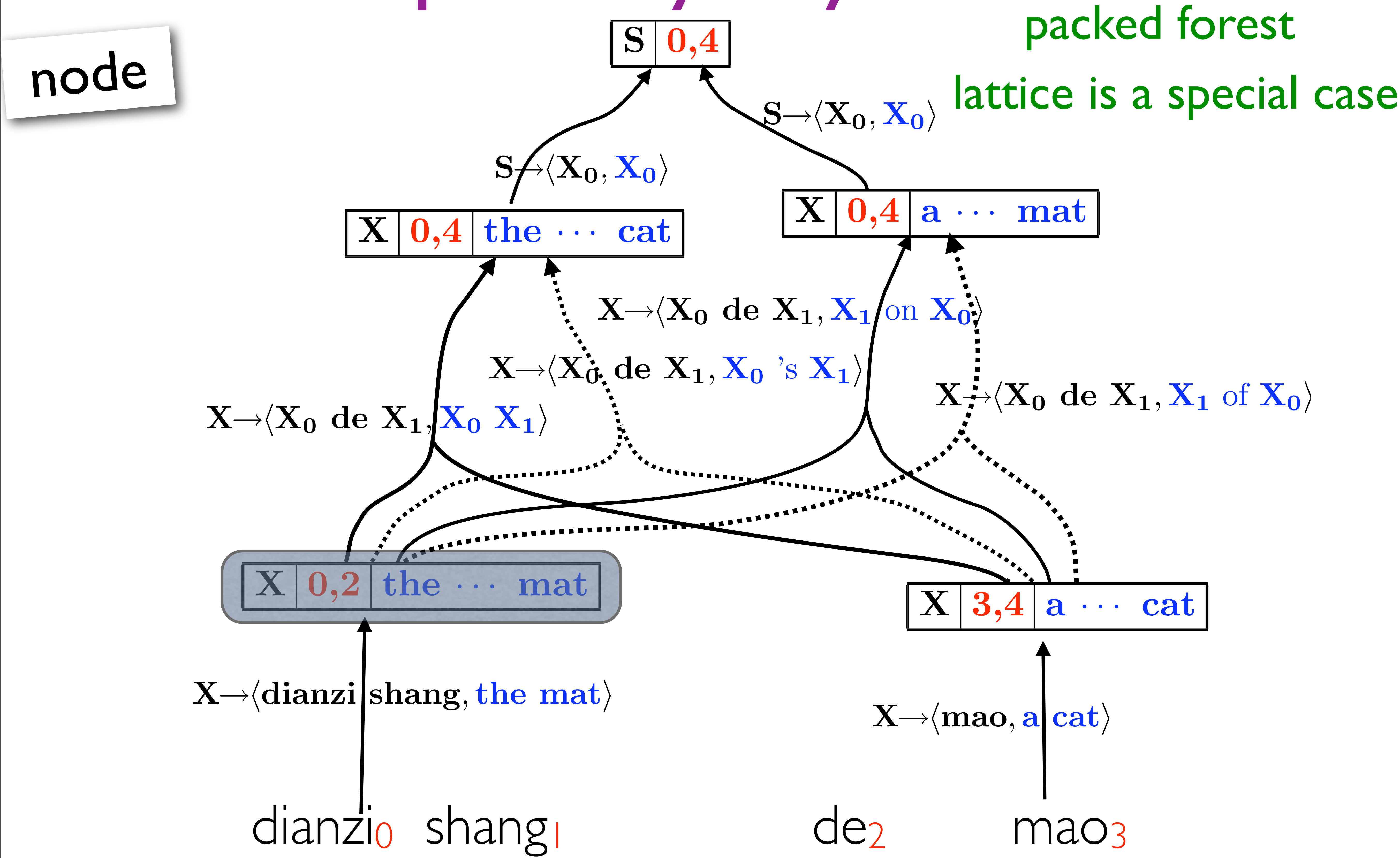
lattice is a special case



A hypergraph is a compact data structure to encode exponentially many trees.



A hypergraph is a compact data structure to encode exponentially many trees.



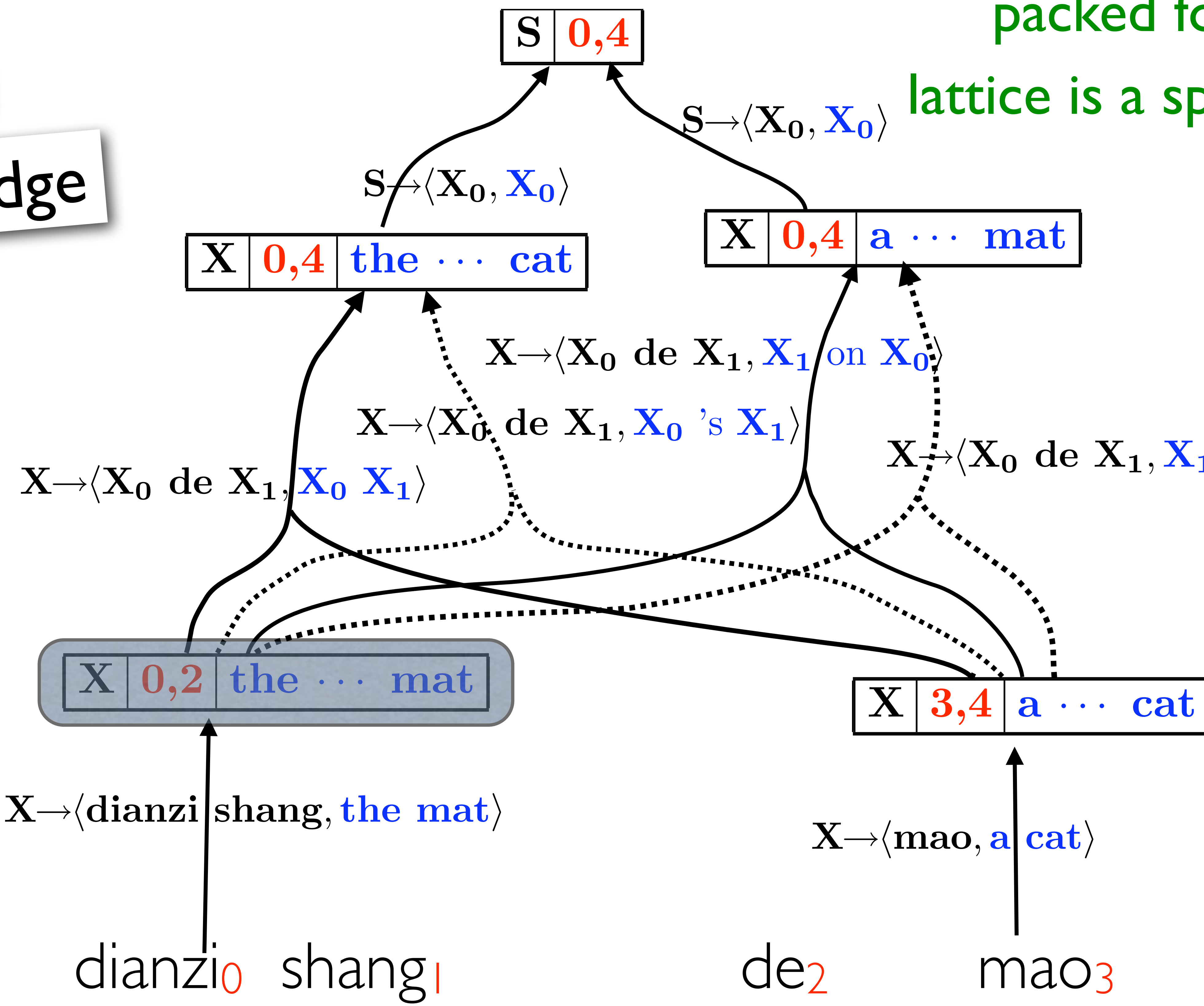
A hypergraph is a compact data structure to encode exponentially many trees.

packed forest

lattice is a special case

node

hyperedge



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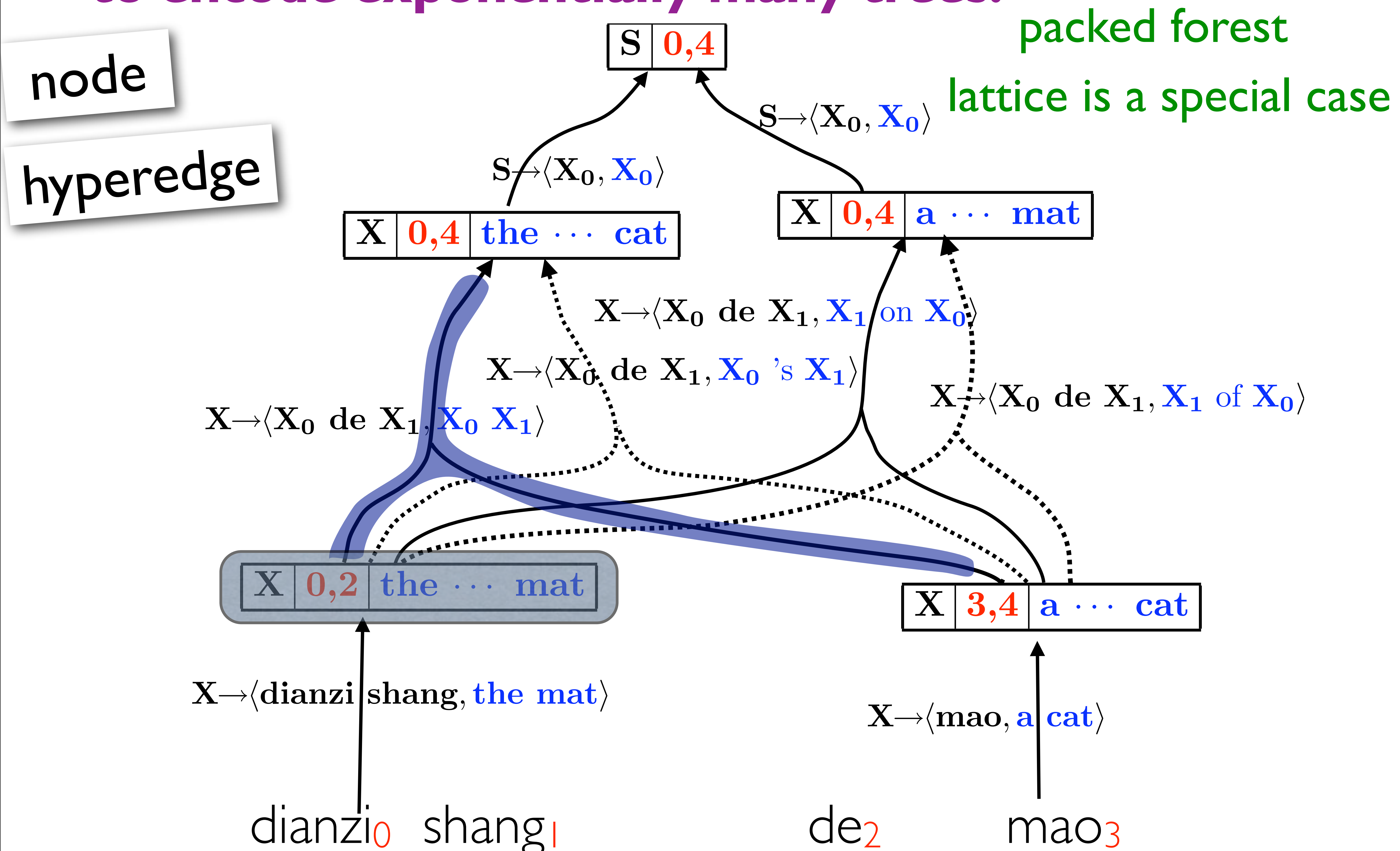

A hypergraph is a compact data structure to encode exponentially many trees.

packed forest

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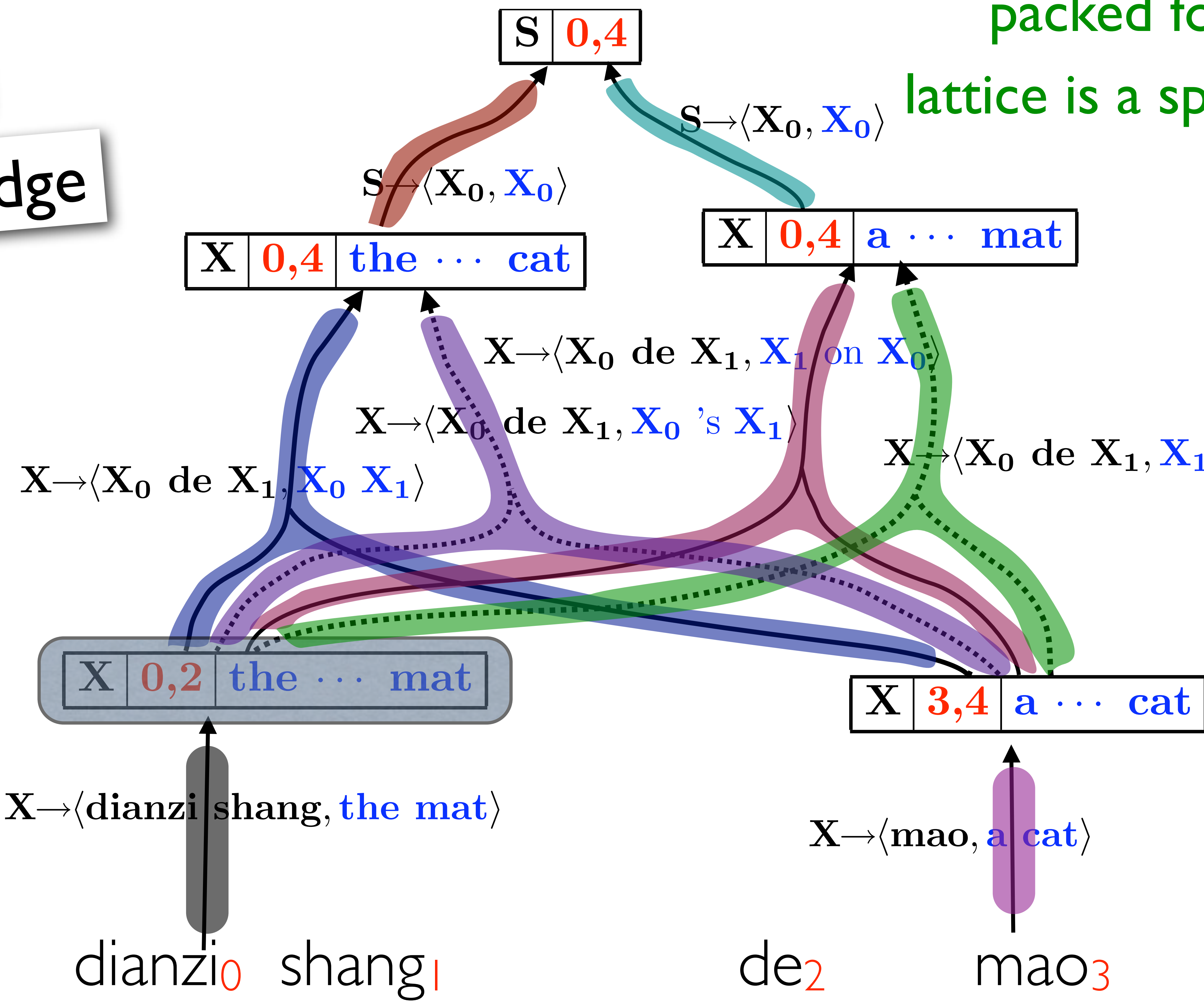
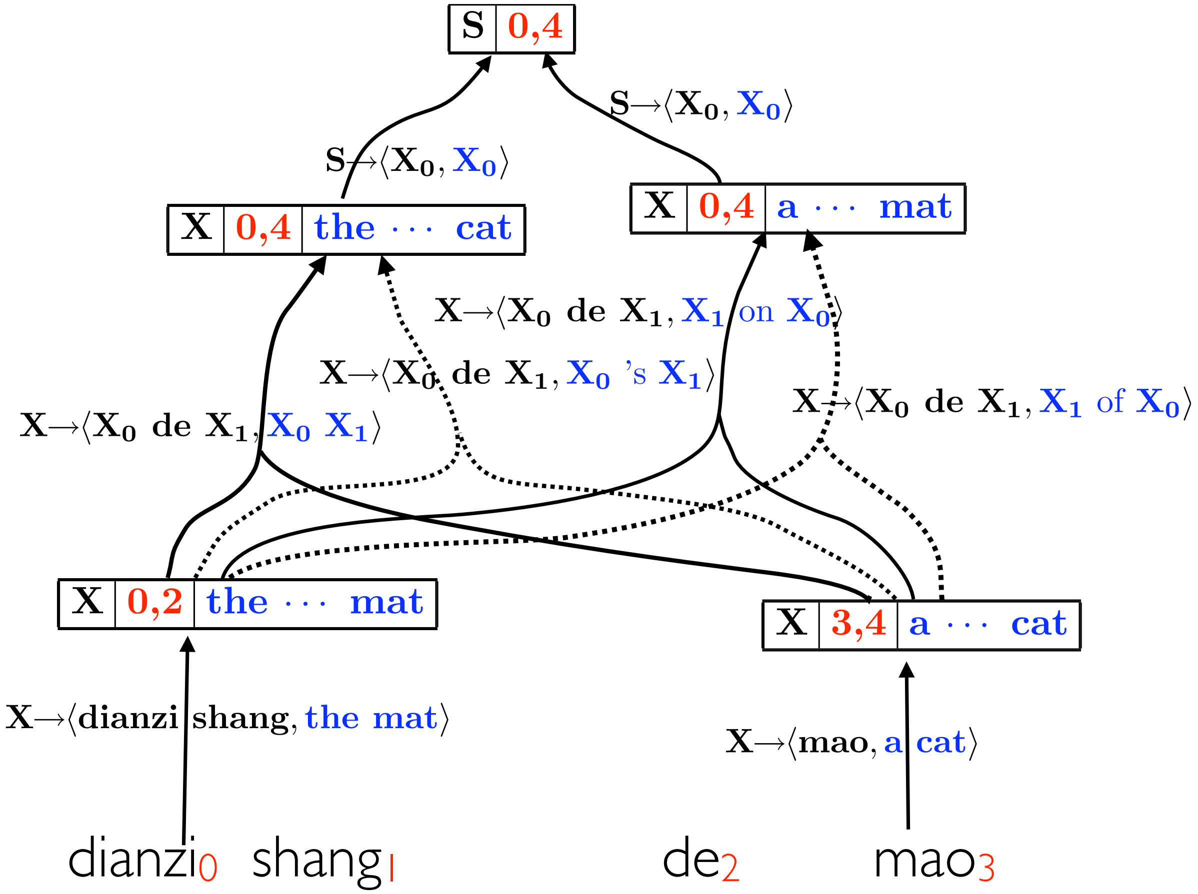
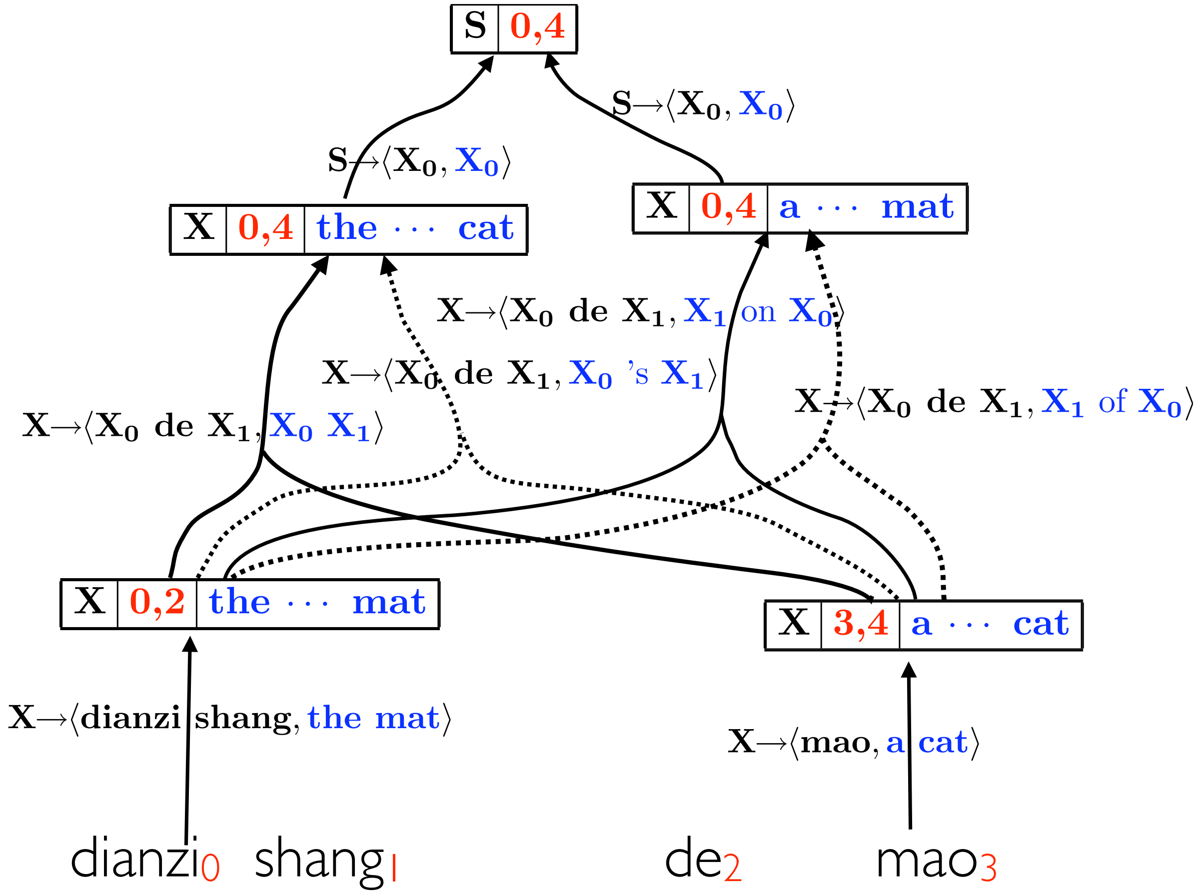
node

hyperedge

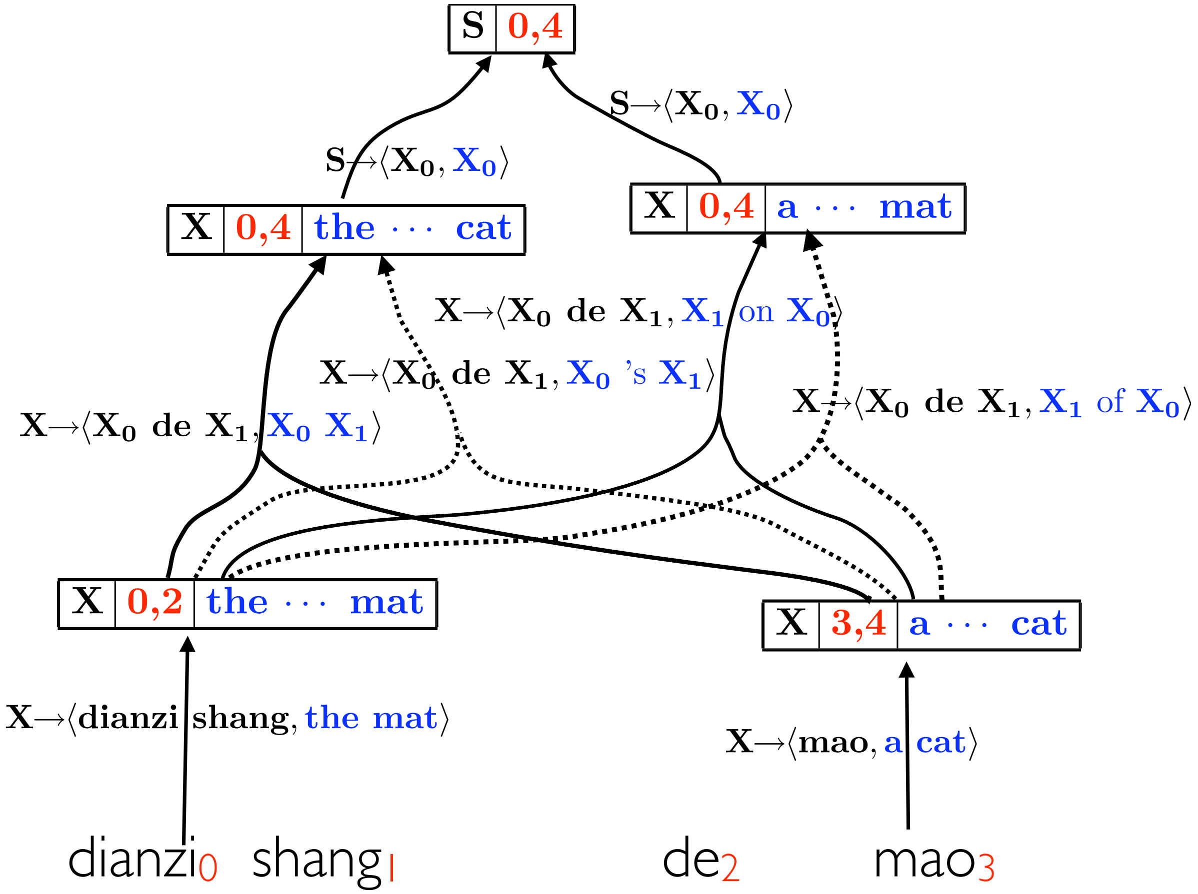


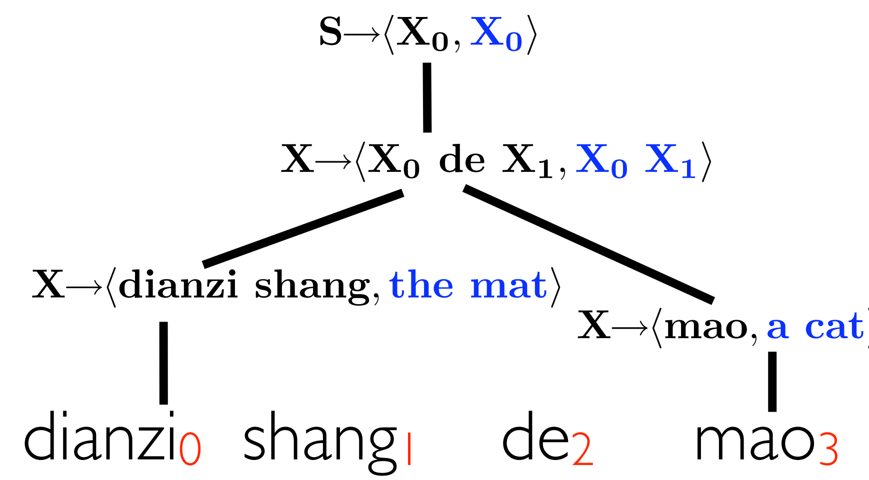

S | 0,4
S→⟨X0, X0⟩
S→⟨X0, X0⟩
X | 0,4 | the ··· cat
X | 0,4 | a ··· mat
X→⟨X0 de X1, X1 on X0⟩
X→⟨X0 de X1, X0 's X1⟩
X→⟨X0 de X1, X1 of X0⟩
X→⟨X0 de X1, X0 X1⟩
X | 0,2 | the ··· mat
X | 3,4 | a ··· cat
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3

How many trees?

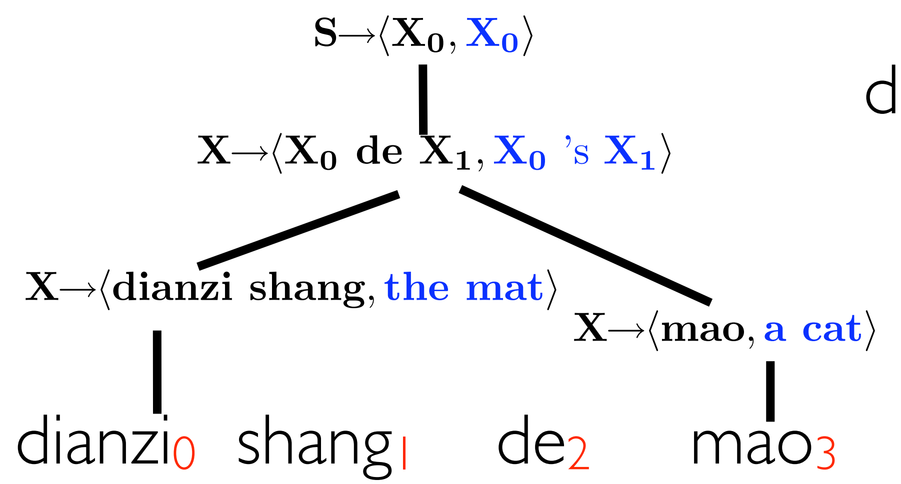


How many trees?

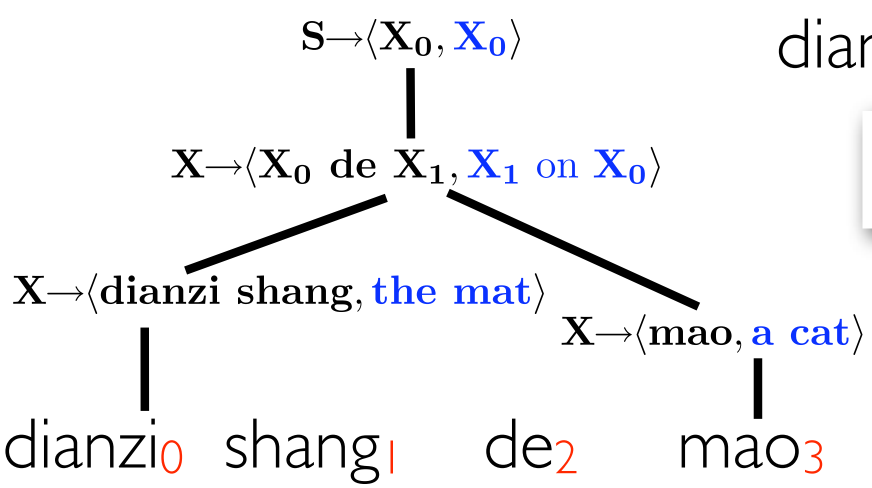




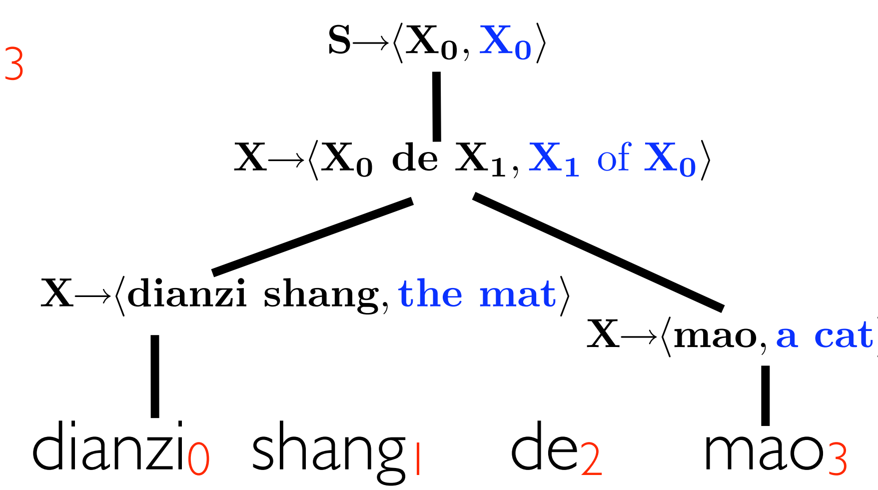
the mat a cat



the mat 's a cat



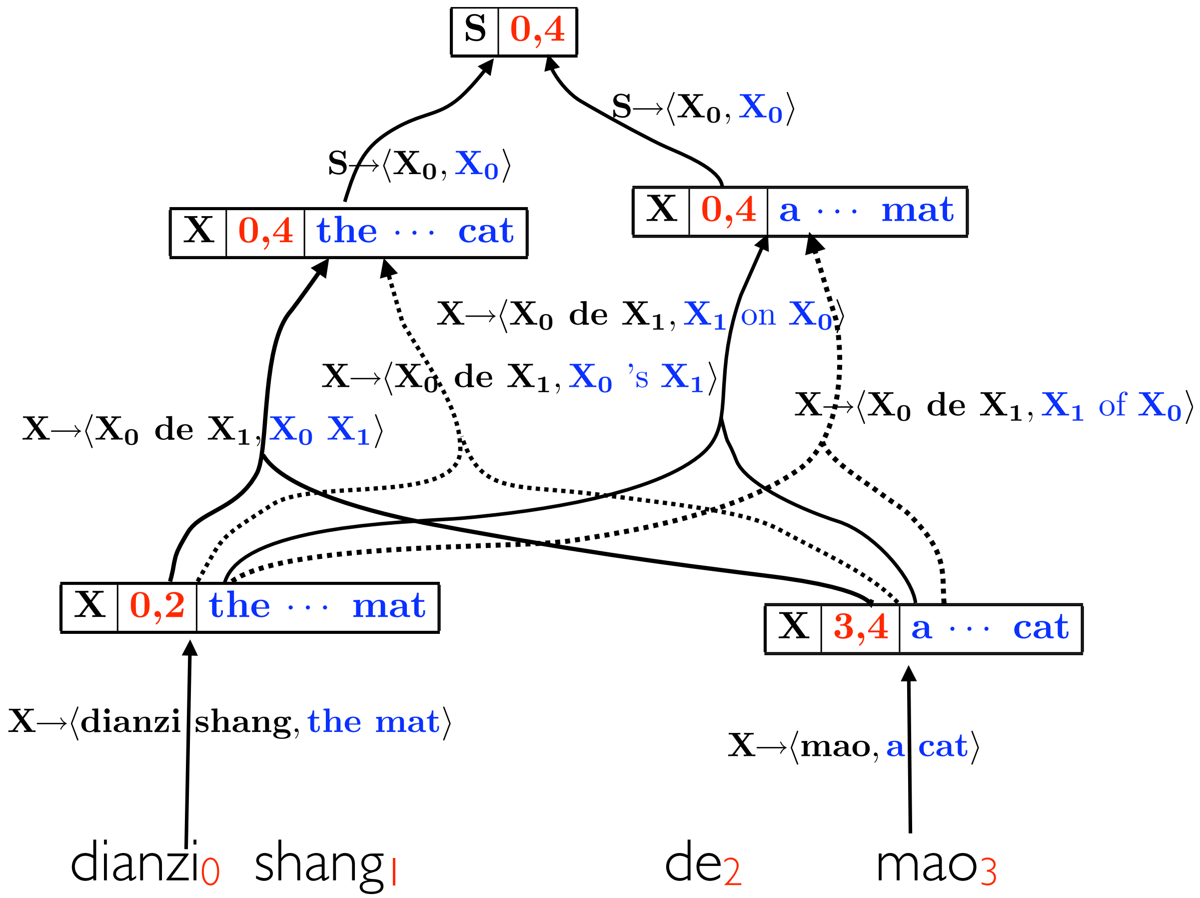
a cat on the mat

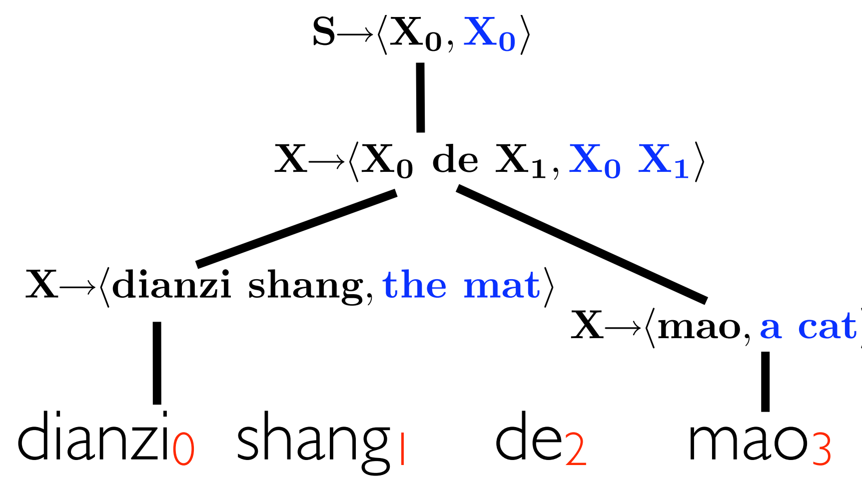


a cat of the mat

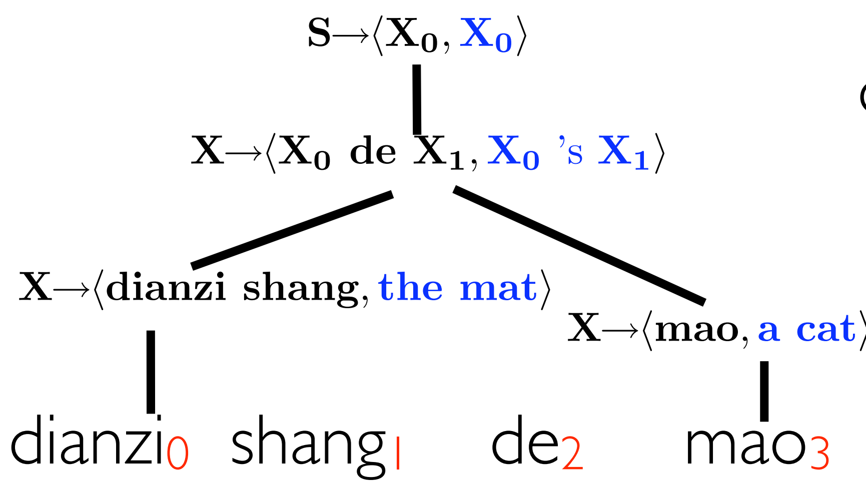
How many trees?

four ☺

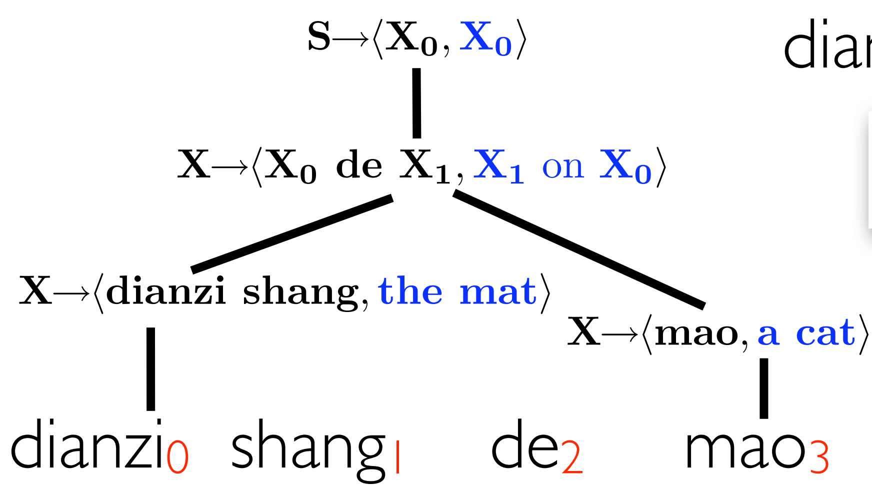




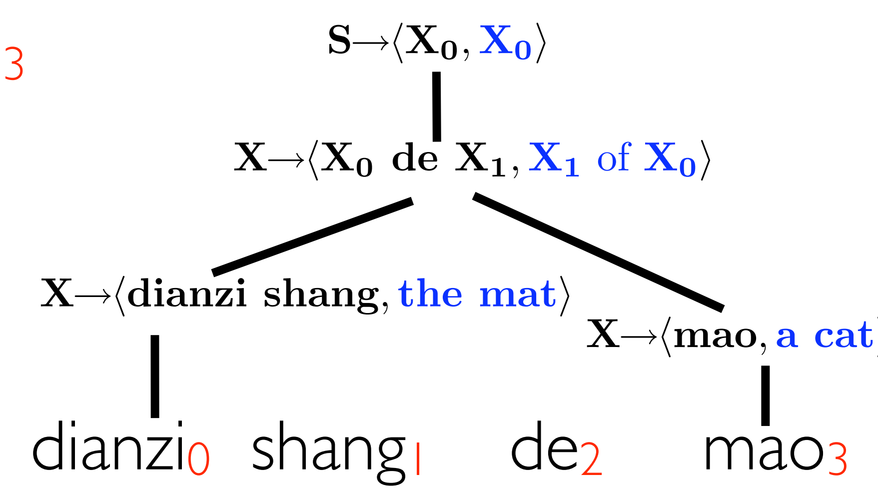
the mat a cat



the mat 's a cat



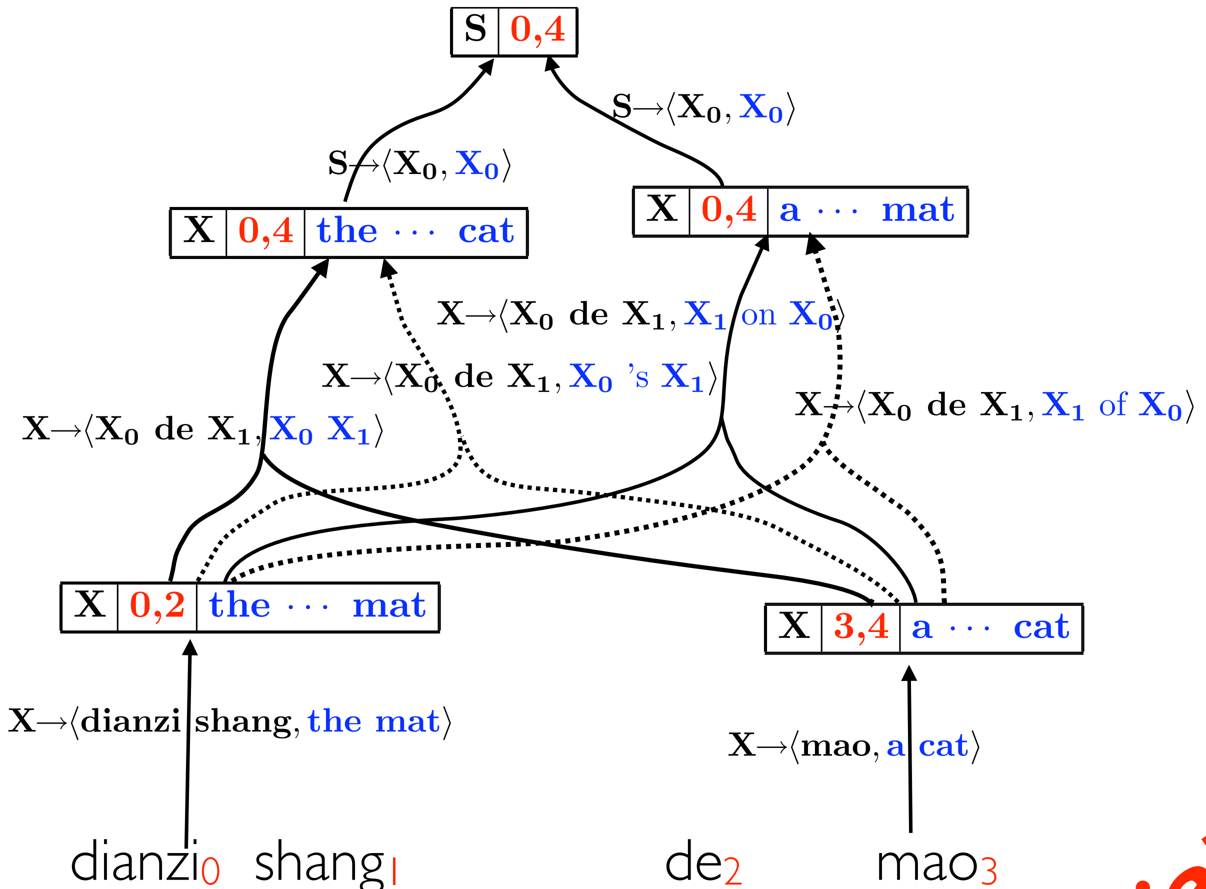
a cat on the mat



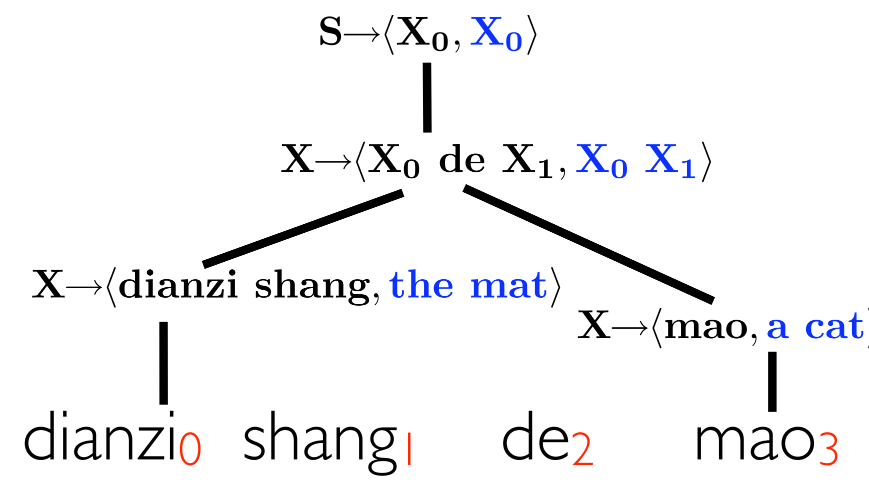
a cat of the mat

How many trees?

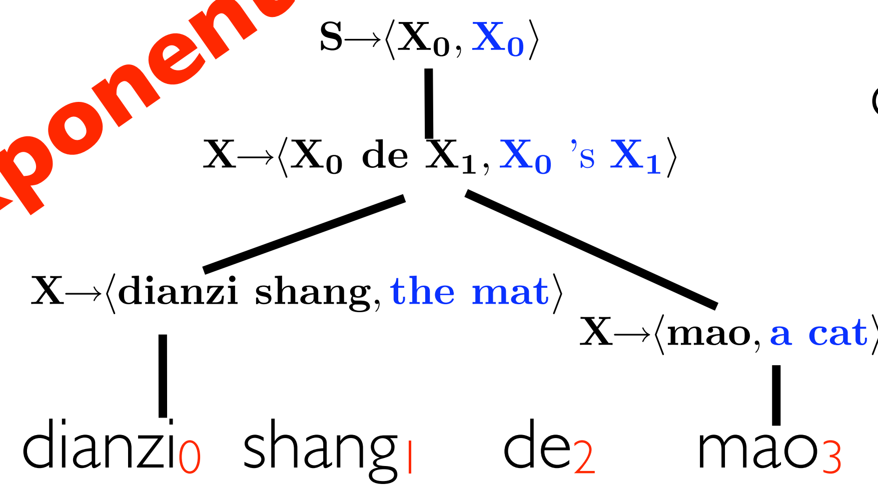
four ☺



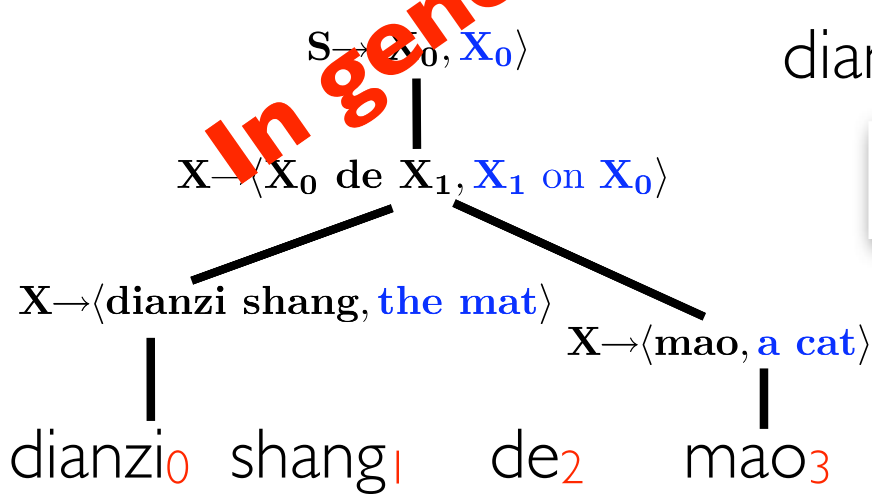
In general, exponentially many trees!



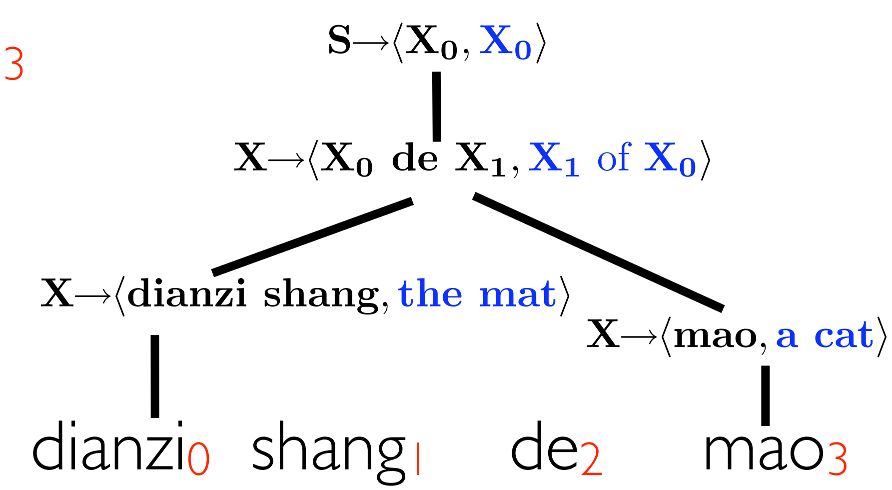
the mat a cat



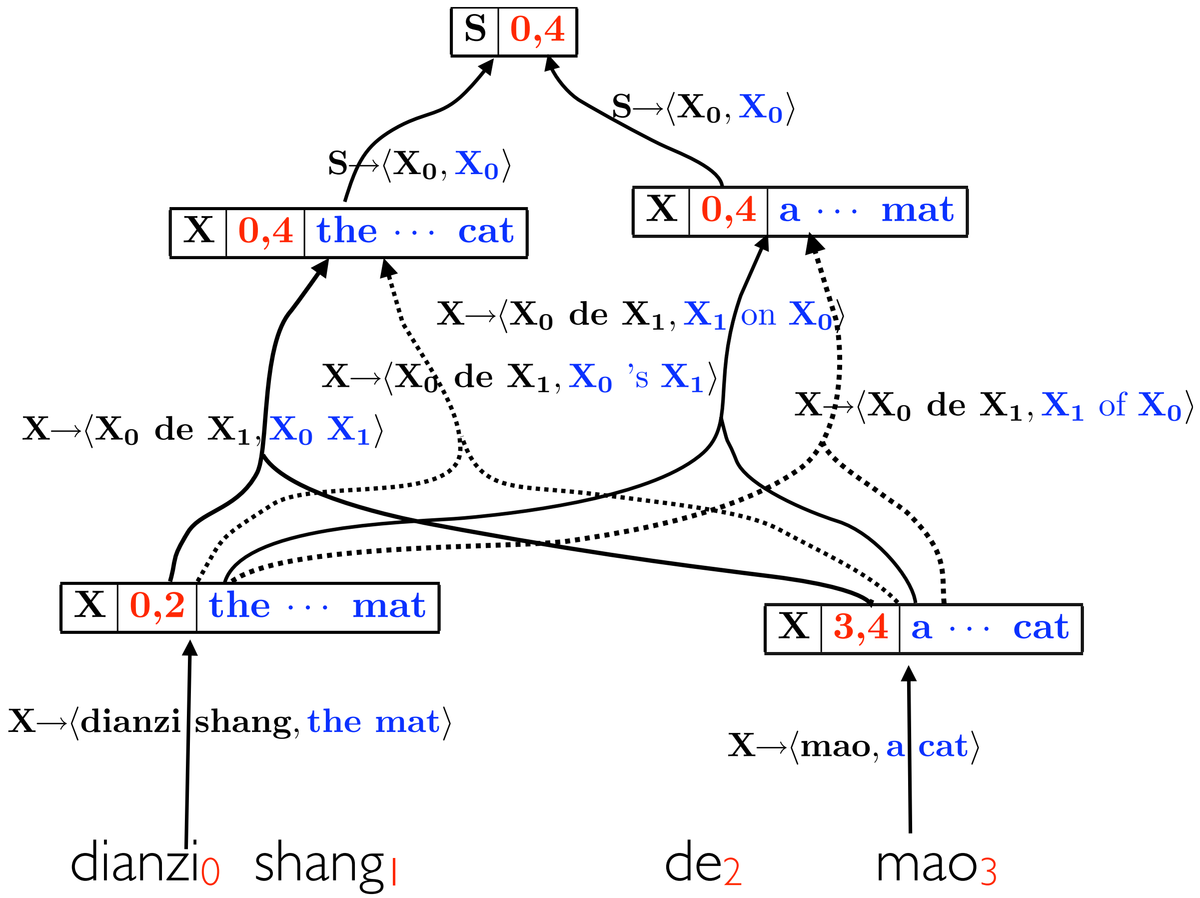
the mat 's a cat

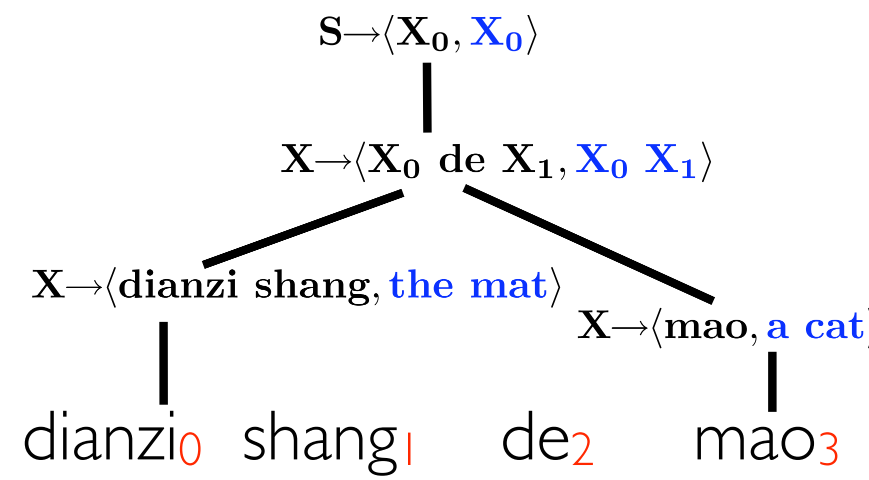


a cat on the mat

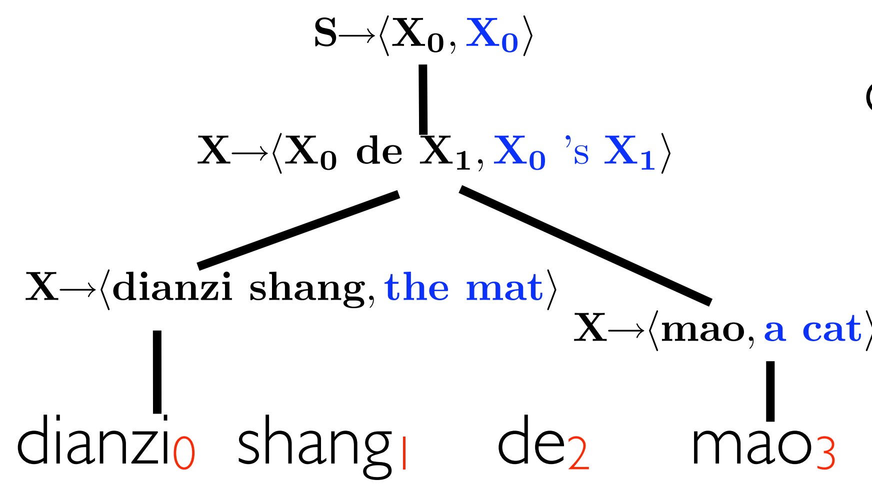


a cat of the mat

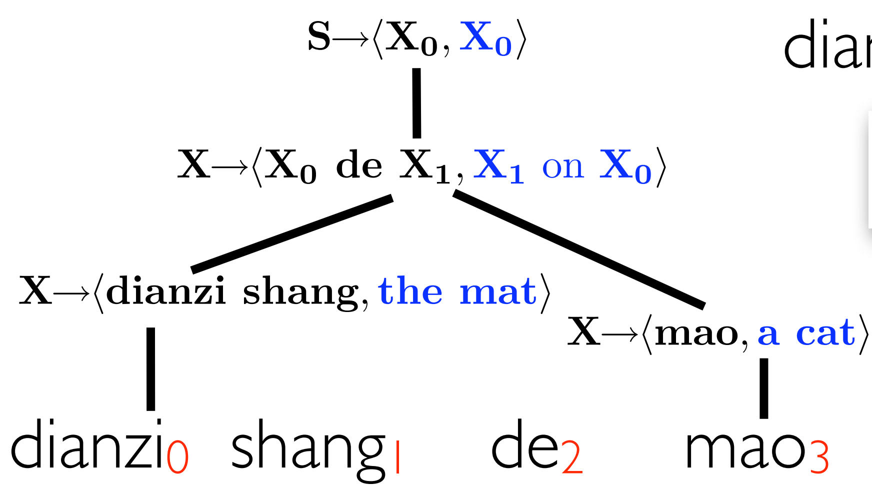




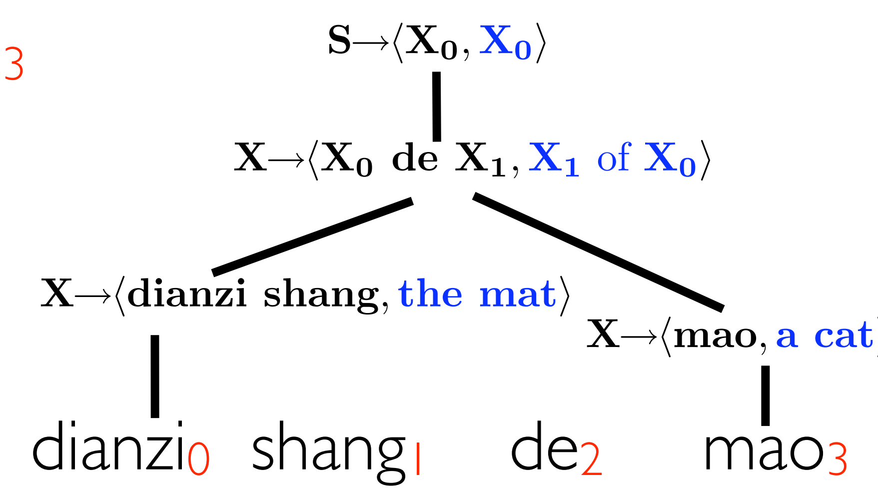
the mat a cat



the mat 's a cat

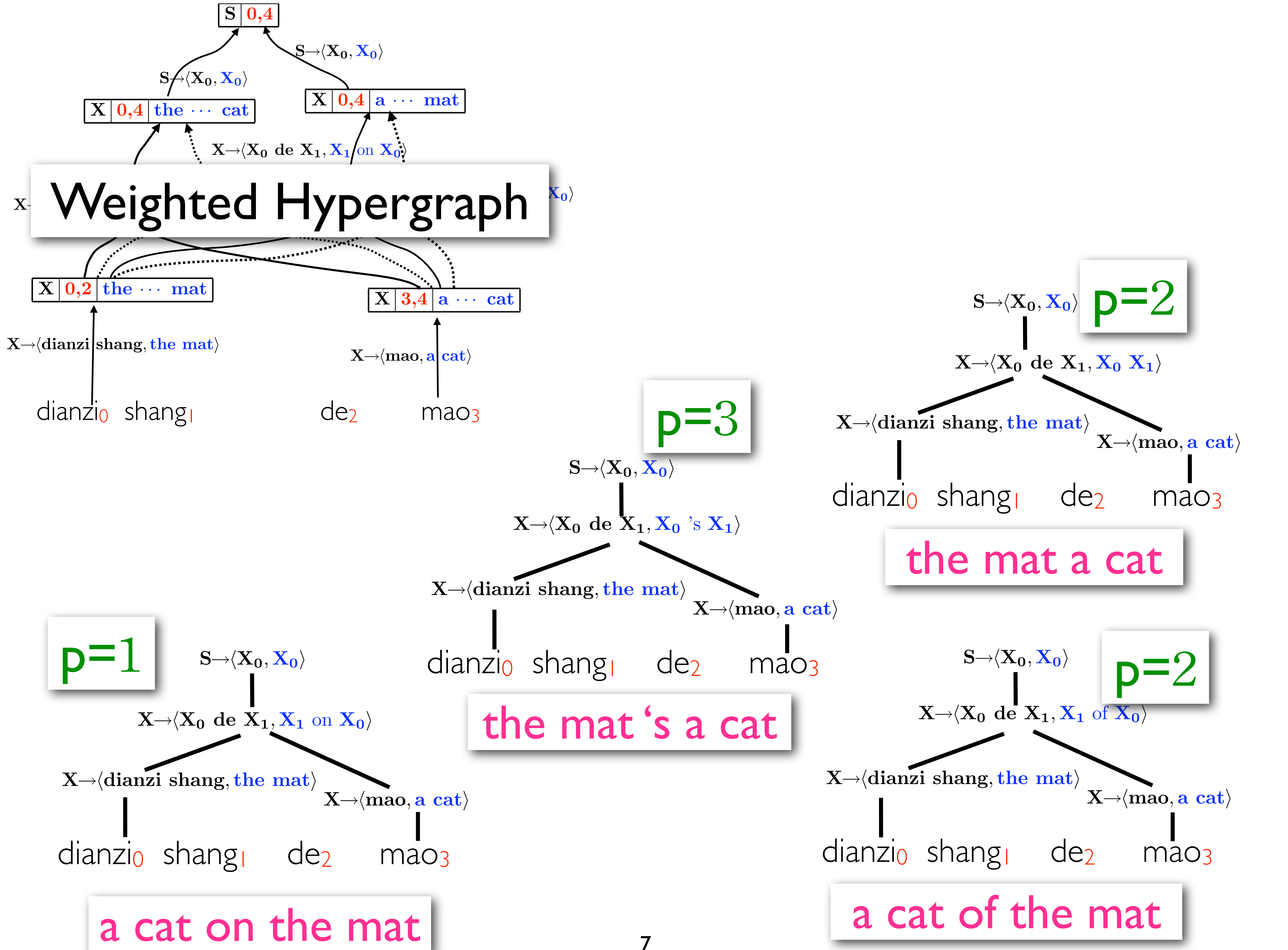


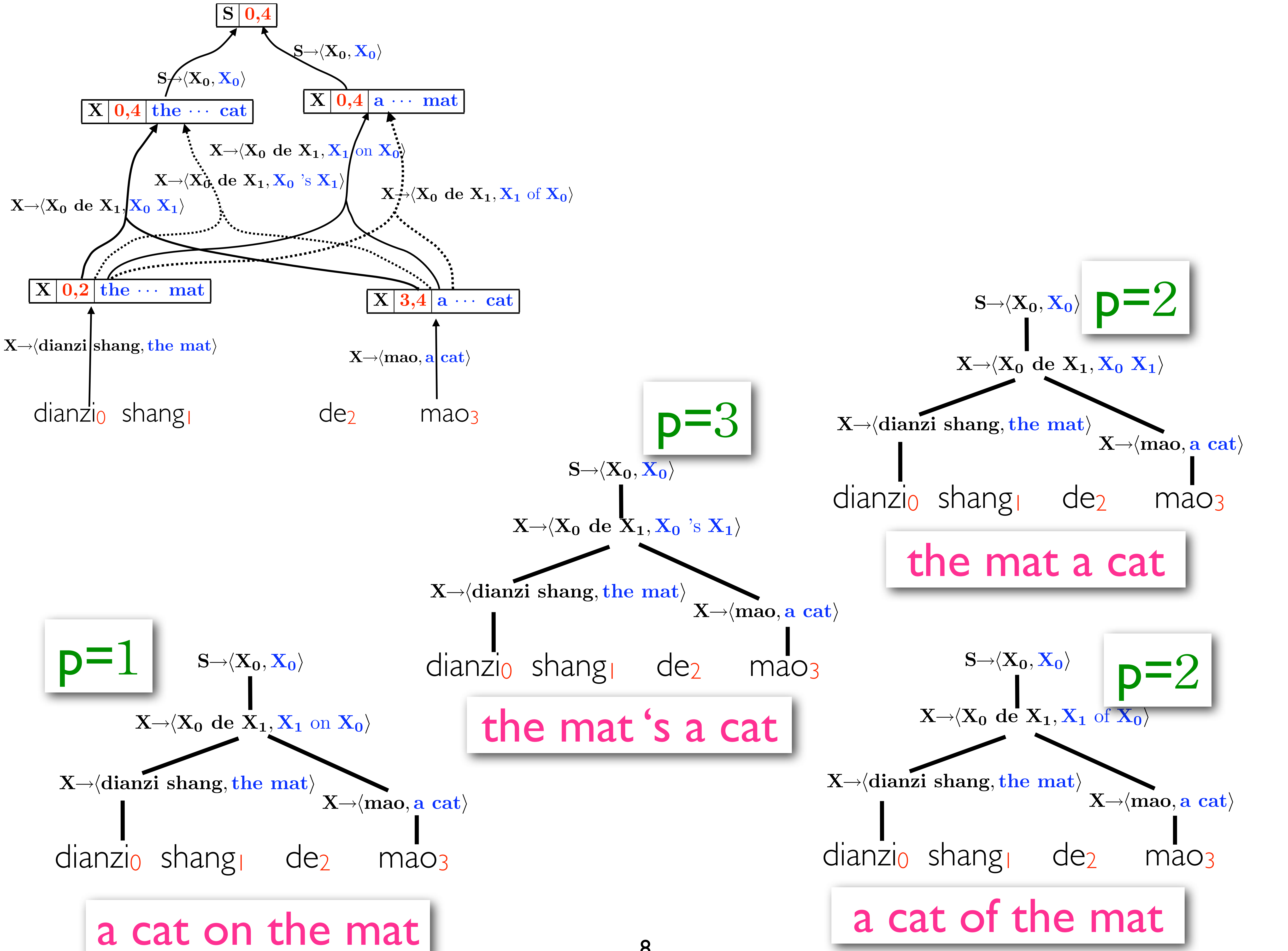
a cat on the mat

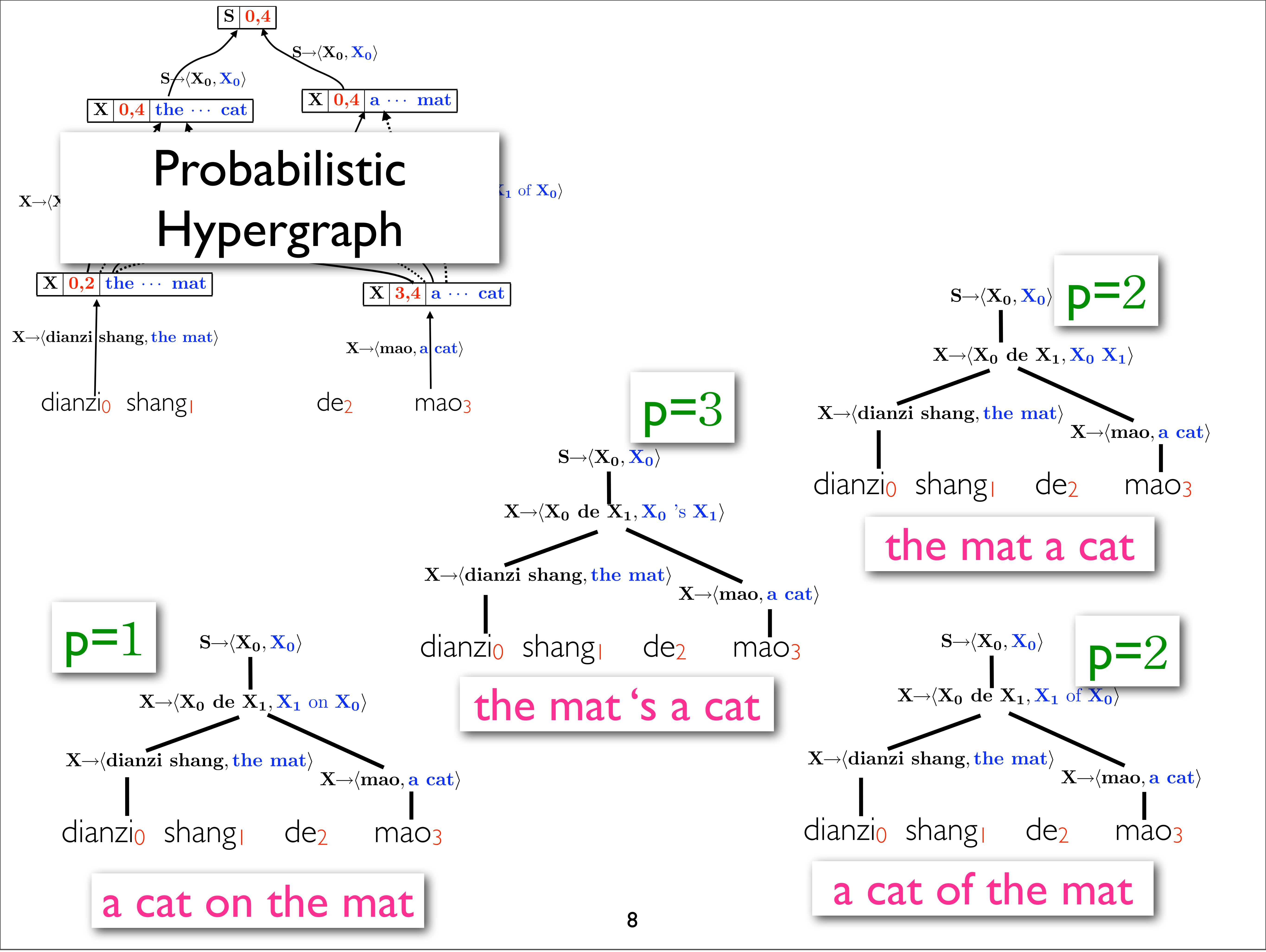


a cat of the mat

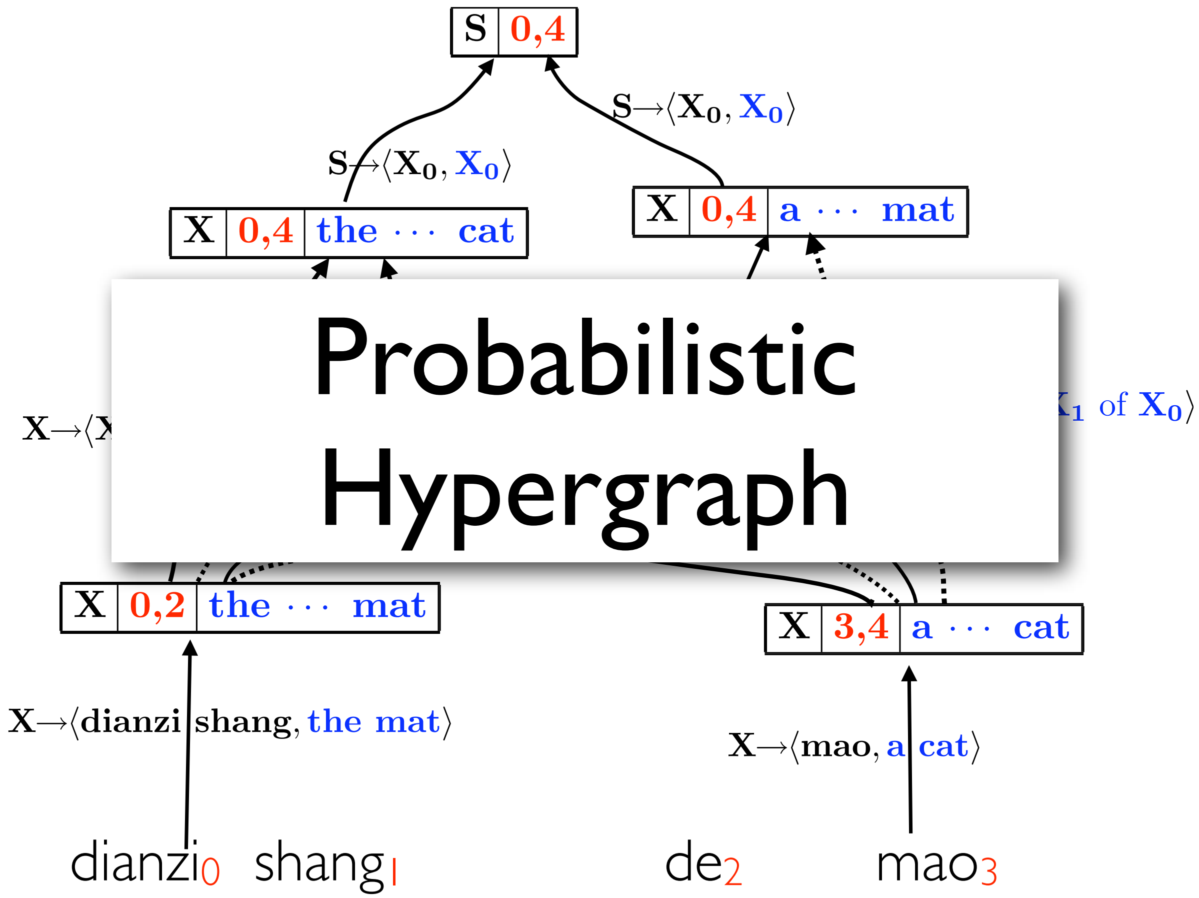
Weighted Hypergraph




S | 0,4
S→⟨X0, X0⟩
S→⟨X0, X0⟩
X | 0,4 | the ··· cat
X | 0,4 | a ··· mat
X→⟨X0 de X1, X1 on X0⟩
X→⟨X0 de X1, X0 's X1⟩
X→⟨X0 de X1, X1 of X0⟩
X→⟨X0 de X1, X0 X1⟩
X | 0,2 | the ··· mat
X | 3,4 | a ··· cat
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
p=1
S→⟨X0, X0⟩
X→⟨X0 de X1, X1 on X0⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
a cat on the mat
p=3
S→⟨X0, X0⟩
X→⟨X0 de X1, X0 's X1⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
the mat 's a cat
p=2
S→⟨X0, X0⟩
X→⟨X0 de X1, X0 X1⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
the mat a cat
p=2
S→⟨X0, X0⟩
X→⟨X0 de X1, X1 of X0⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
a cat of the mat

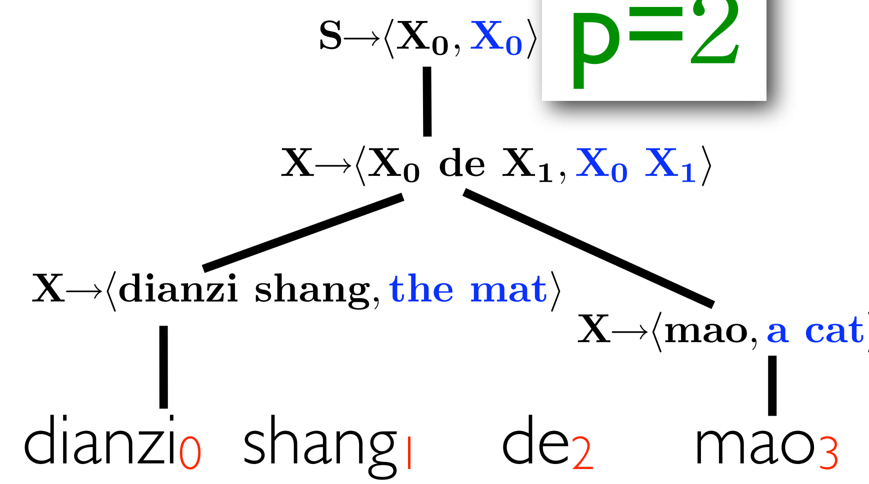

Probabilistic Hypergraph
S 0,4
S→⟨X0, X0⟩
S→⟨X0, X0⟩
X 0,4 the ··· cat
X 0,4 a ··· mat
X 0,2 the ··· mat
X 3,4 a ··· cat
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
p=1
S→⟨X0, X0⟩
X→⟨X0 de X1, X1 on X0⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
a cat on the mat
p=3
S→⟨X0, X0⟩
X→⟨X0 de X1, X0 's X1⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
the mat 's a cat
p=2
S→⟨X0, X0⟩
X→⟨X0 de X1, X0 X1⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
the mat a cat
p=2
S→⟨X0, X0⟩
X→⟨X0 de X1, X1 of X0⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
a cat of the mat

Probabilistic Hypergraph



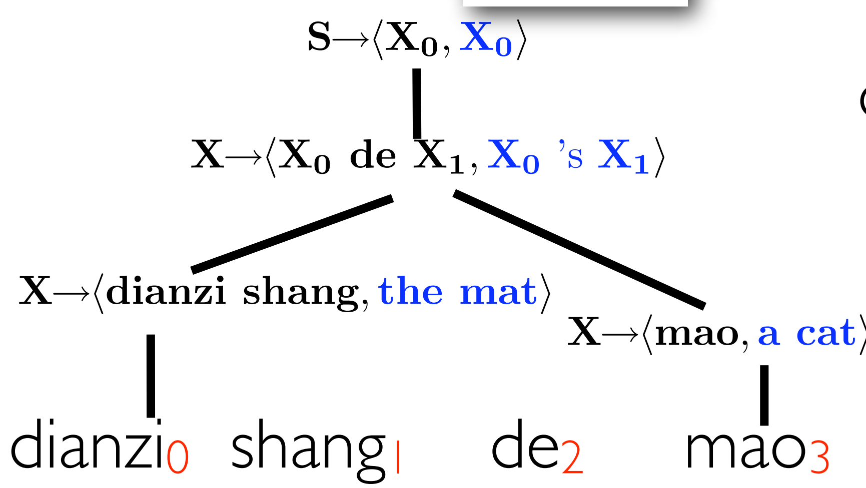
Z=2+1+3+2=8

p=2



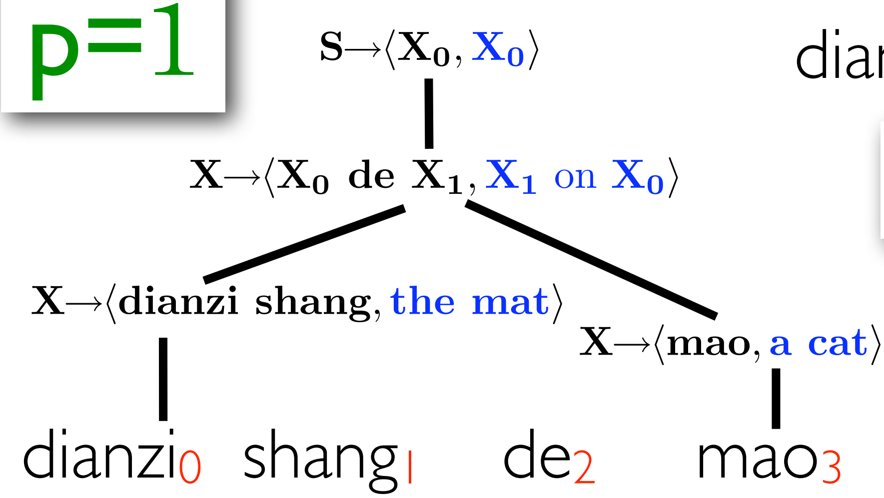
the mat a cat

p=3



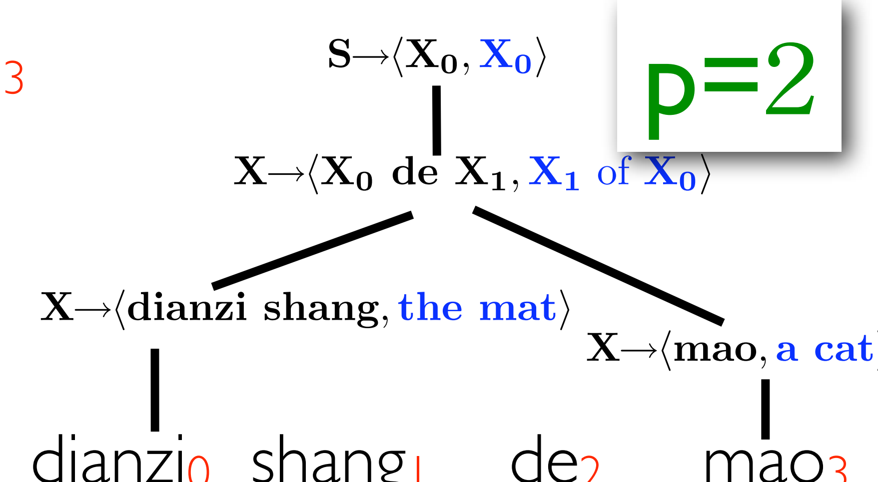
the mat 's a cat

p=1

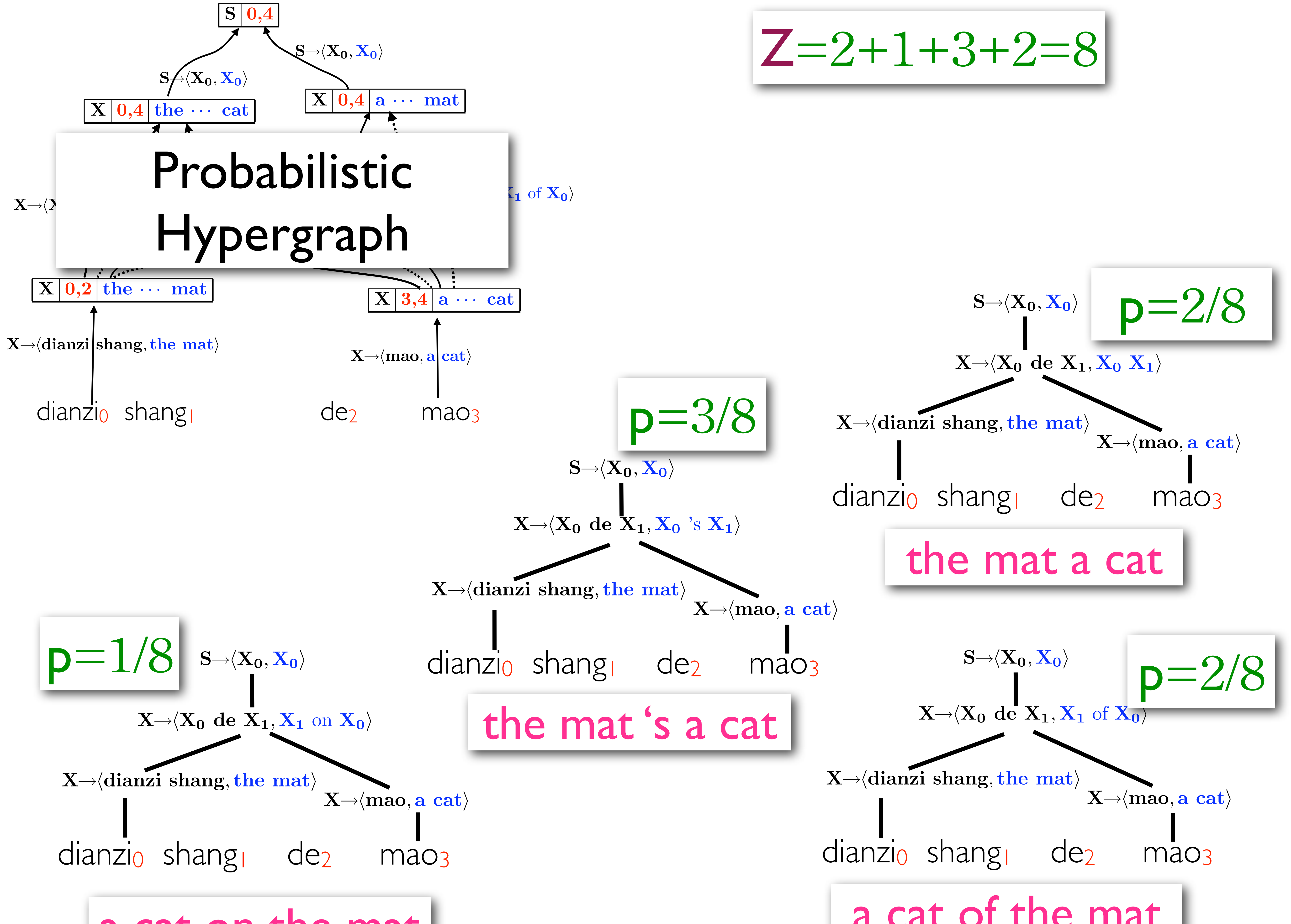


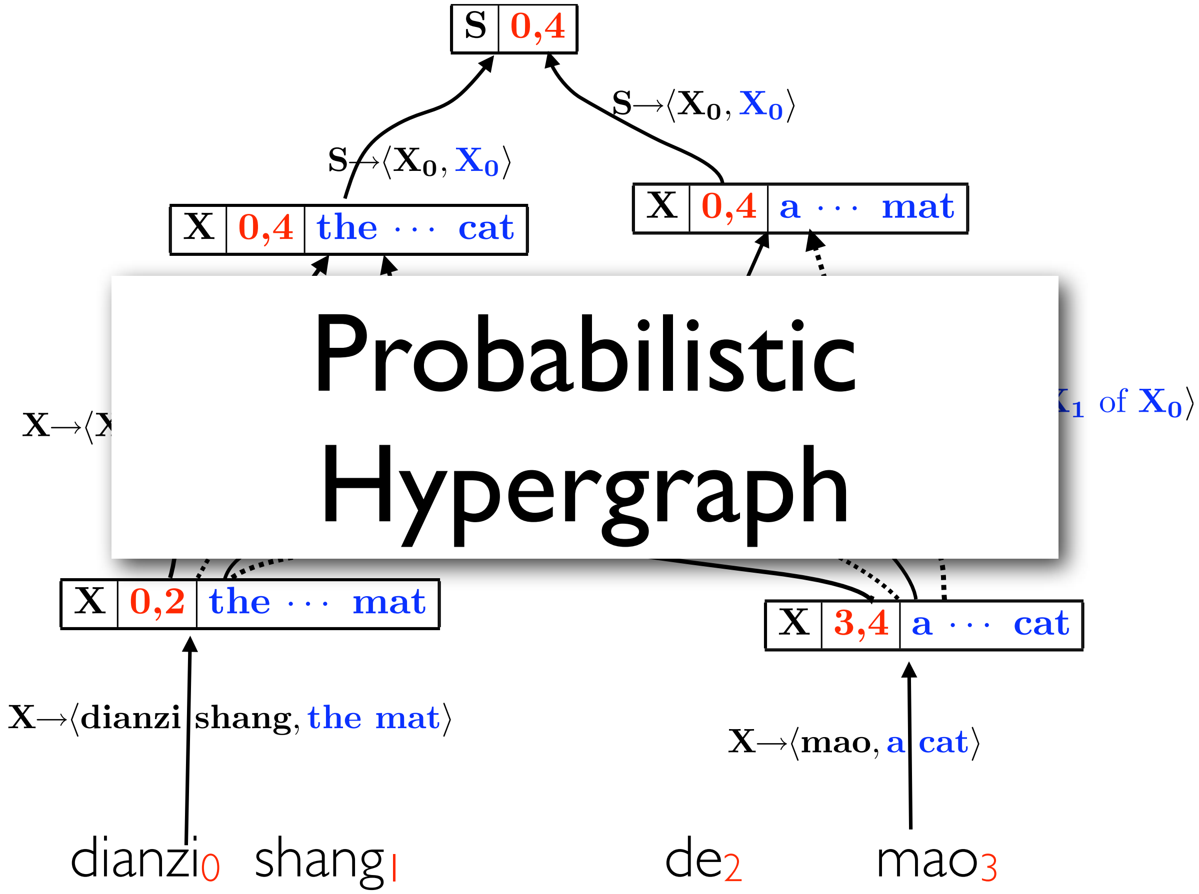
a cat on the mat

p=2



a cat of the mat

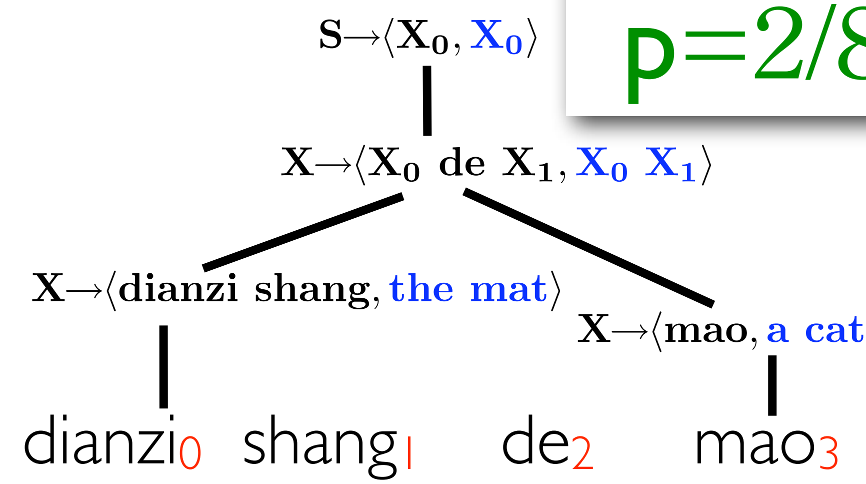

Z=2+1+3+2=8
Probabilistic Hypergraph
S 0,4
S→⟨X0, X0⟩
S→⟨X0, X0⟩
X 0,4 the ··· cat
X 0,4 a ··· mat
X1 of X0⟩
X 0,2 the ··· mat
X 3,4 a ··· cat
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
p=2/8
S→⟨X0, X0⟩
X→⟨X0 de X1, X0 X1⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
the mat a cat
p=3/8
S→⟨X0, X0⟩
X→⟨X0 de X1, X0 's X1⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
the mat 's a cat
p=1/8
S→⟨X0, X0⟩
X→⟨X0 de X1, X1 on X0⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
a cat on the mat
p=2/8
S→⟨X0, X0⟩
X→⟨X0 de X1, X1 of X0⟩
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3
a cat of the mat



Z=2+1+3+2=8

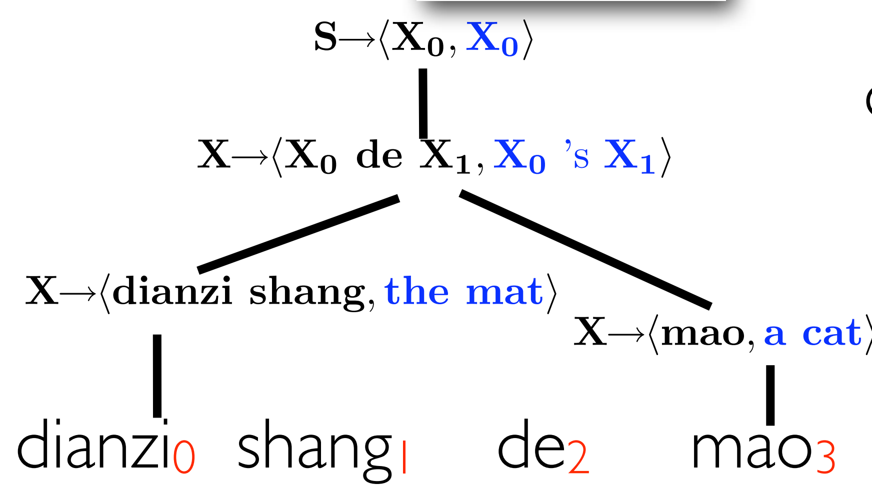
The hypergraph defines a probability distribution over **trees**!

p=2/8



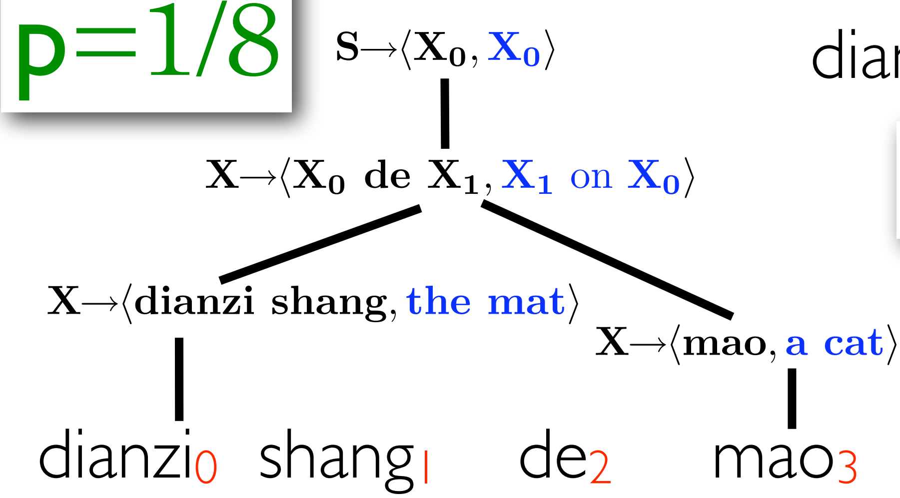
the mat a cat

p=3/8



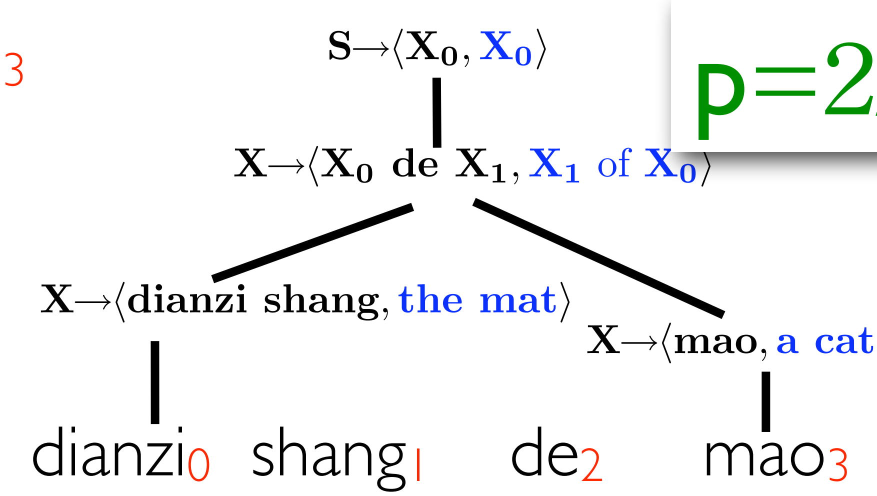
the mat 's a cat

p=1/8



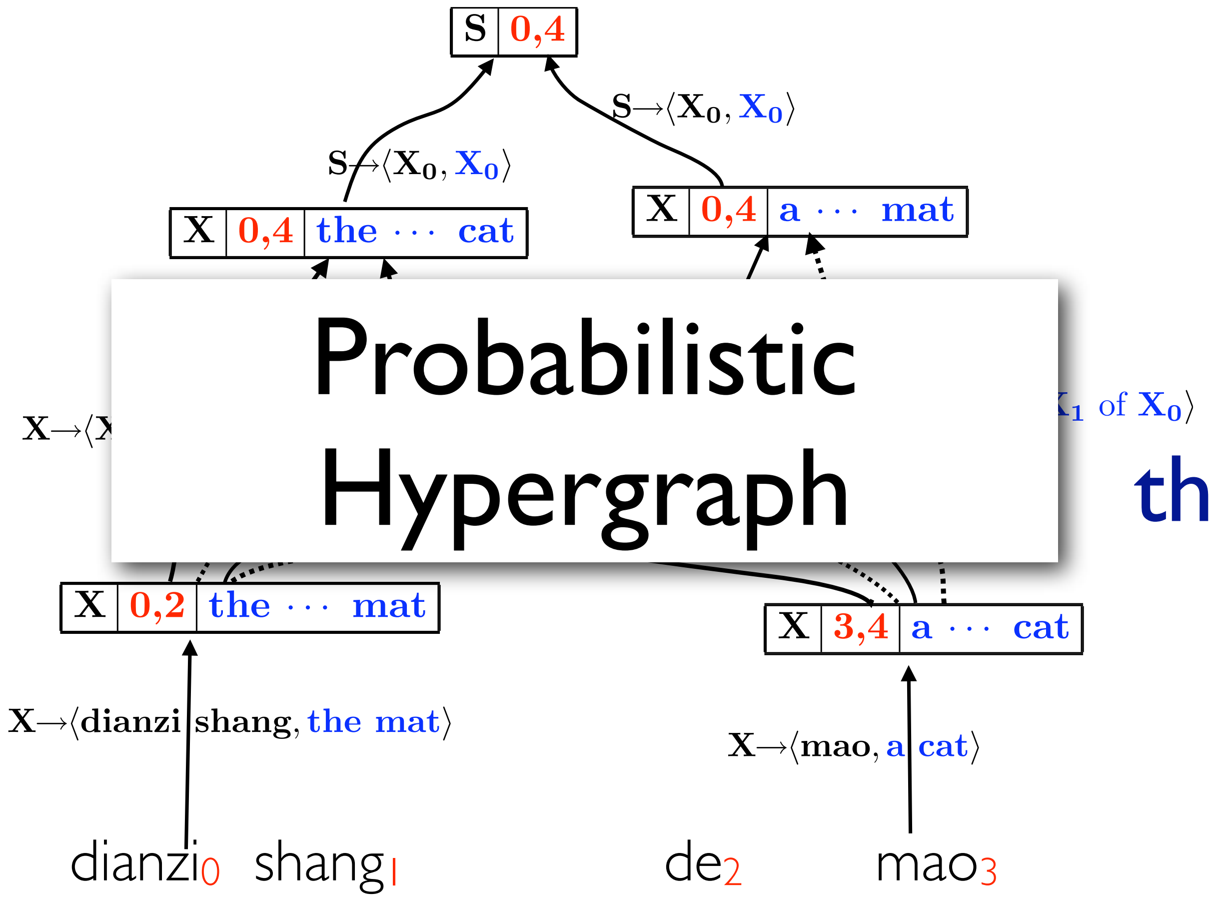
a cat on the mat

p=2/8



a cat of the mat

Probabilistic Hypergraph

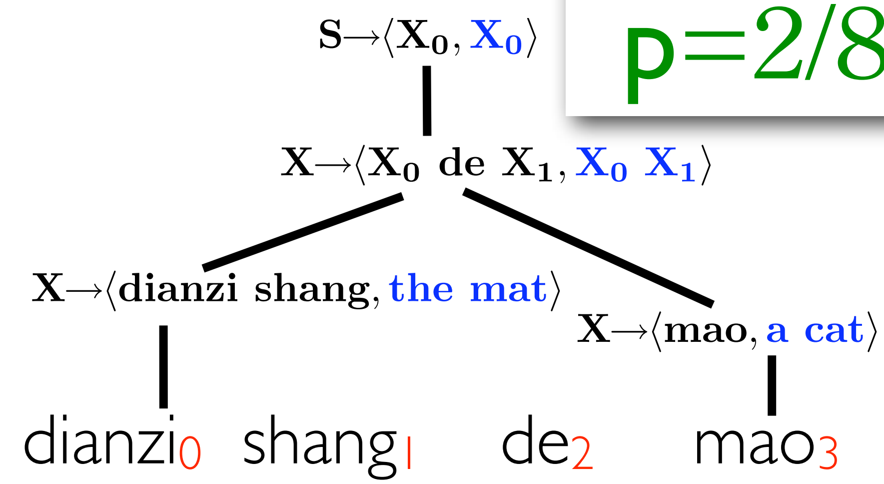


Z=2+1+3+2=8

The hypergraph defines a probability distribution over **trees**!

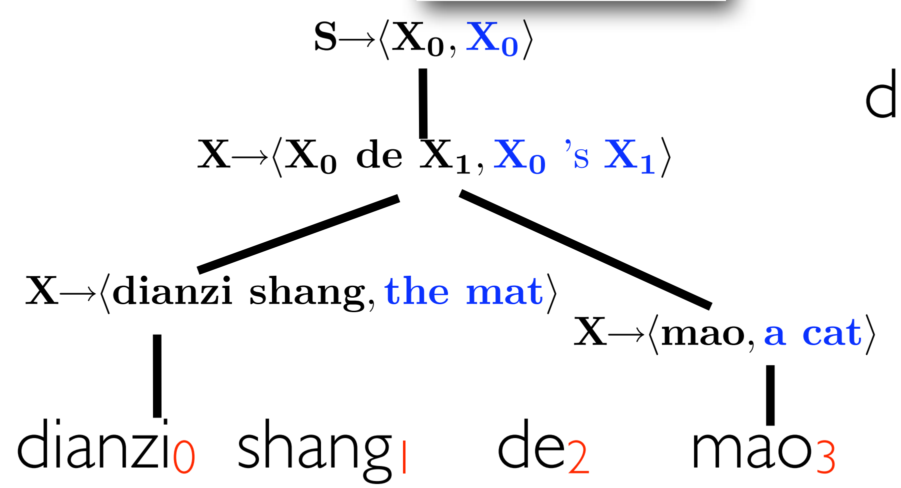
the distribution is parameterized by Θ

p=2/8



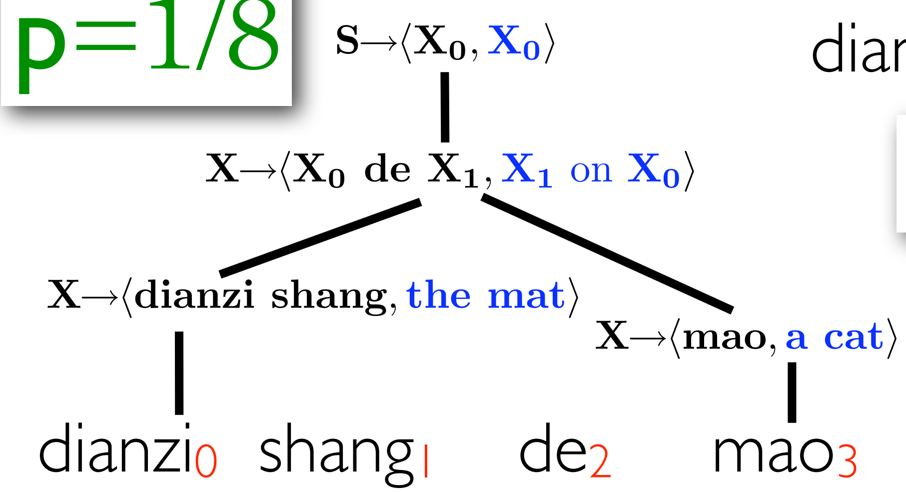
the mat a cat

p=3/8



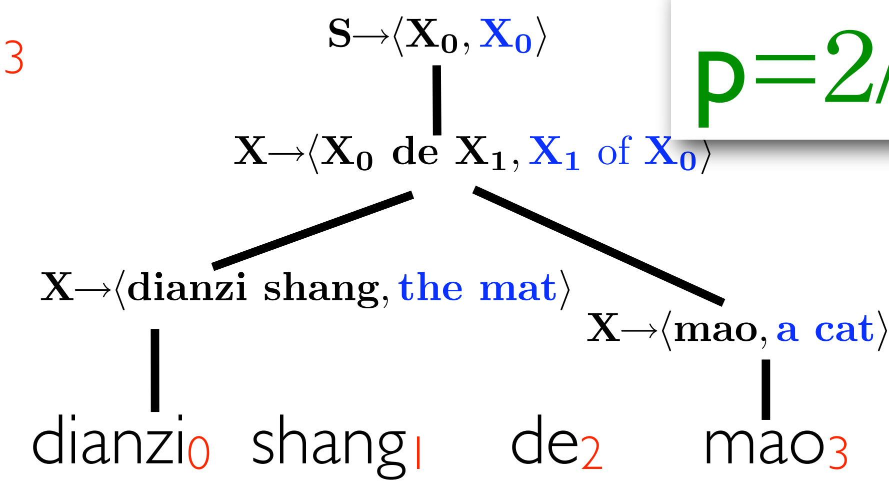
the mat 's a cat

p=1/8

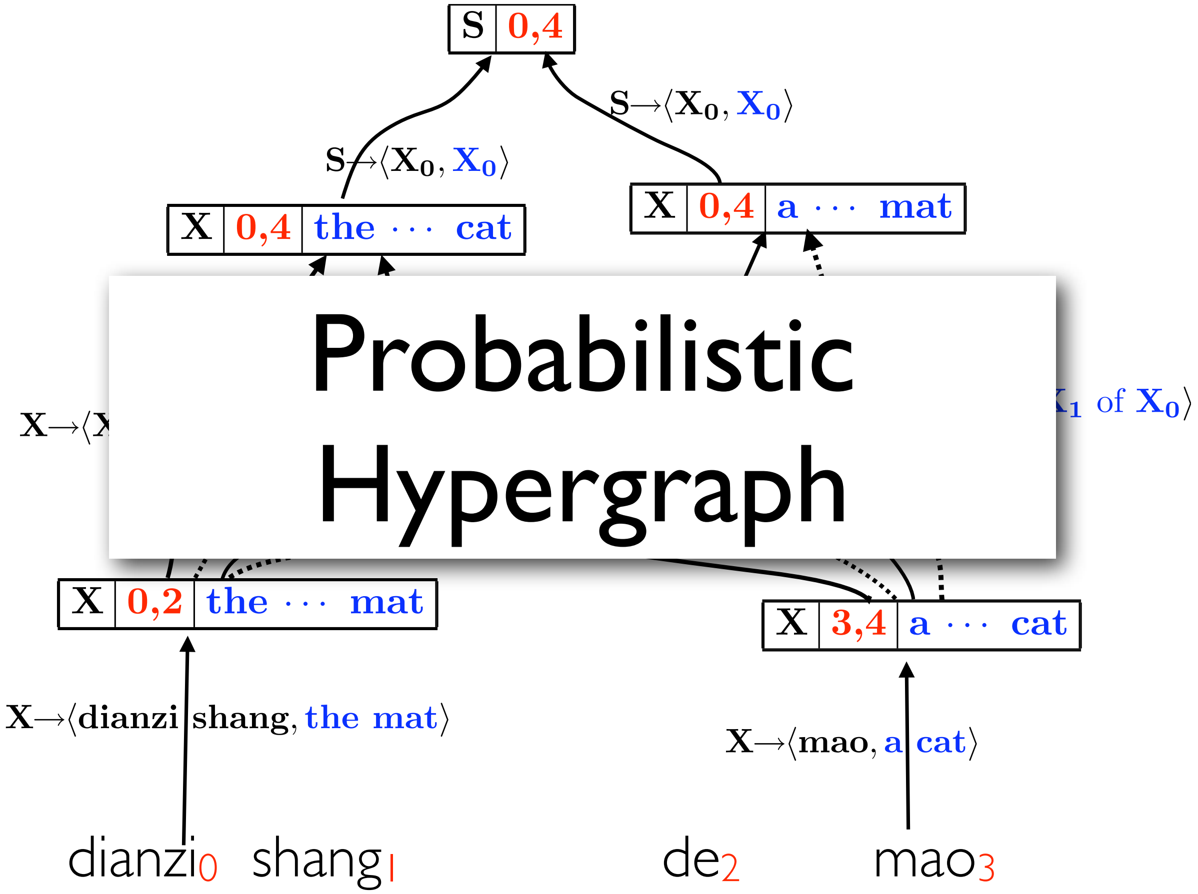


a cat on the mat

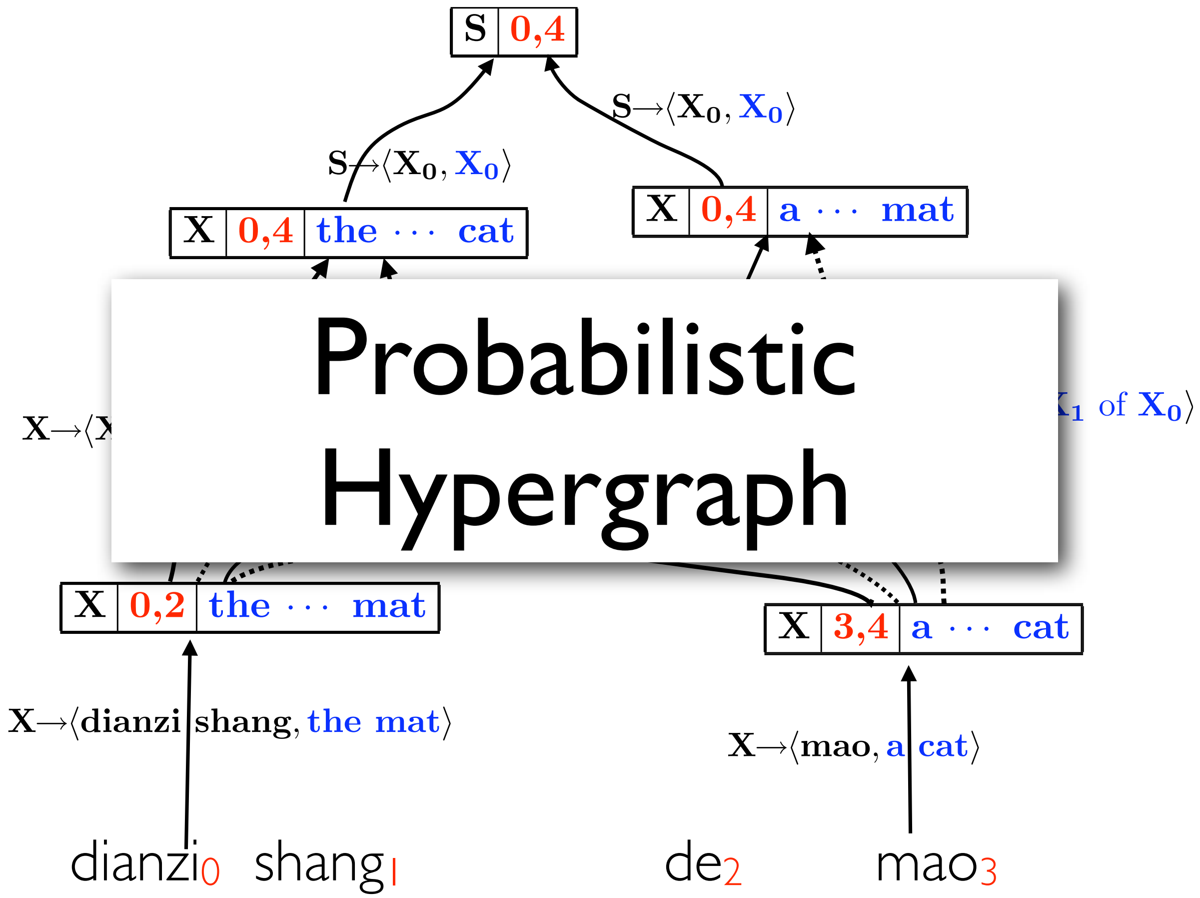
p=2/8



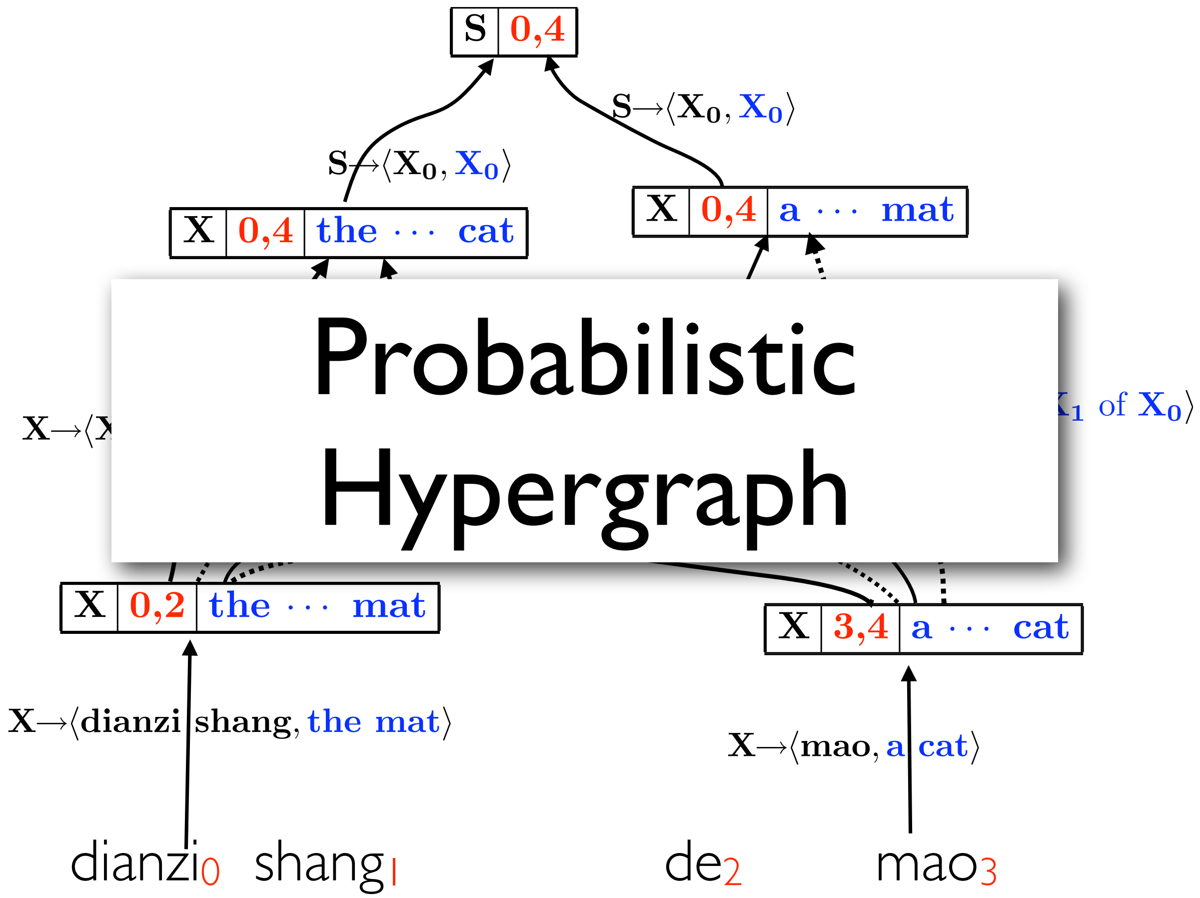
a cat of the mat



Probabilistic Hypergraph


Probabilistic Hypergraph
S 0,4
S→⟨X0, X0⟩
S→⟨X0, X0⟩
X 0,4 the ··· cat
X 0,4 a ··· mat
X→⟨X
X1 of X0⟩
X 0,2 the ··· mat
X 3,4 a ··· cat
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3

Entropy?



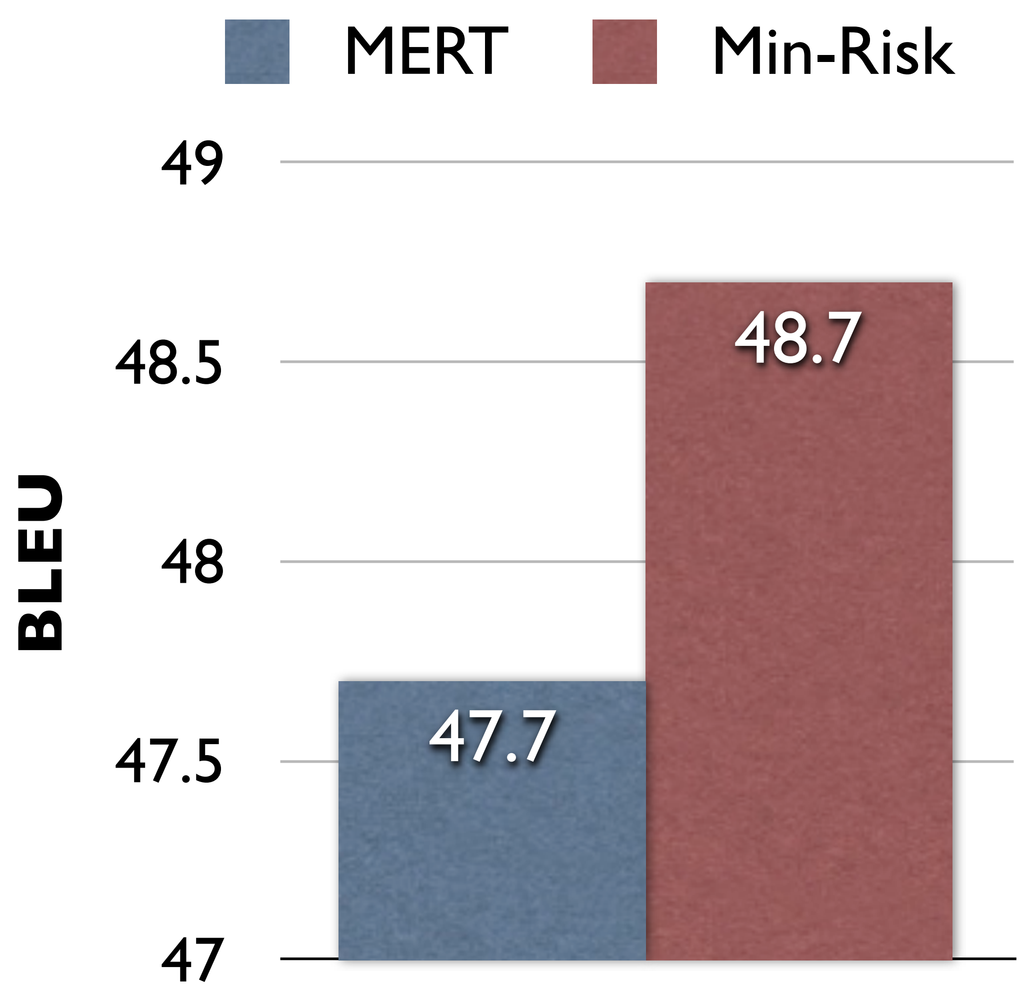
Entropy?

Bayes Risk?

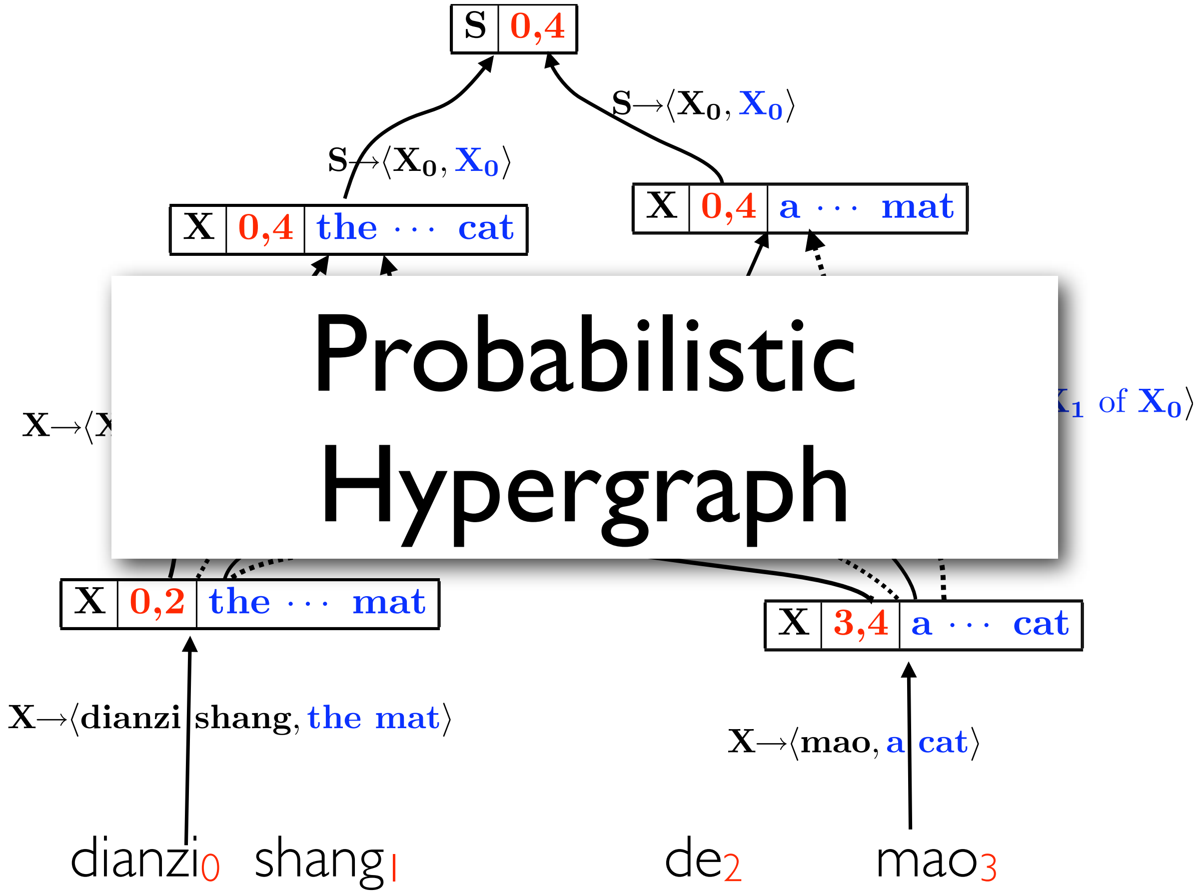
Minimum Risk Training on Translation Forests

Finding optimal parameters:

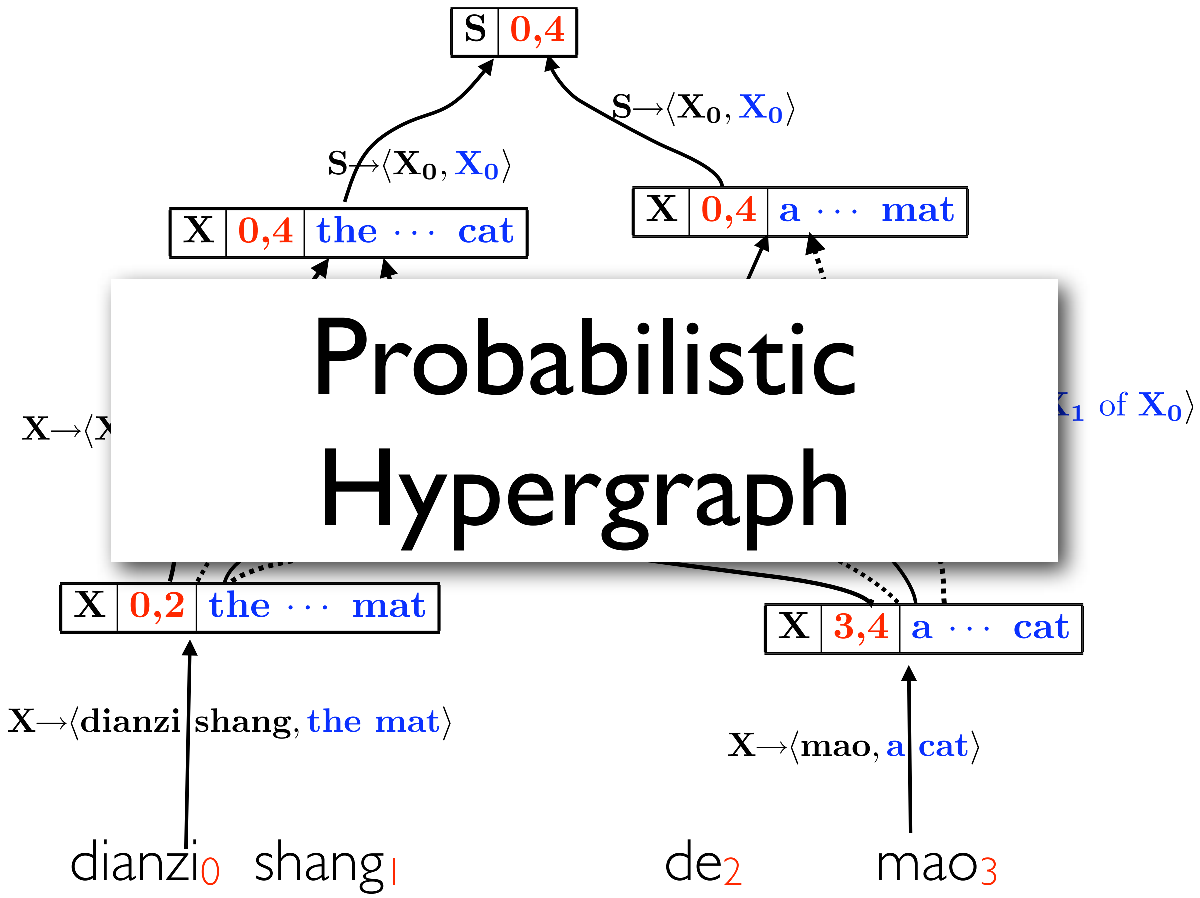
$$\theta^* = \textit{argmin}\ \text{Risk}(P_\theta) - \text{Temperature} \times \text{Entropy}(P_\theta)$$



BLEU scores on an IWSLT Chinese-English test set



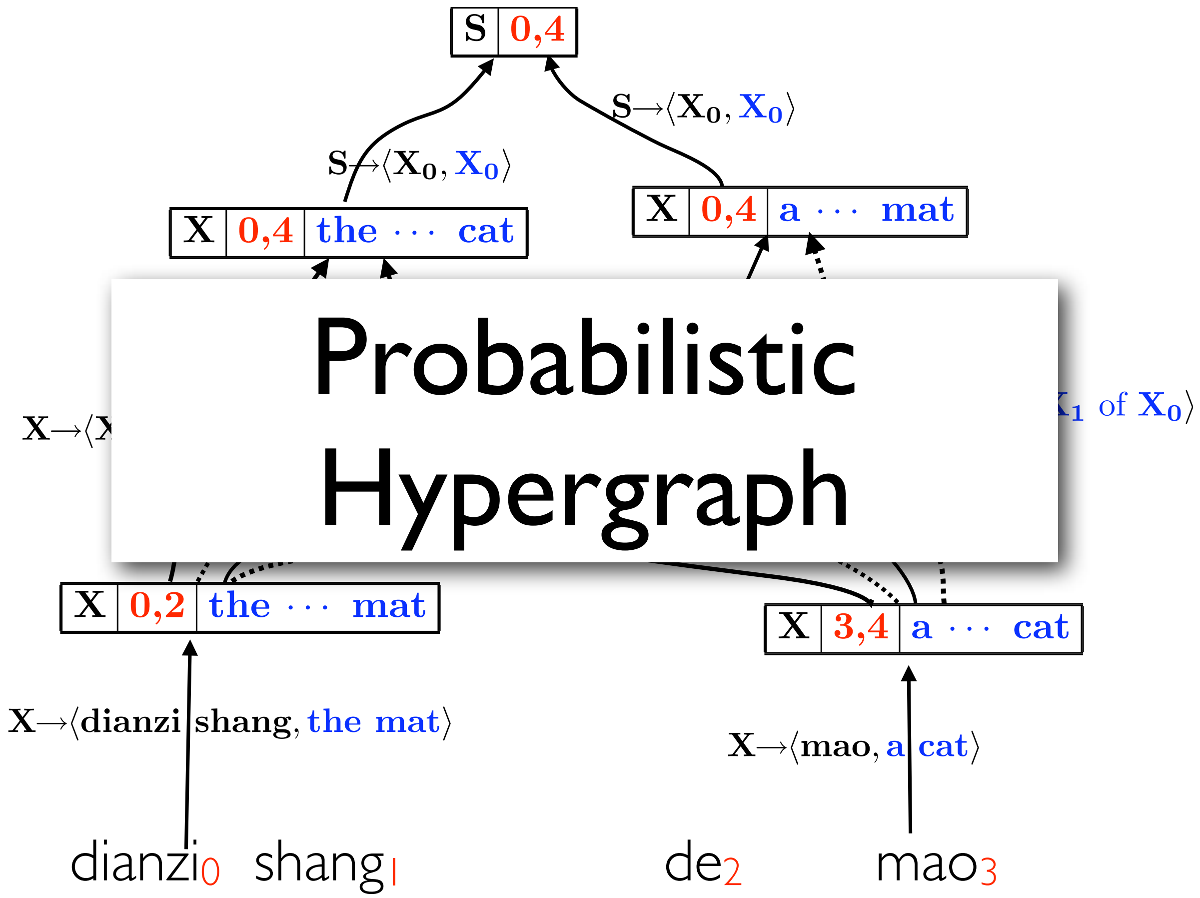
Probabilistic Hypergraph



Probabilistic Hypergraph

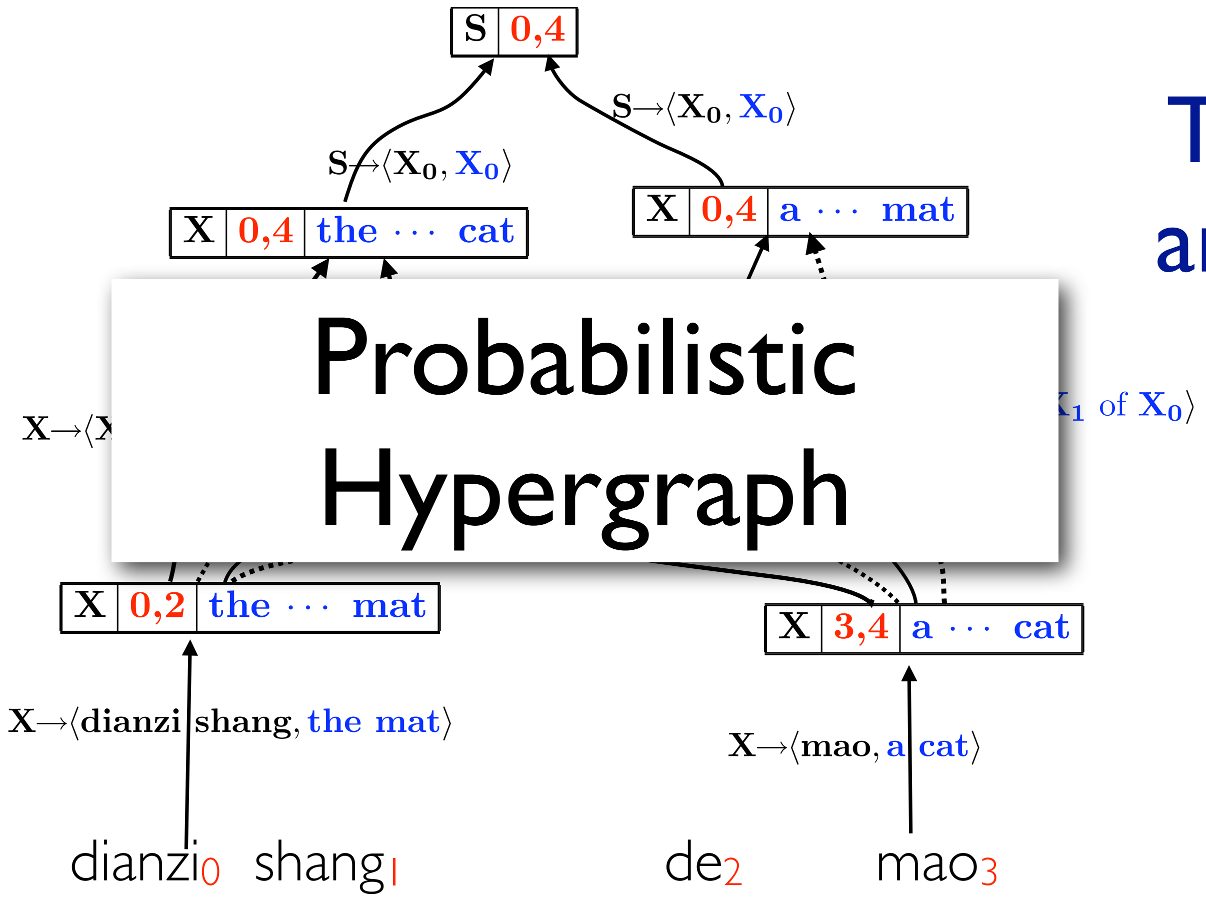
- First-order quantities:
  - expectation
    - entropy
    - Bayes risk
    - cross-entropy
    - KL divergence
    - feature expectations
  - first-order gradient of Z

Probabilistic Hypergraph



- First-order quantities:
  - expectation
    - entropy
    - Bayes risk
    - cross-entropy
    - KL divergence
    - feature expectations
  - first-order gradient of Z

- Second-order quantities:
  - Expectation over product
    - interaction between features
  - Hessian matrix of Z
    - second-order gradient descent
  - gradient of expectation
    - gradient of entropy or Bayes risk



This work provides a unified, elegant, and efficient framework in computing all of these!

- First-order quantities:
  - expectation
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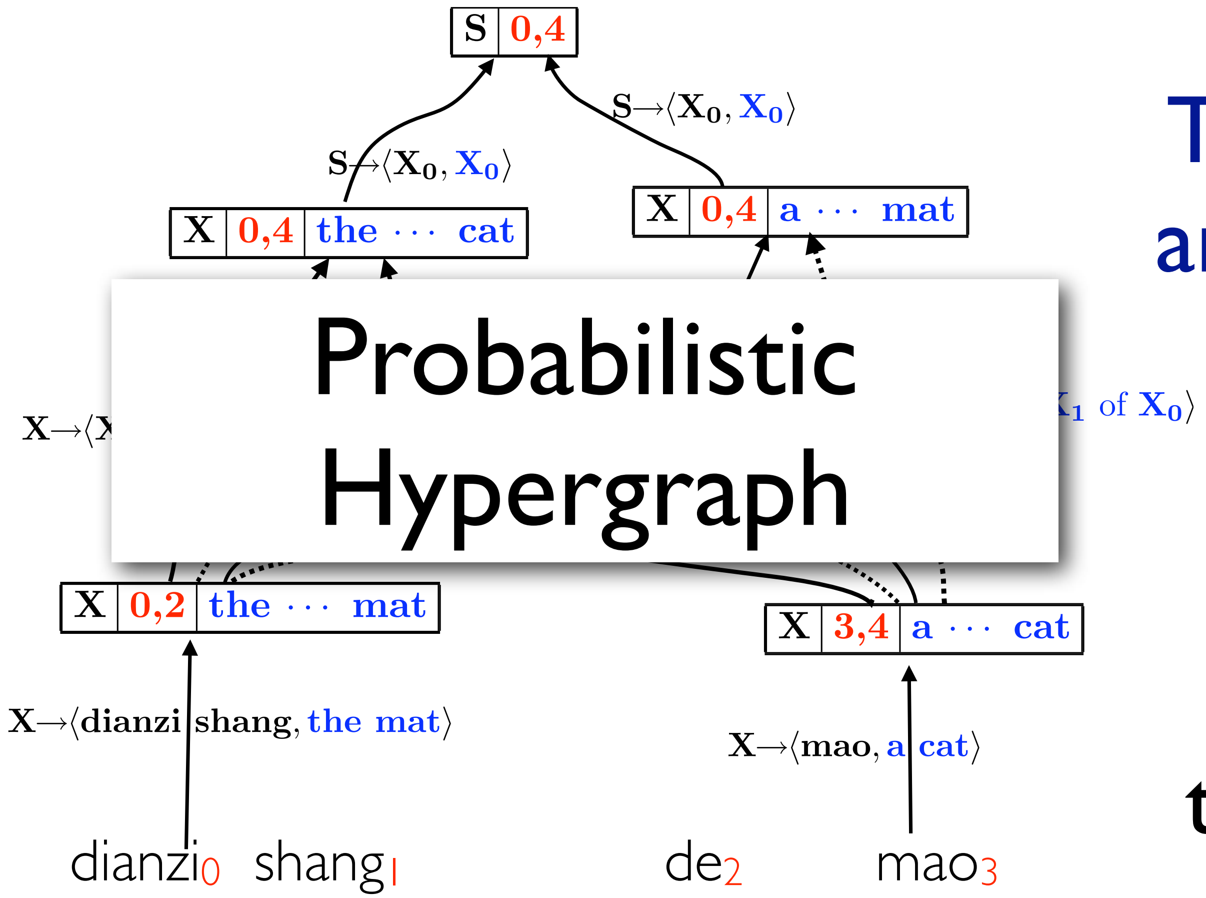

This work provides a unified, elegant, and efficient framework in computing all of these!

useful in applying machine learning techniques into machine translation

- First-order quantities:
  - expectation
    - entropy
    - Bayes risk
    - cross-entropy
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  - gradient of expectation
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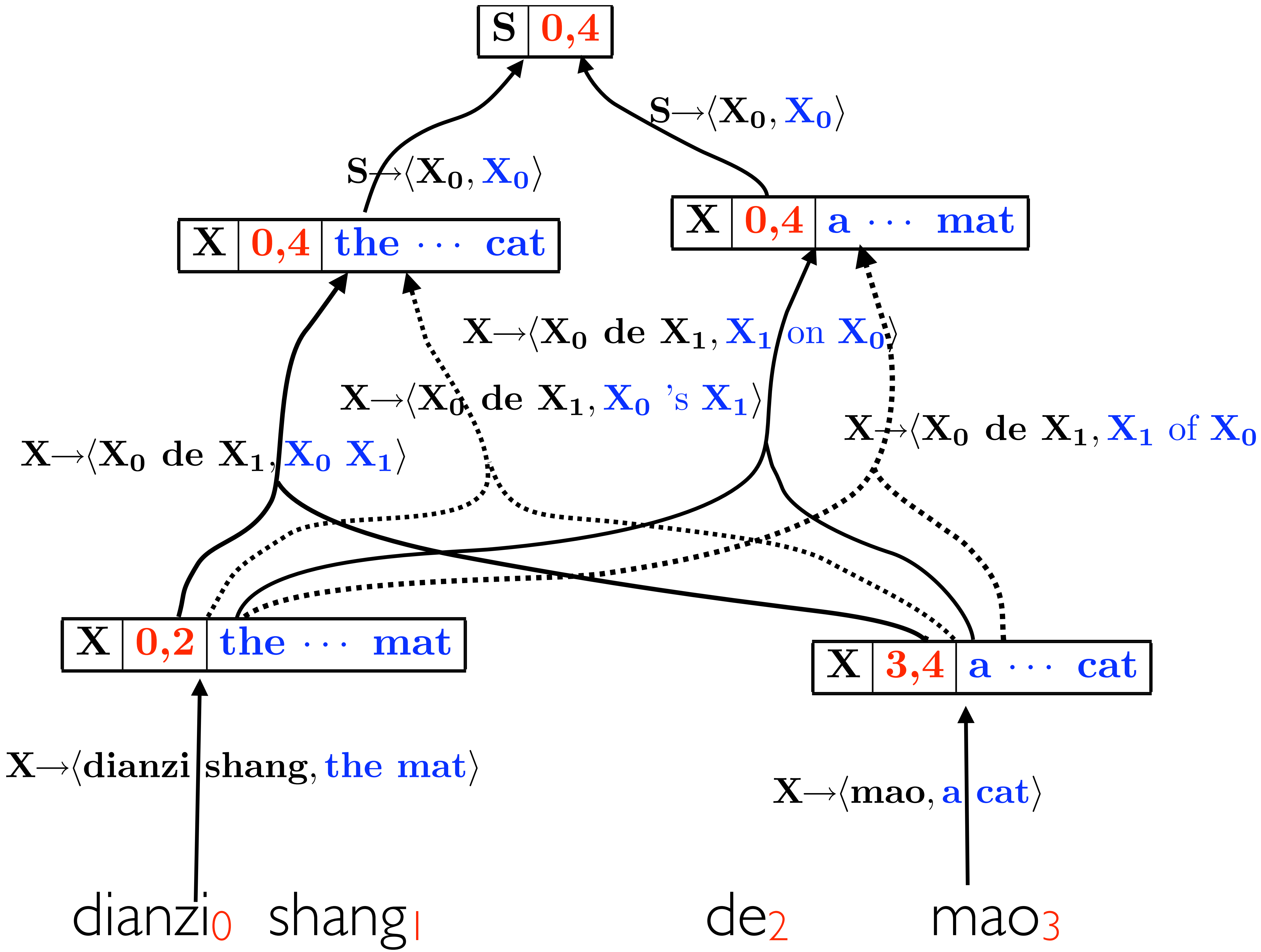
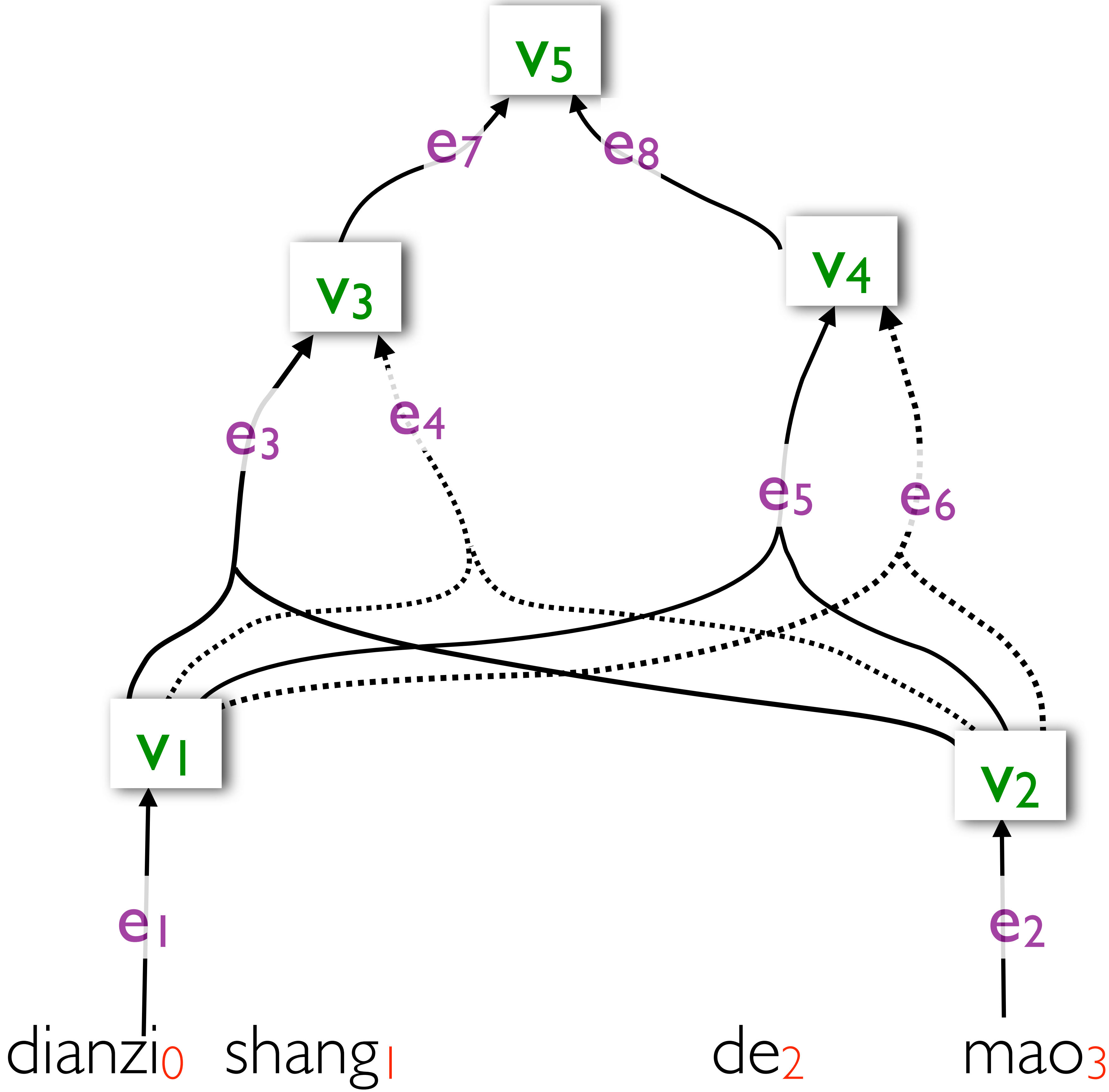
Outline

- Semiring-weighted Inside Algorithm
  - counting semiring (Goodman, 1999)
  - expectation semirings (Eisner, 2002)
  - second-order expectation semirings (new)
- Applications of the Semirings (new)
- Speed-up with Inside-outside (new)
- Minimum Risk Training over **Forests** (new)

Semiring-weighted Inside Algorithm



S 0,4
S→⟨X0, X0⟩
S→⟨X0, X0⟩
X 0,4 the ··· cat
X 0,4 a ··· mat
X→⟨X0 de X1, X1 on X0⟩
X→⟨X0 de X1, X0 's X1⟩
X→⟨X0 de X1, X0 X1⟩
X→⟨X0 de X1, X1 of X0⟩
X 0,2 the ··· mat
X 3,4 a ··· cat
X→⟨dianzi shang, the mat⟩
X→⟨mao, a cat⟩
dianzi0 shang1 de2 mao3


v5
e7
e8
v3
v4
e3
e4
e5
e6
v1
v2
e1
e2
dianzi0 shang1 de2 mao3

What is a Semiring?

What is a Semiring?

$$\langle K, \oplus, \otimes \rangle$$

a **set** with **plus** and **times** operations

Compute the Number of Derivation Trees

Compute the Number of Derivation Trees

Counting semiring: ordinary integers

Compute the Number of Derivation Trees

Counting semiring: ordinary integers

Inside algorithm:

Compute the Number of Derivation Trees

Counting semiring: ordinary integers

Inside algorithm:

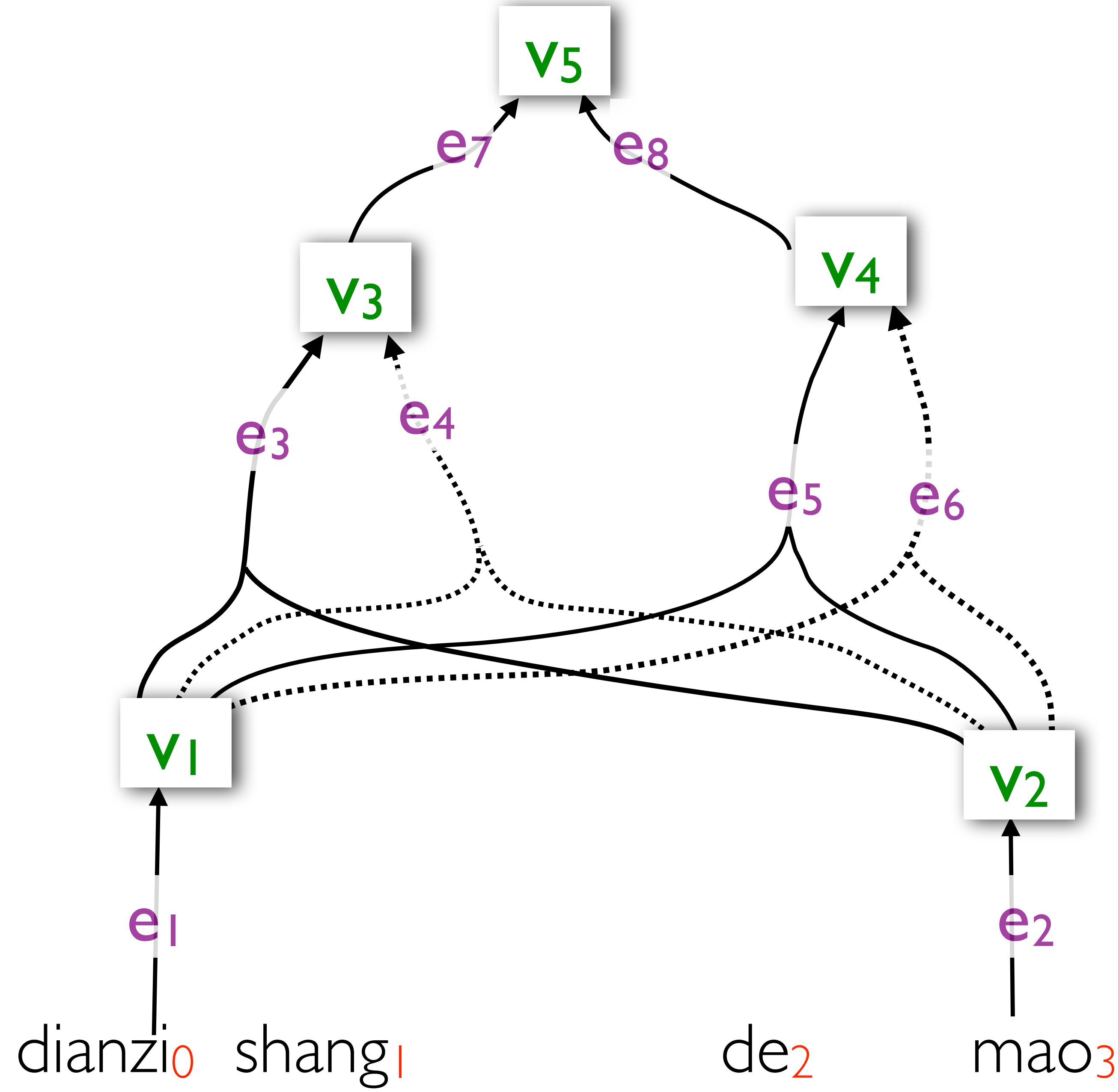
Inputs:

Compute the Number of Derivation Trees

Counting semiring: ordinary integers

Inside algorithm:

Inputs:
- a hypergraph

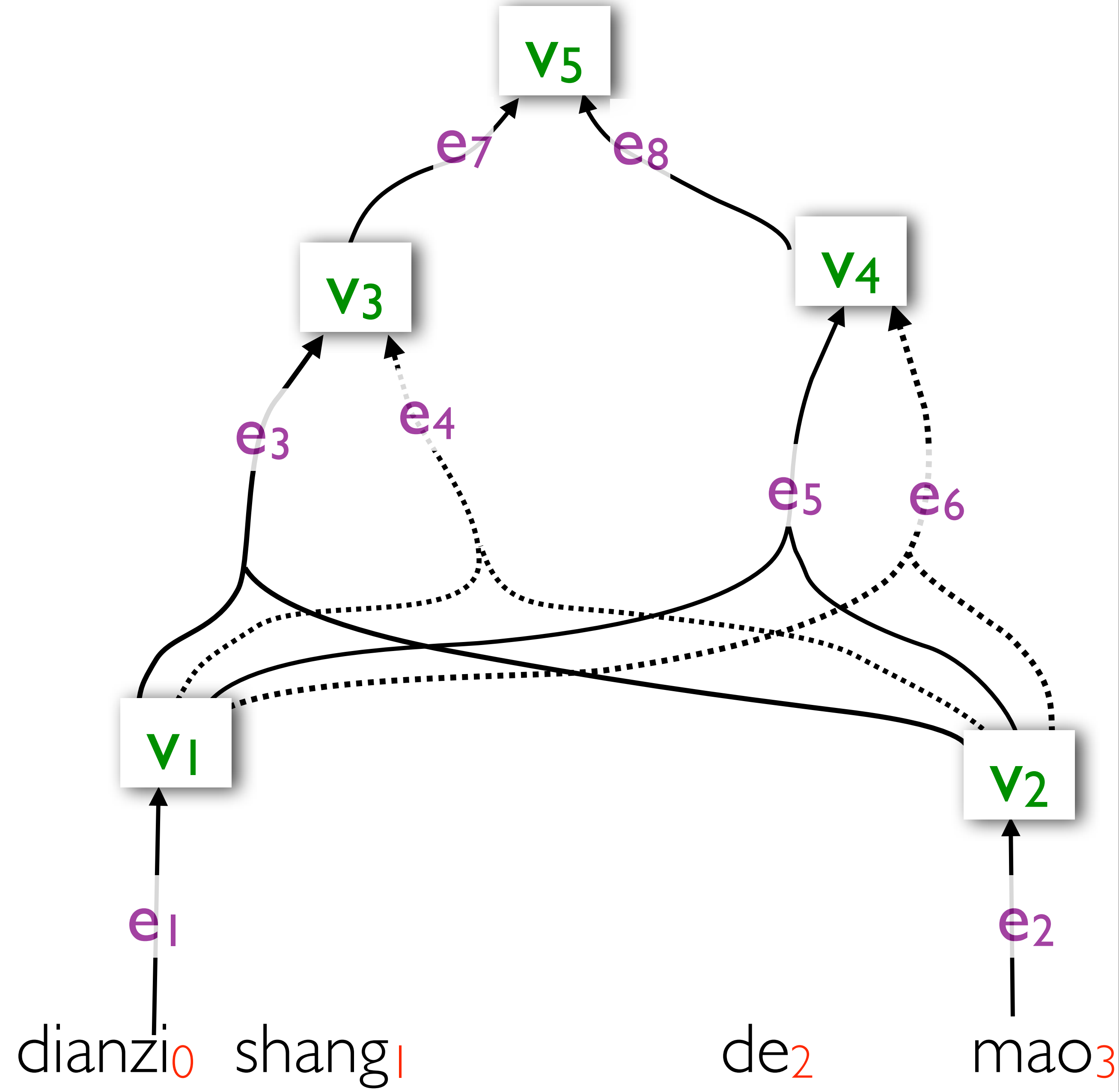


Compute the Number of Derivation Trees

Counting semiring: ordinary integers

Inside algorithm:

Inputs:
- a hypergraph
- a weight for each hyperedge



Compute the Number of Derivation Trees

Counting semiring: ordinary integers

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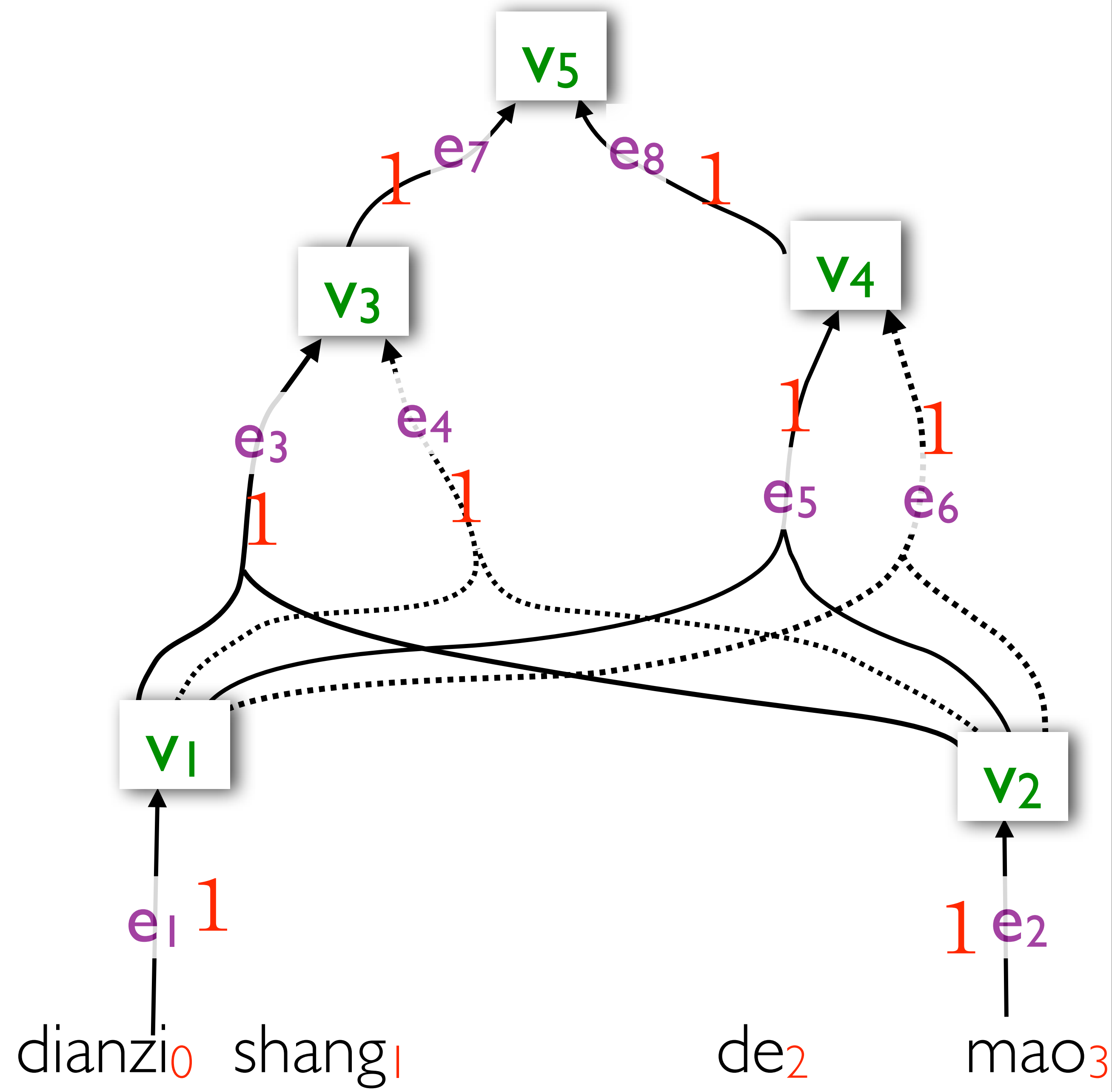

Compute the Number of Derivation Trees

Counting semiring: ordinary integers

Inside algorithm:

Inputs:
- a hypergraph
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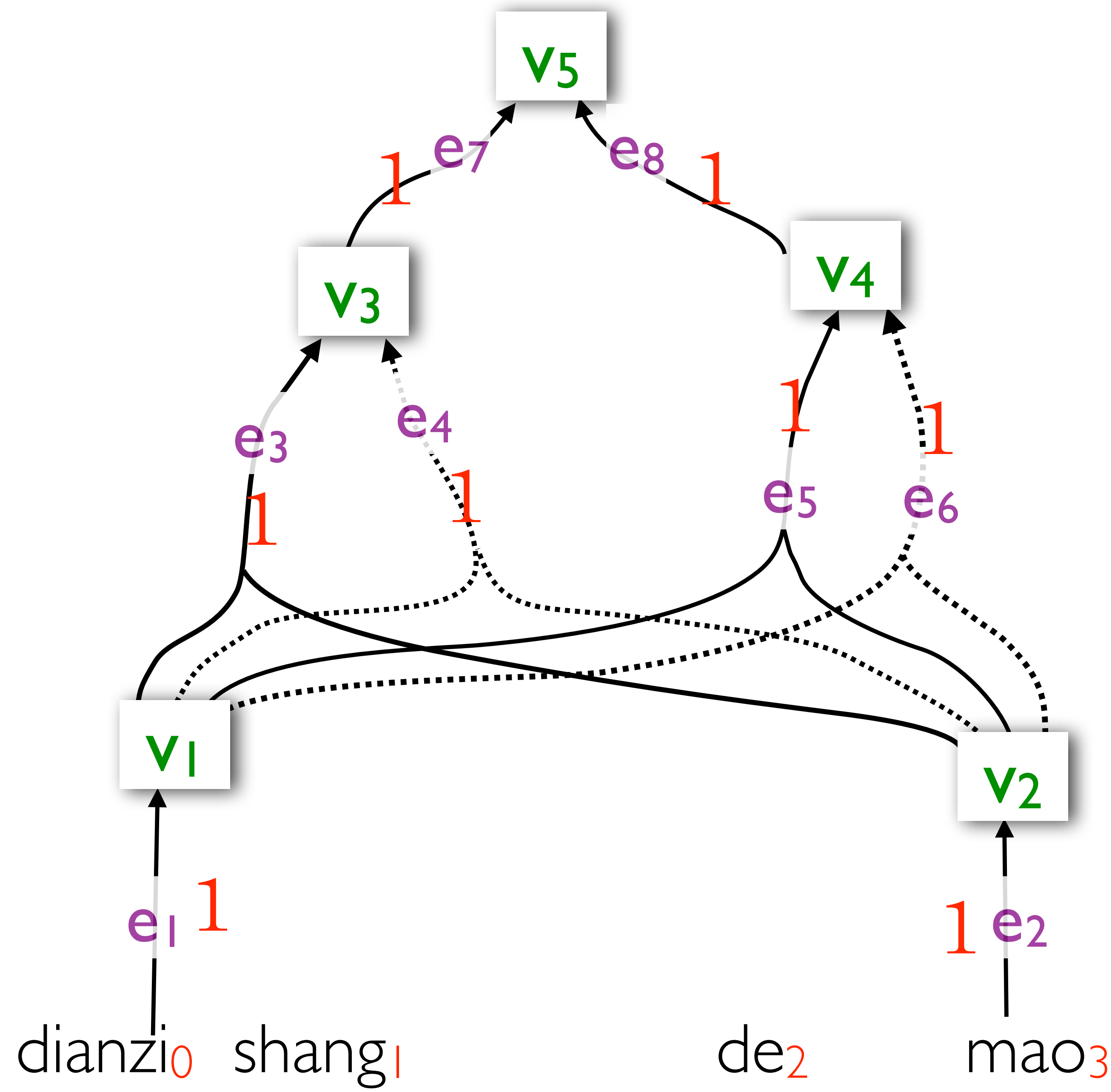
Output: k(root)



Compute the Number of Derivation Trees

Counting semiring: ordinary integers

Inside algorithm:

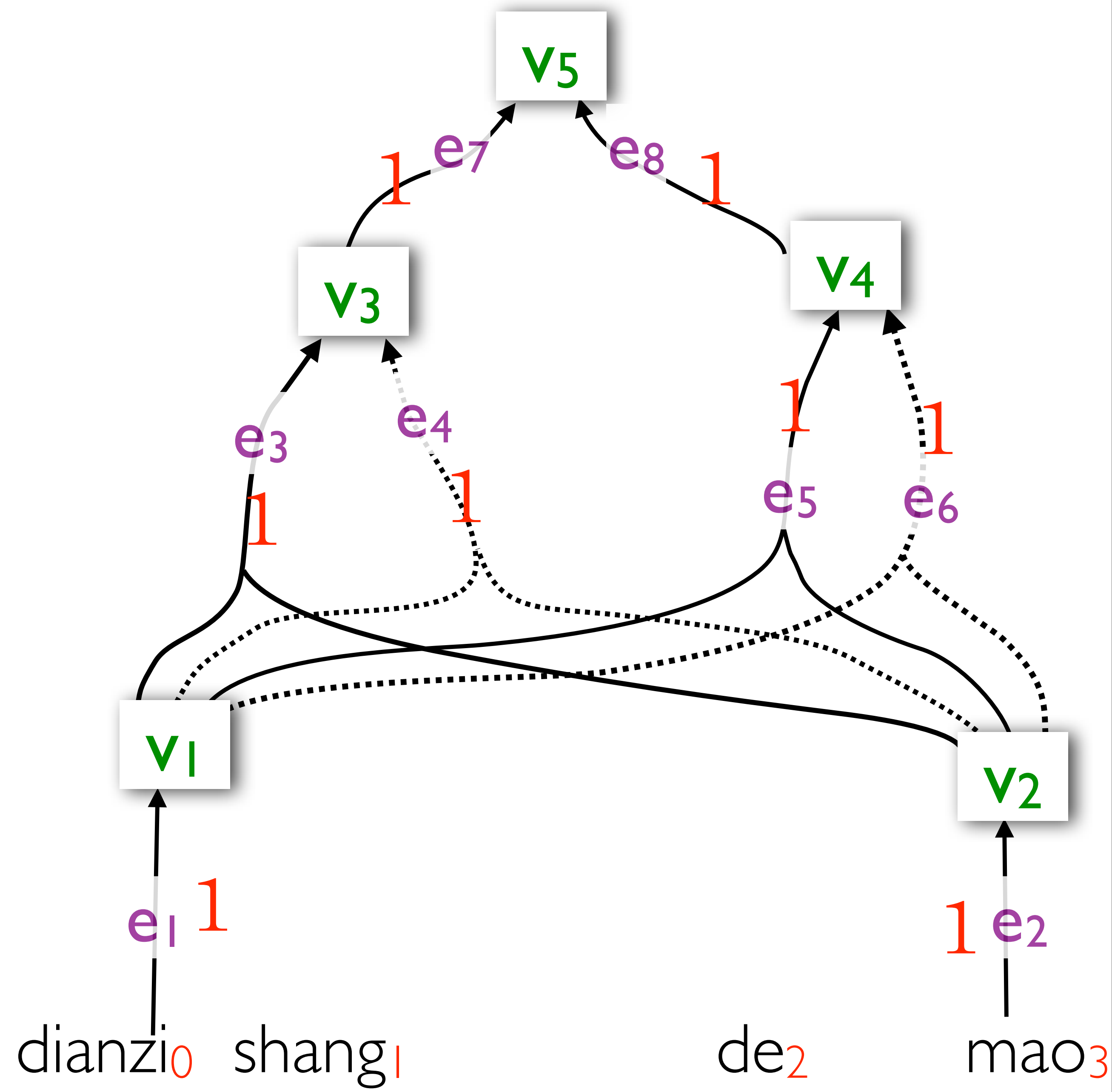
Inputs:

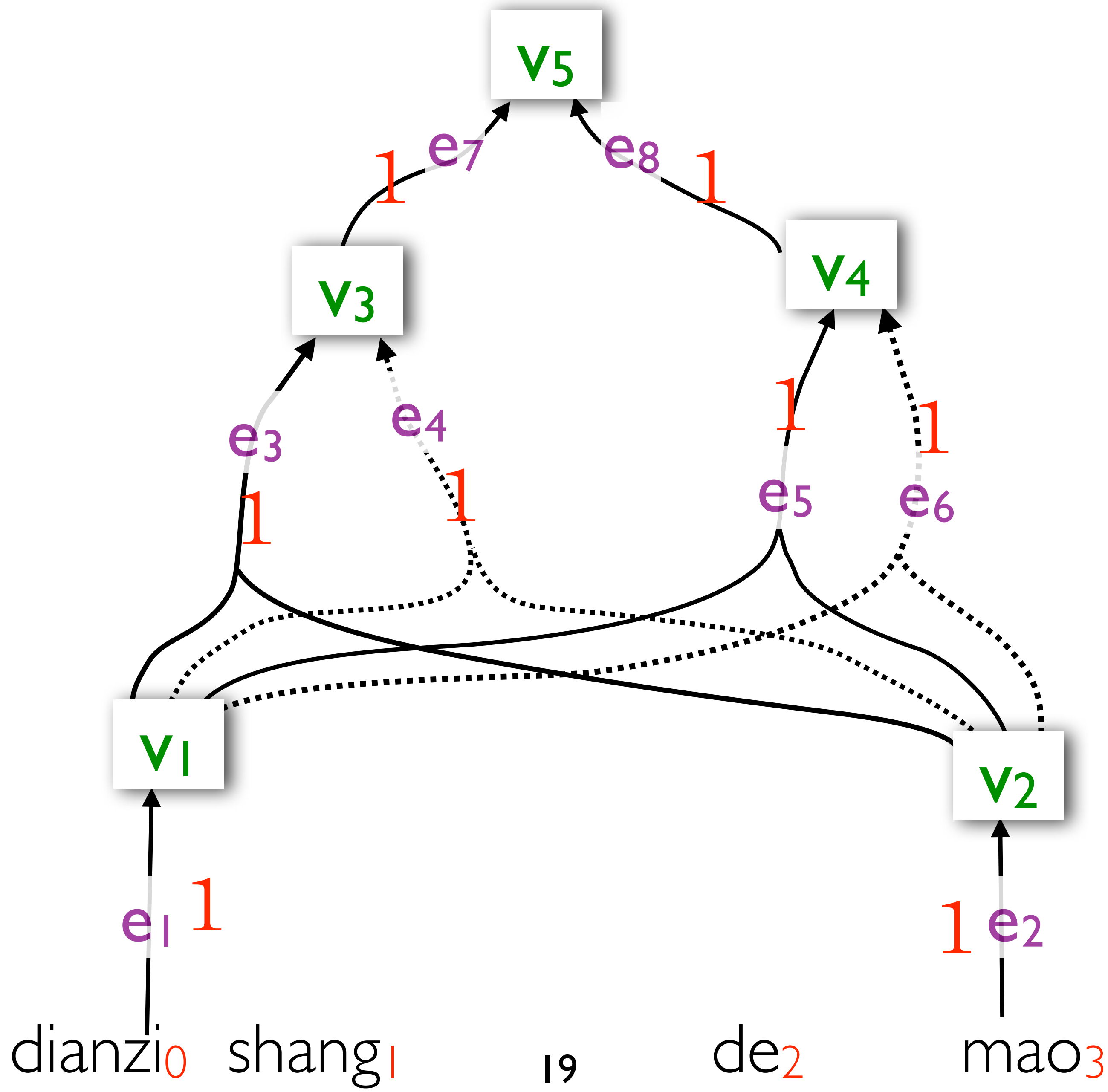
- a hypergraph
- a weight for each hyperedge

Output: k(root)

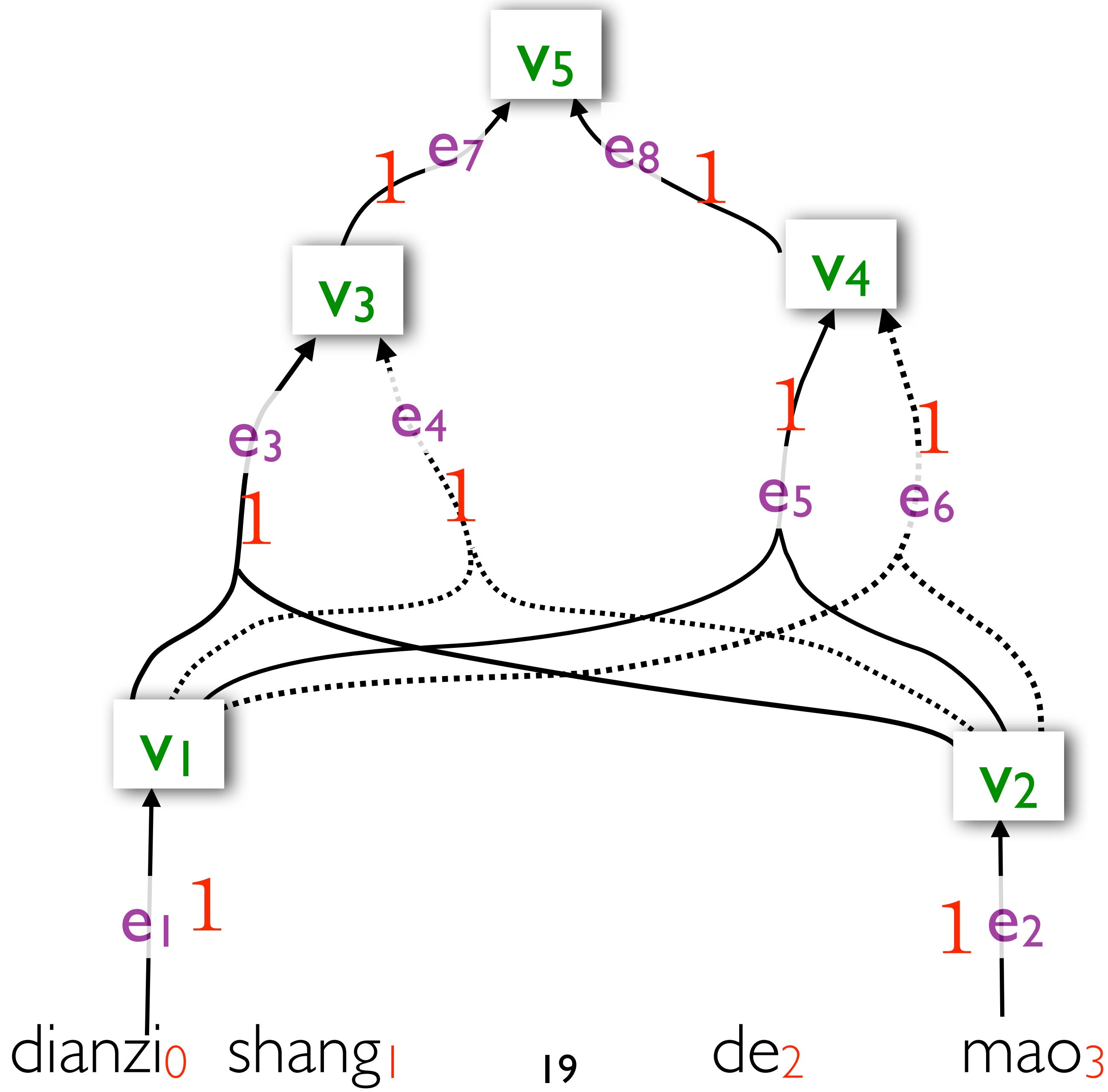
Complexity:

O(size of the hypergraph)

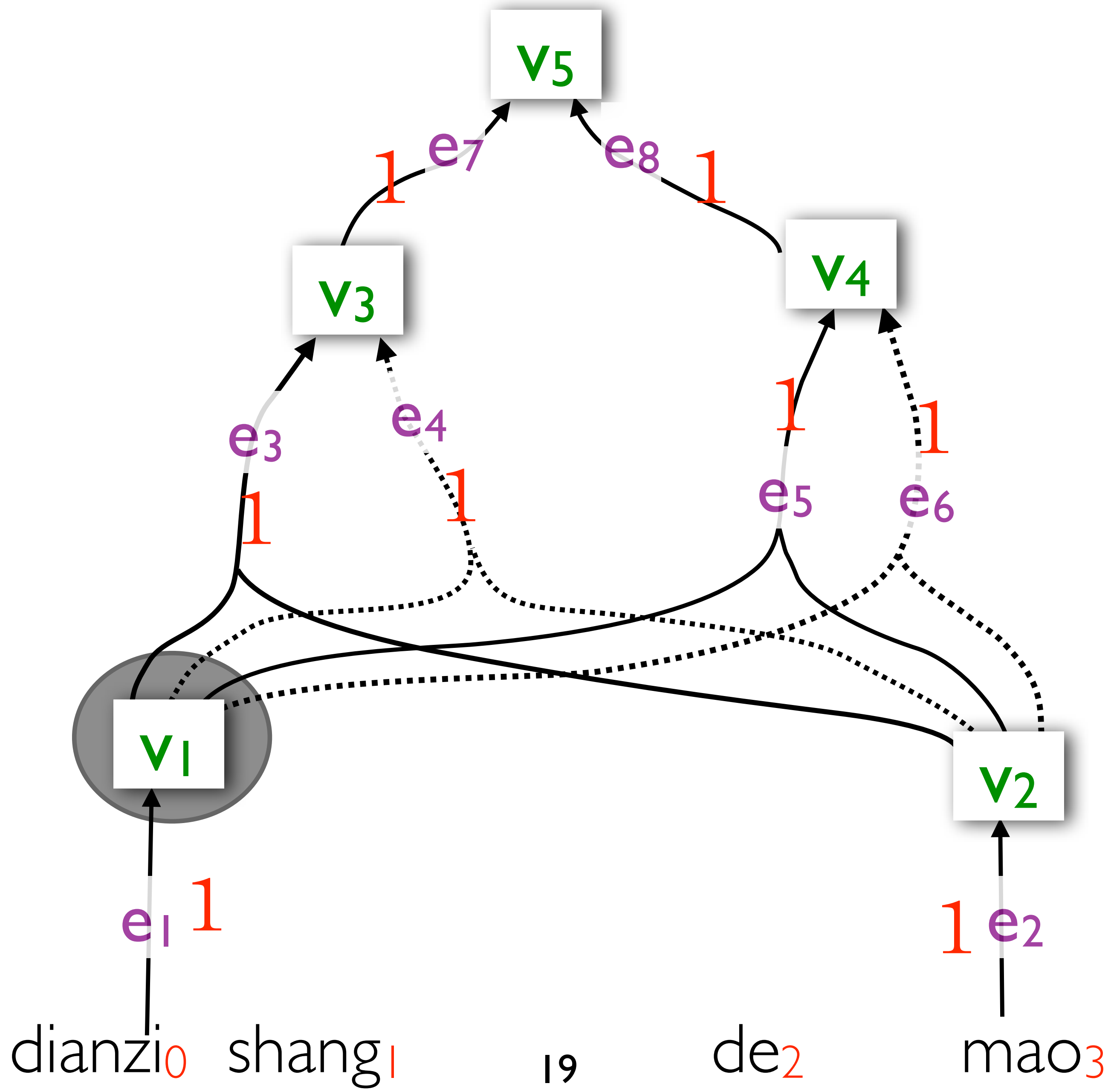



v5
e7
1
e8
1
v3
v4
e3
1
e4
1
1
e5
1
e6
v1
v2
e1
1
1
e2
dianzi0
shang1
de2
mao3

Bottom-up process in computing the number of trees

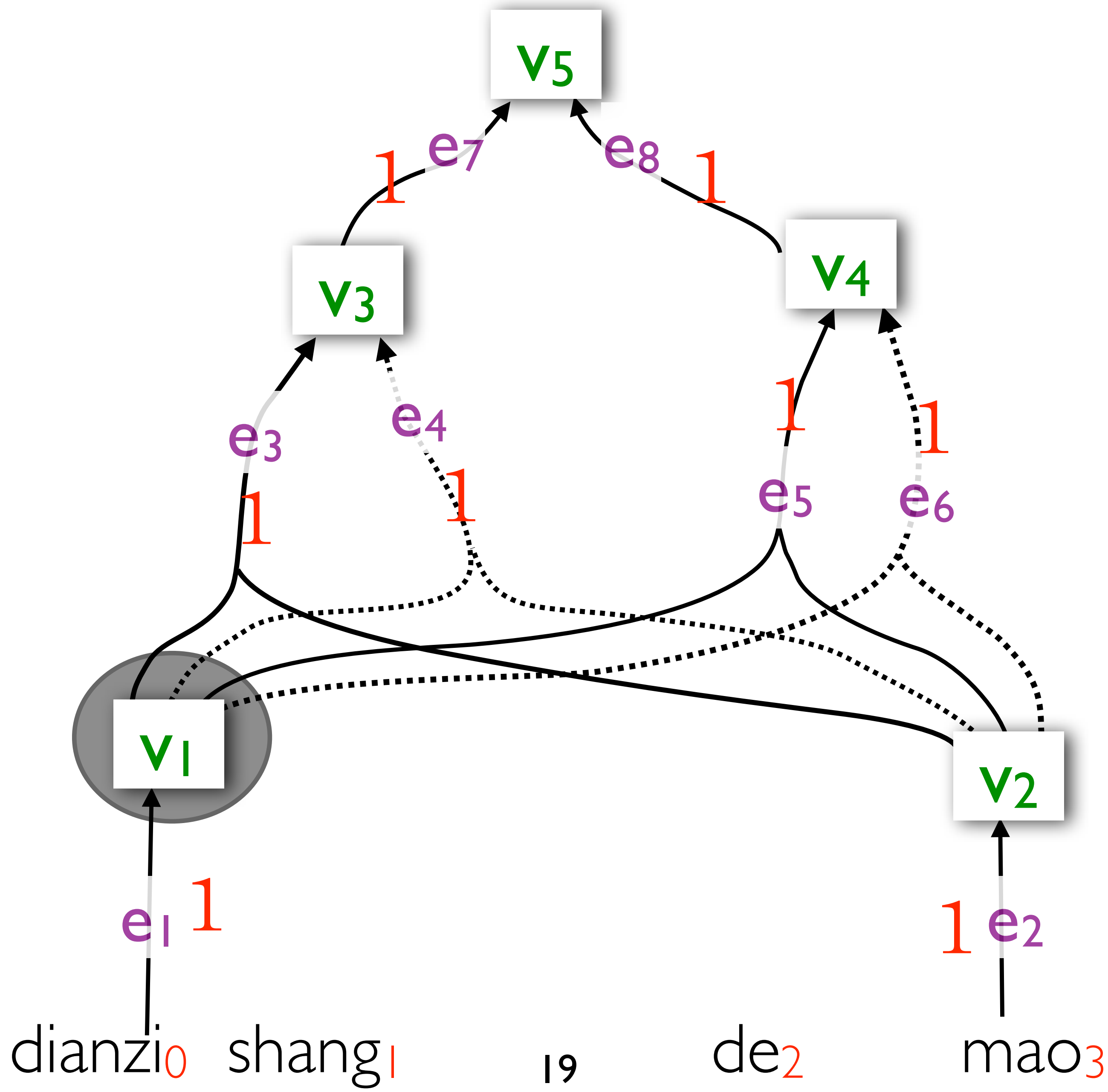


Bottom-up process in computing the number of trees



$k(v_1) = k(e_1)$

Bottom-up process in computing the number of trees



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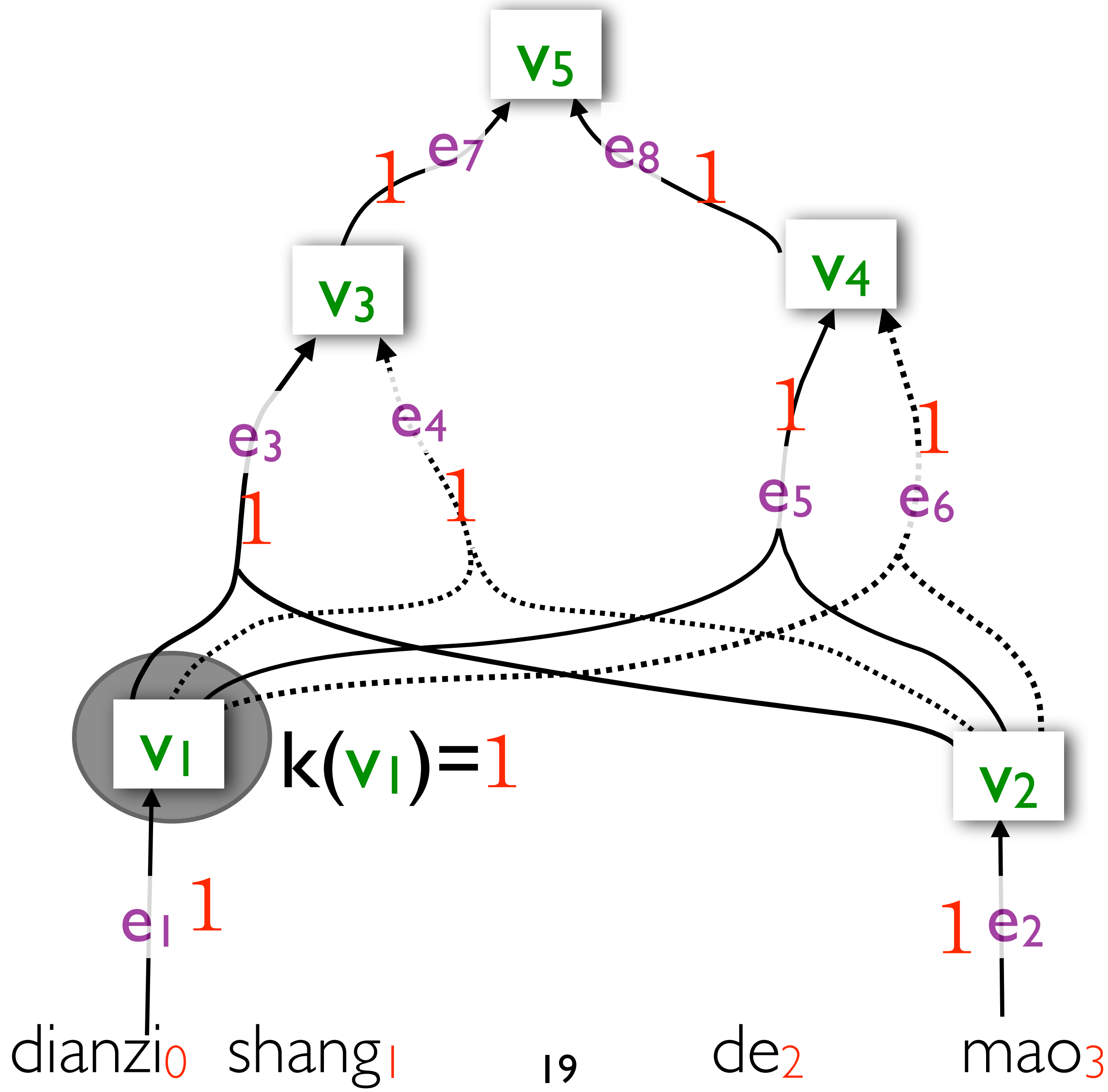
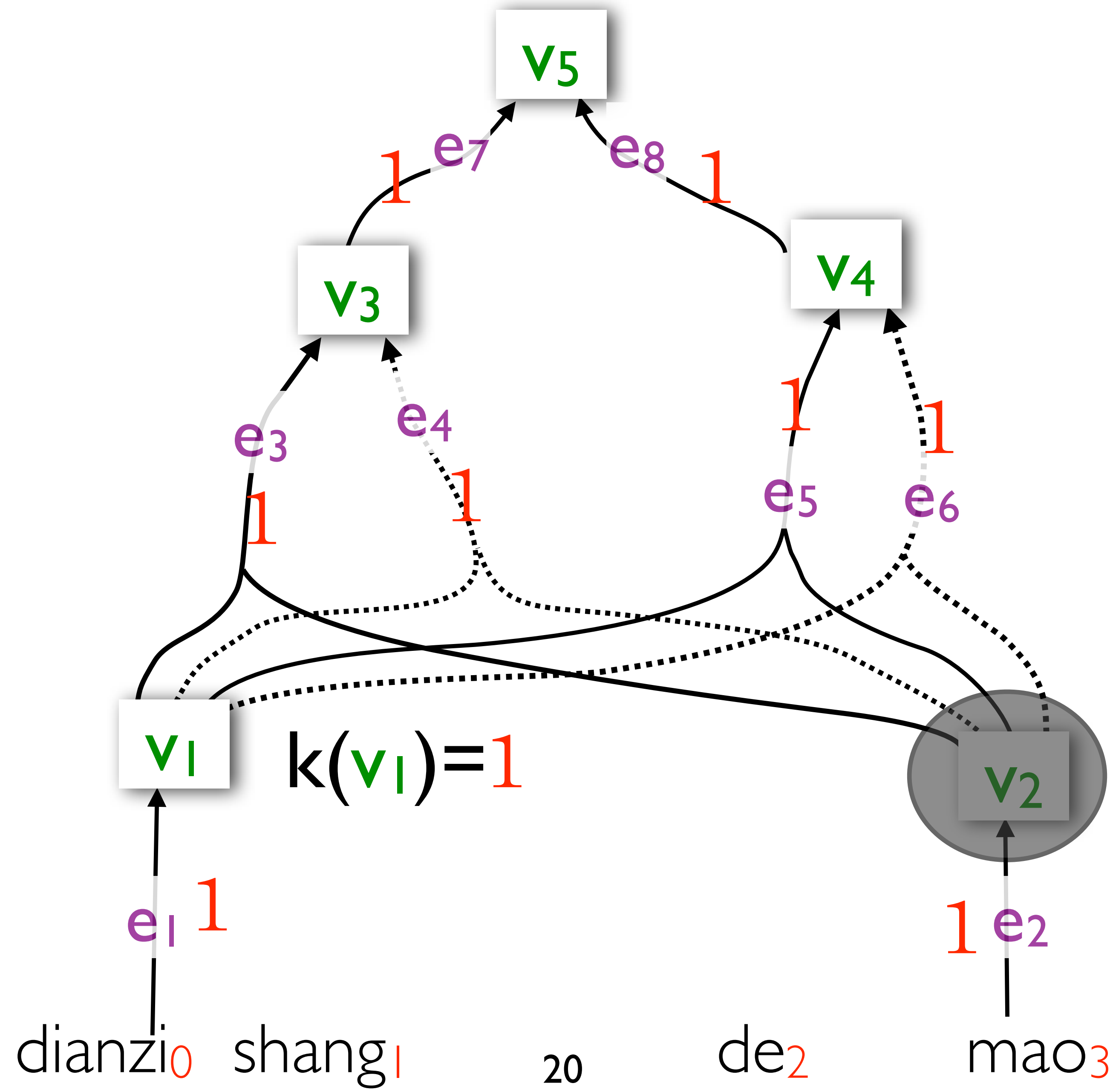
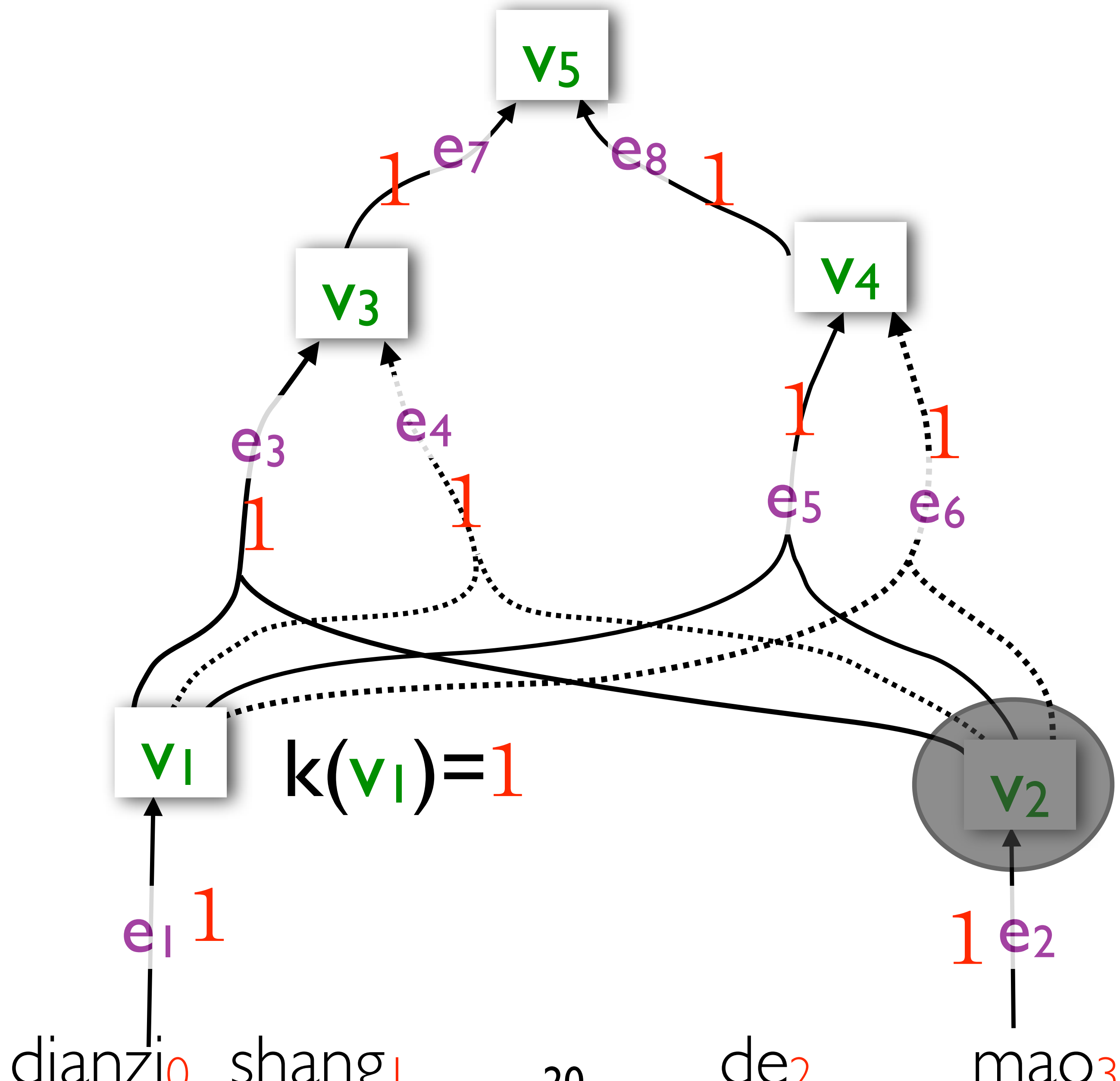


$k(v_1) = k(e_1)$



Bottom-up process in computing the number of trees

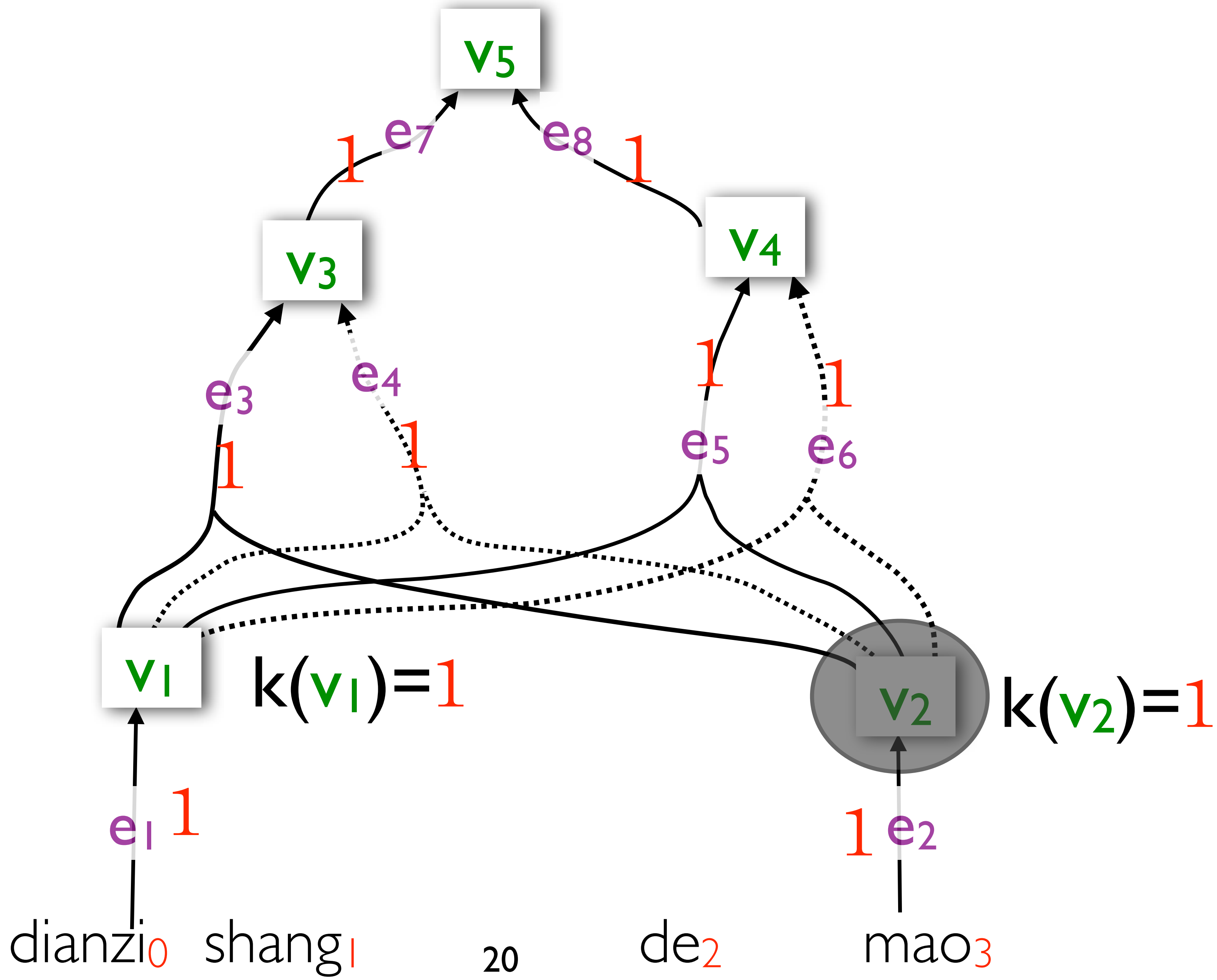
$k(v_1) = k(e_1)$ $\quad$ $k(v_2) = k(e_2)$



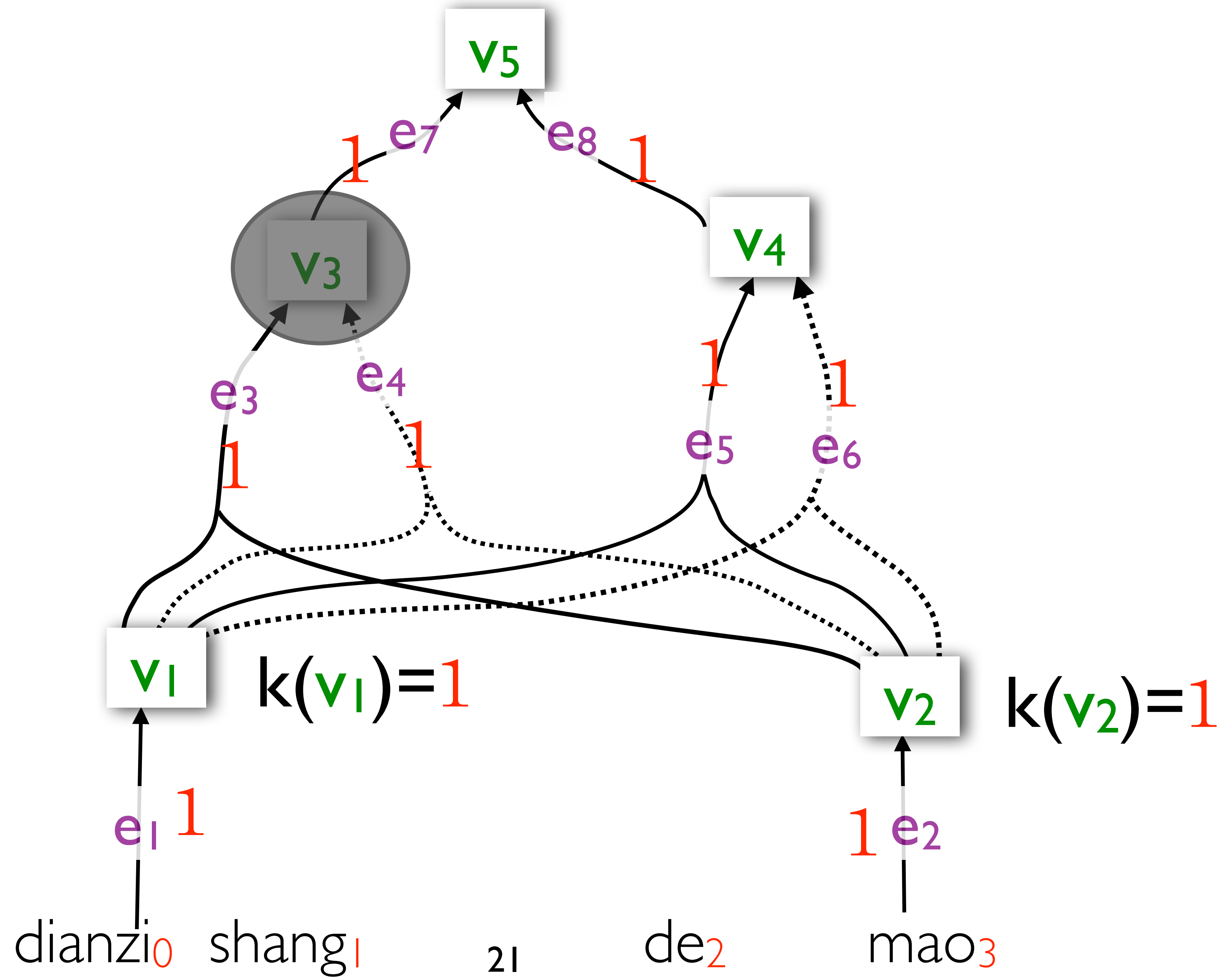
Bottom-up process in computing the number of trees

$k(v_1) = k(e_1)$ $k(v_2) = k(e_2)$

Bottom-up process in computing the number of trees



Compute $k(v_3)$: the weight at node v_3

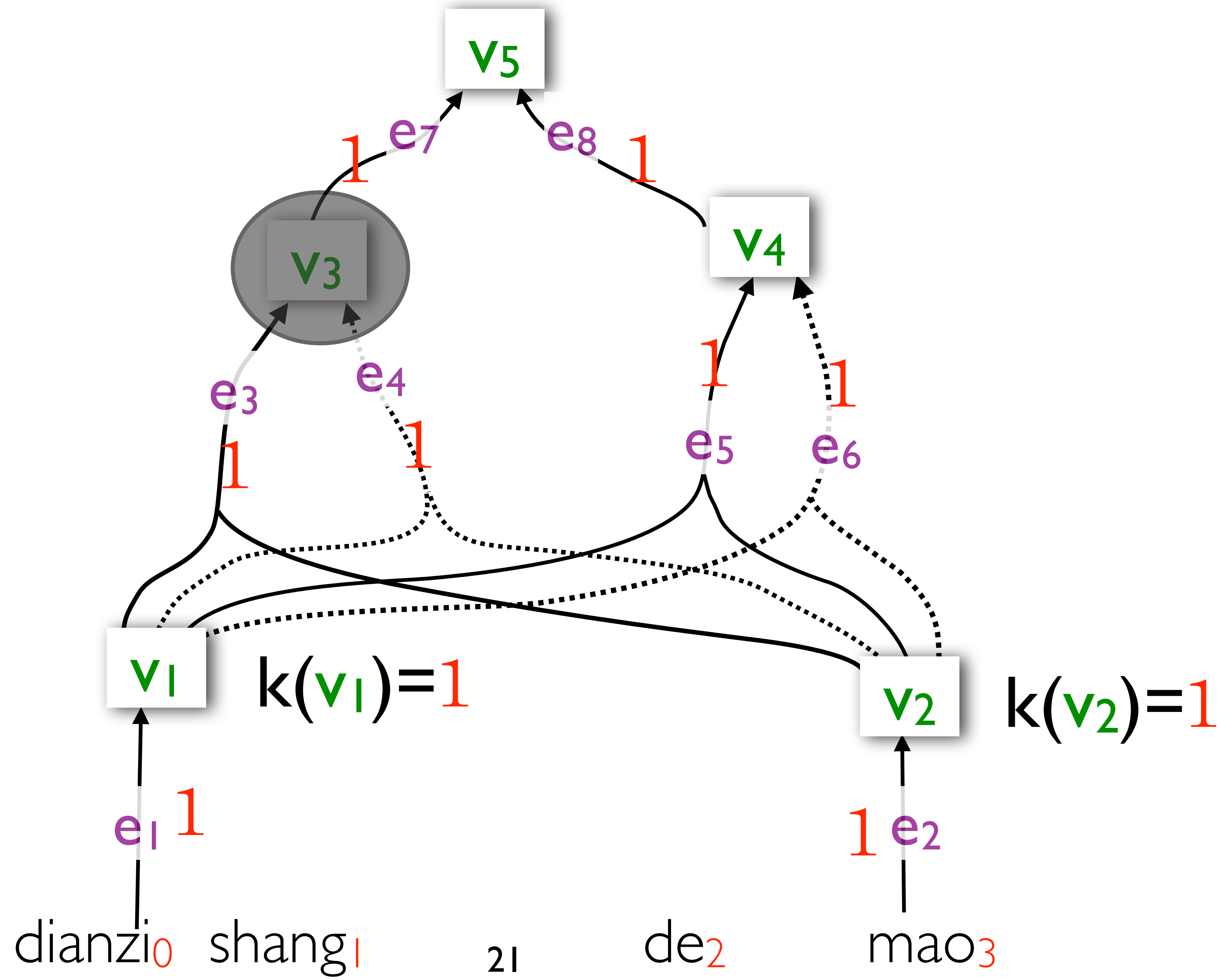


Bottom-up process in computing the number of trees

Compute $k(v_3)$: the weight at node v_3

Hyperedge e_3:

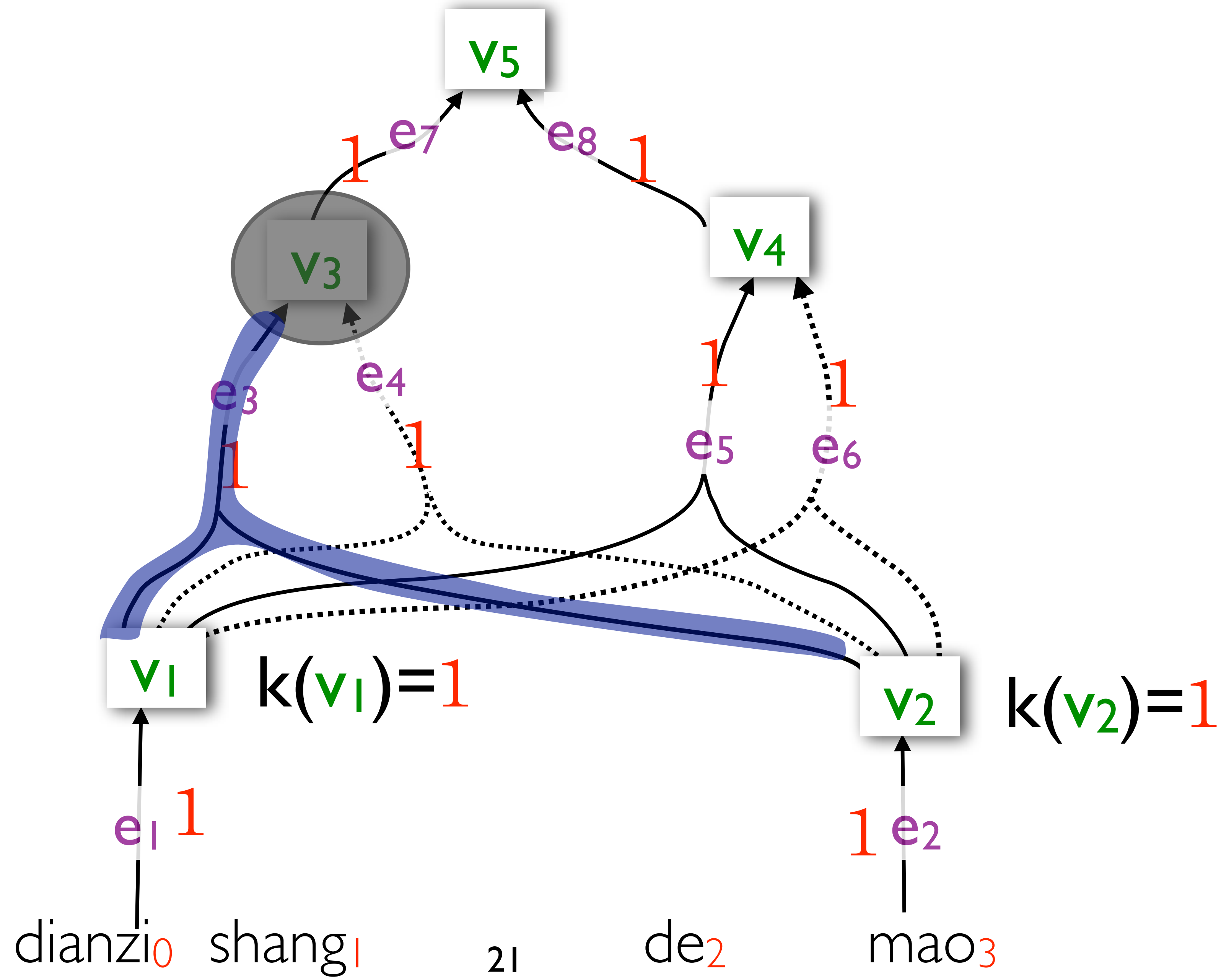
Bottom-up process in computing the number of trees



Compute $k(v_3)$: the weight at node v_3

Hyperedge e_3:

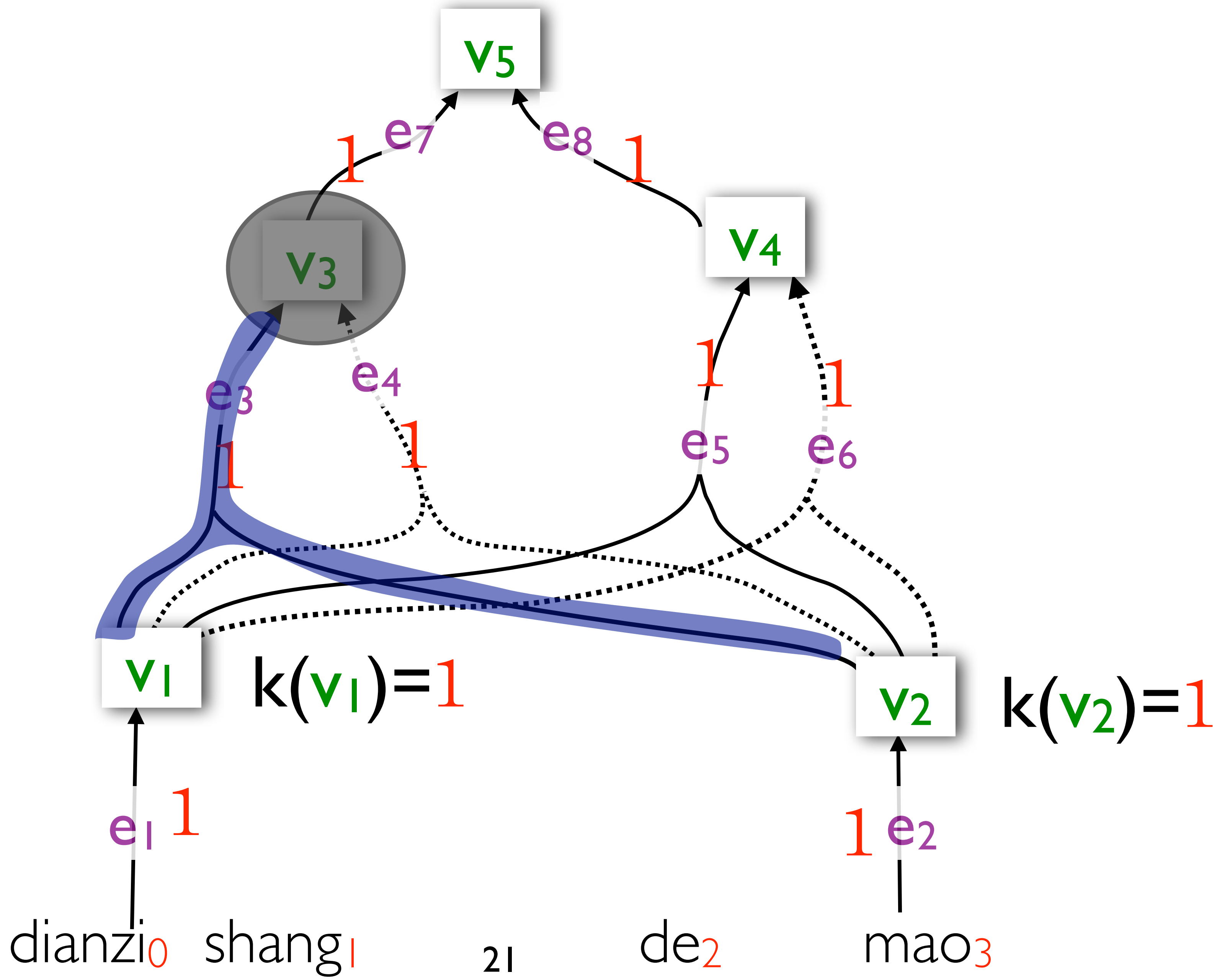
Bottom-up process in computing the number of trees



Compute k(v_3): the weight at node v_3

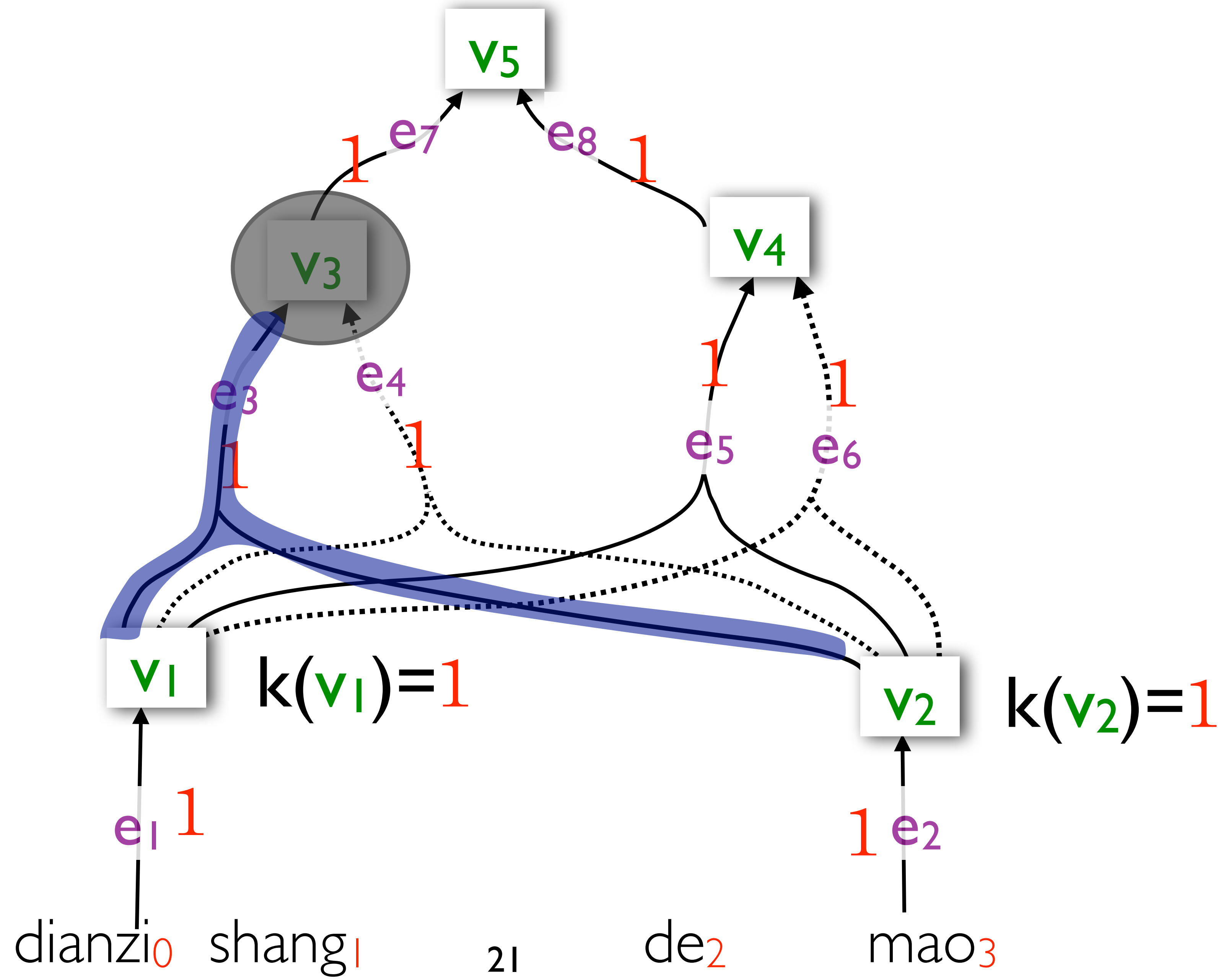
Hyperedge e_3: $k(e_3) \otimes k(v_1) \otimes k(v_2)$

Bottom-up process in computing the number of trees



Compute $k(v_3)$: the weight at node v_3

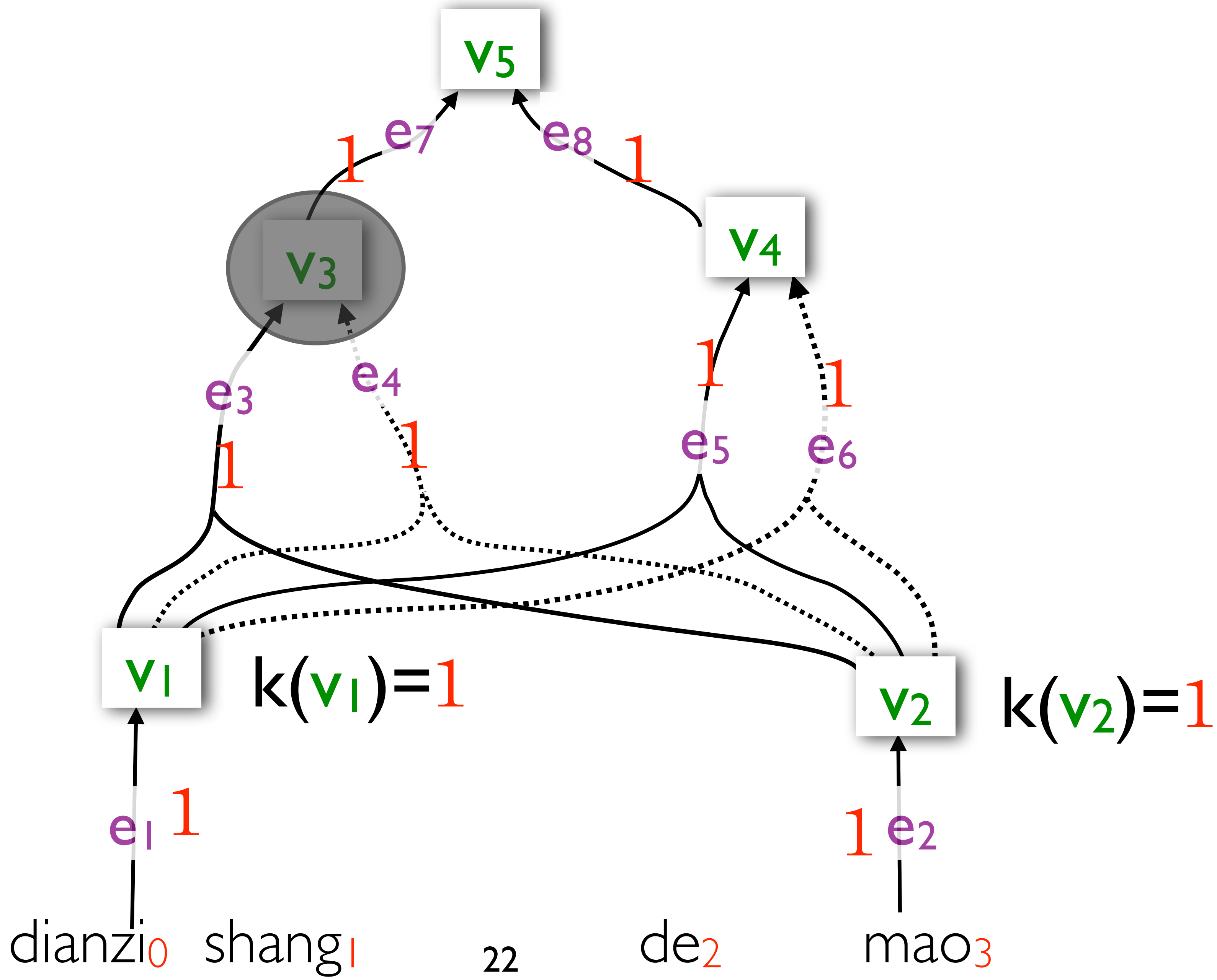
Hyperedge e_3: $k(e_3) \bigotimes k(v_1) \bigotimes k(v_2) = 1 \otimes 1 \otimes 1 = 1$



Bottom-up process in computing the number of trees

Compute k(v3): the weight at node v3

Bottom-up process in computing the number of trees

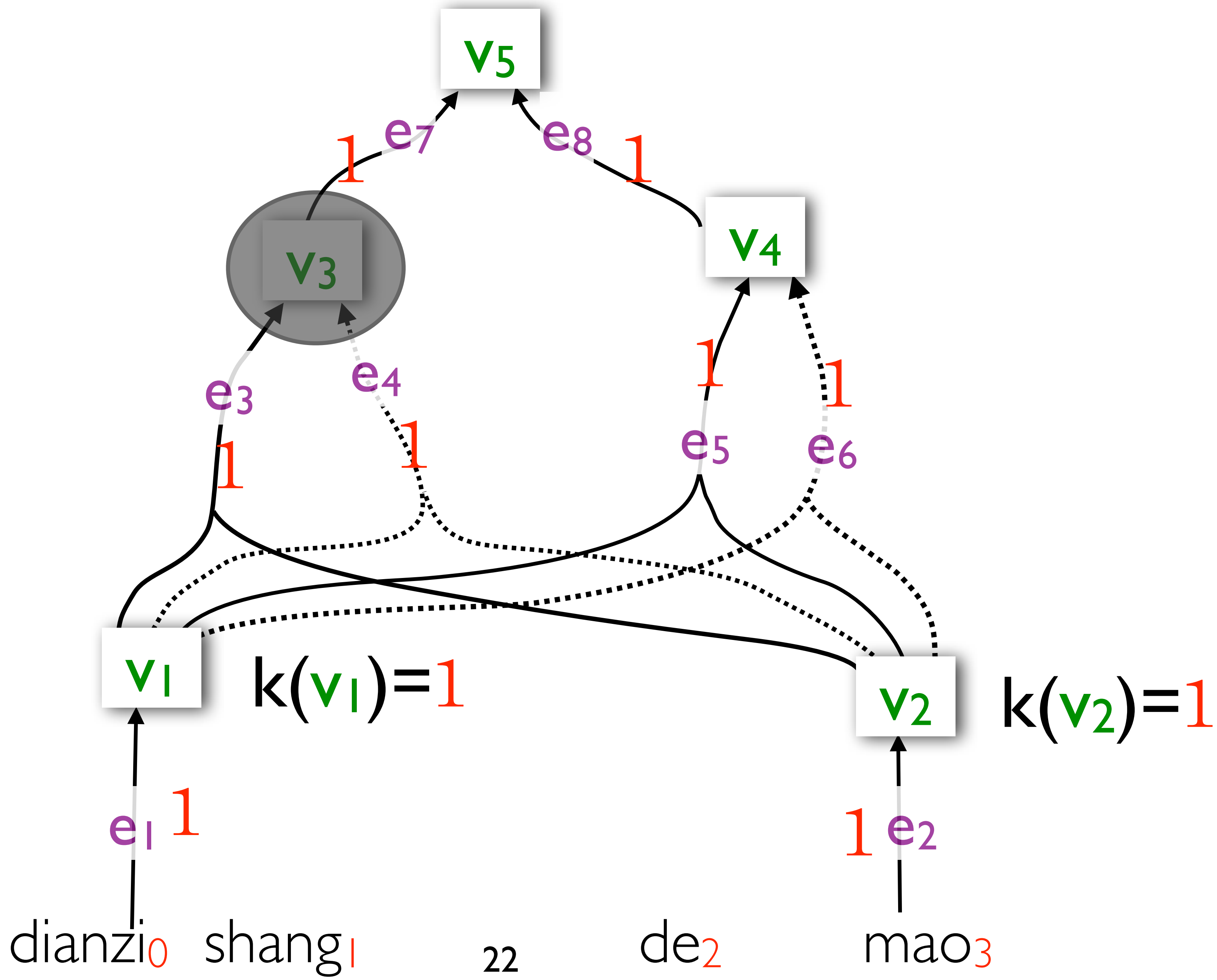


Wednesday, August 19, 2009

Compute $k(v_3)$: the weight at node v_3

Hyperedge e_4:

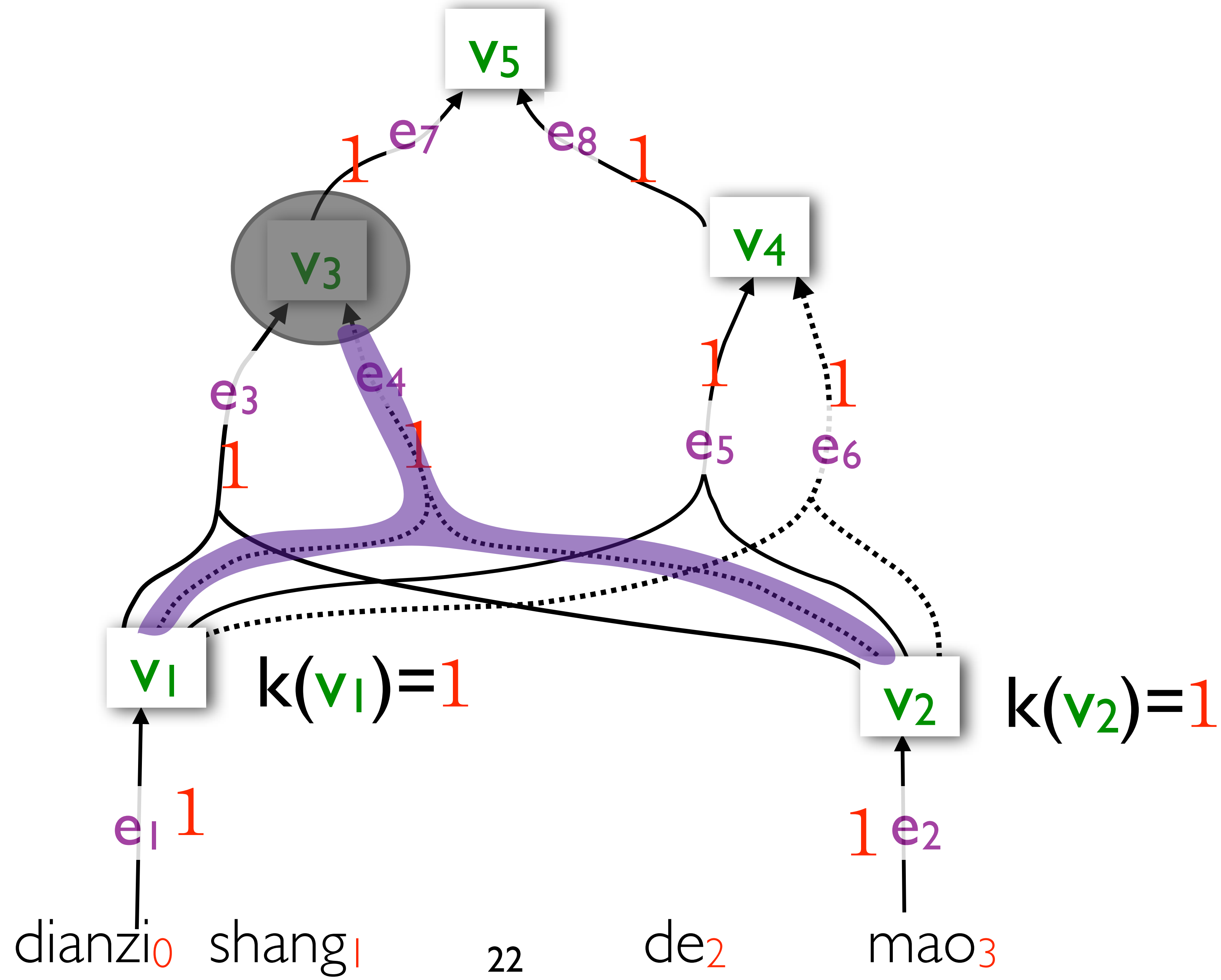
Bottom-up process in computing the number of trees



Compute k(v_3): the weight at node v_3

Hyperedge e_4:

Bottom-up process in computing the number of trees

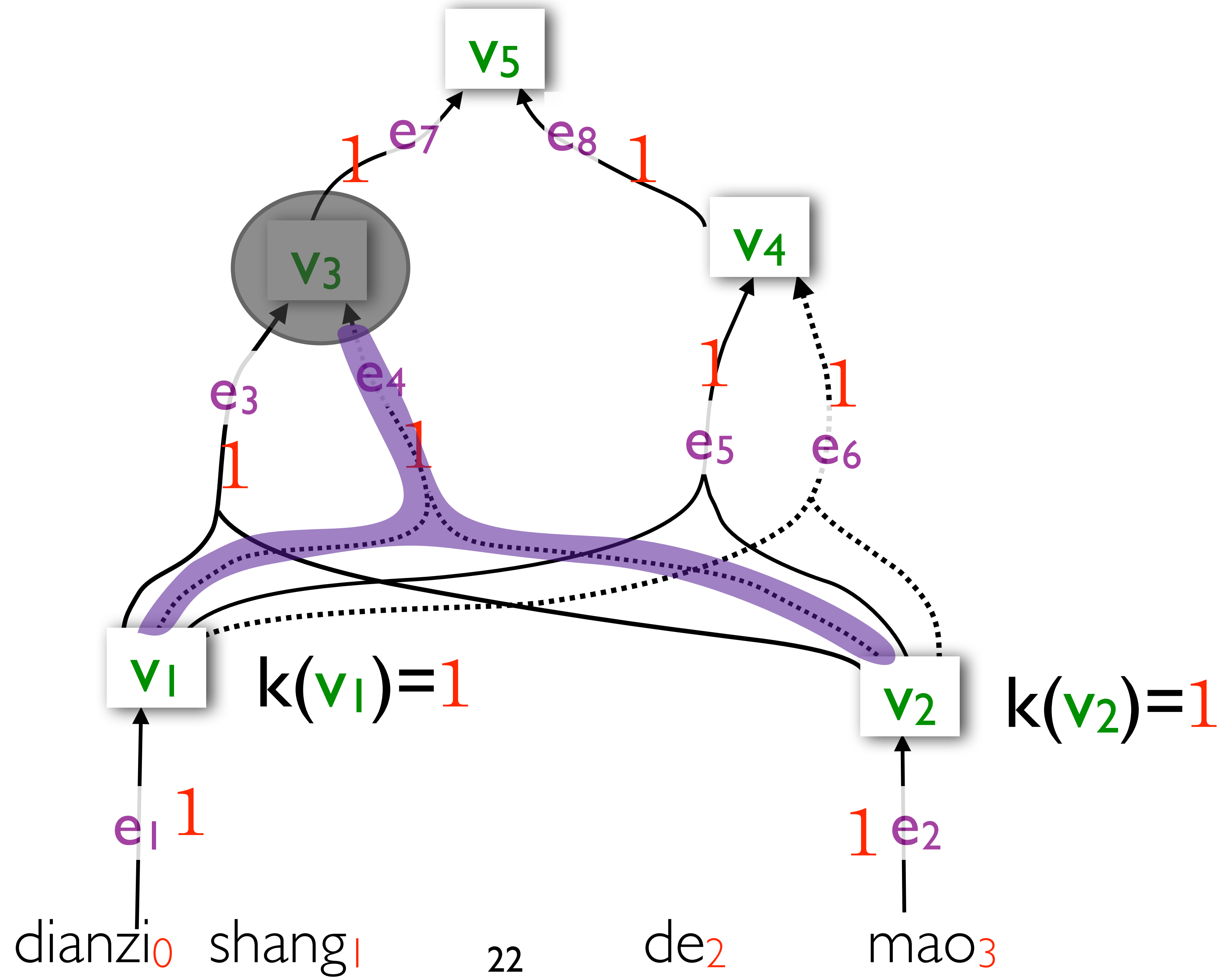


Compute $k(v_3)$: the weight at node v_3

Hyperedge e_4: $k(e_4) \bigotimes k(v_1) \bigotimes k(v_2) = 1 \otimes 1 \otimes 1 = 1$

Bottom-up

process in computing the number of trees

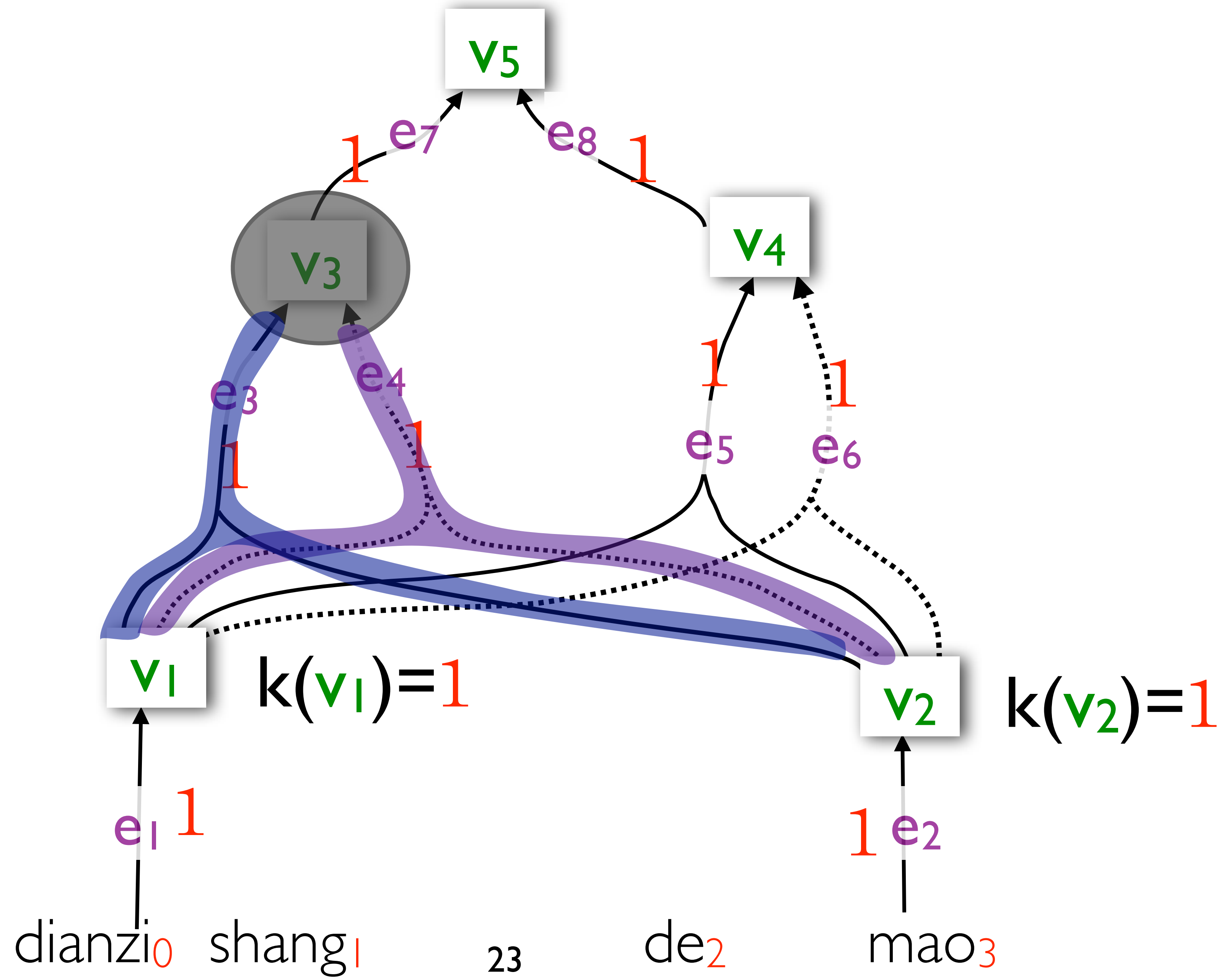


Compute $k(v_3)$: the weight at node v_3

Hyperedge e_3: $k(e_3) \otimes k(v_1) \otimes k(v_2) = 1$

Hyperedge e_4: $k(e_4) \otimes k(v_1) \otimes k(v_2) = 1$

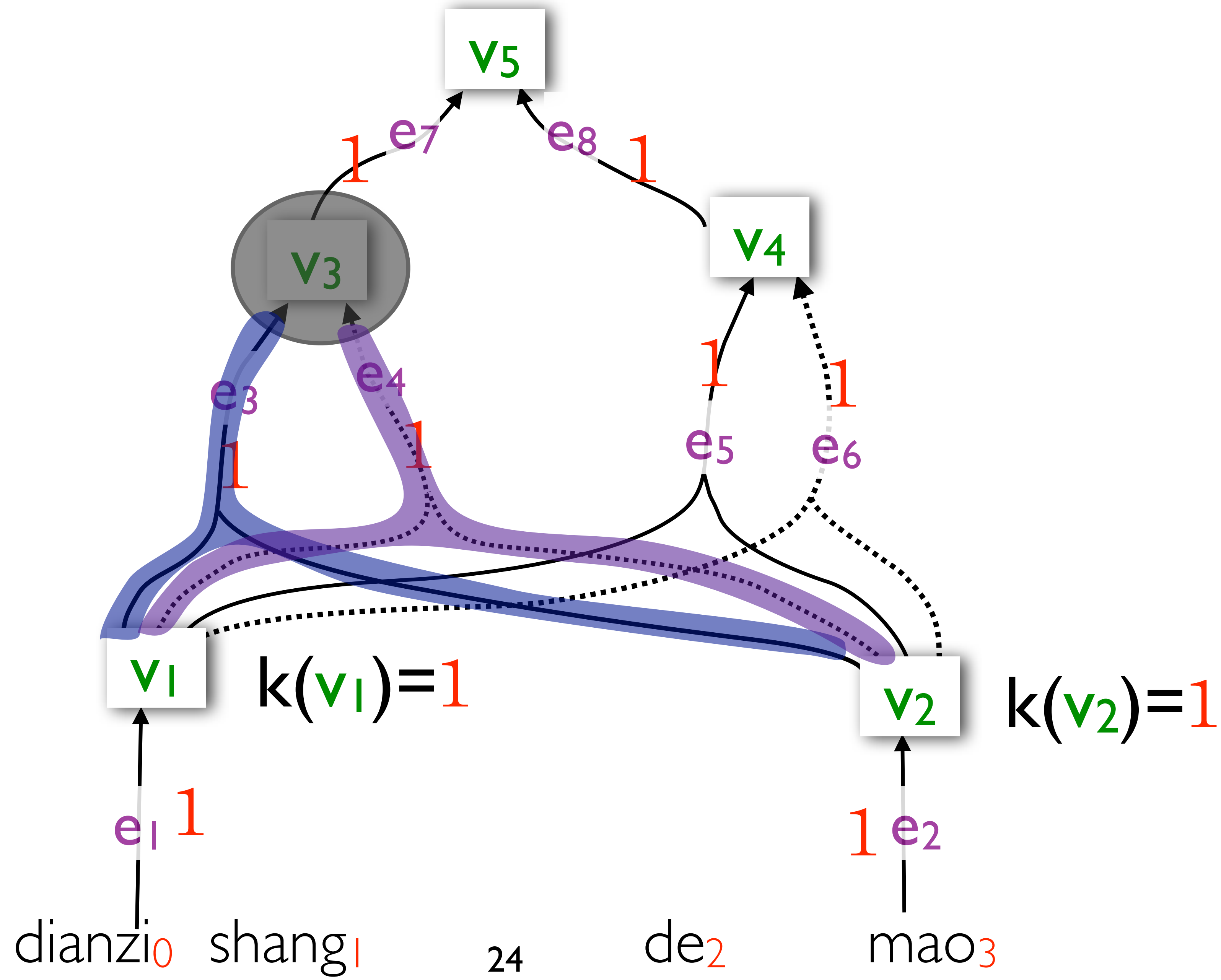
Bottom-up process in computing the number of trees



Compute $k(v_3)$: the weight at node v_3

$k(e_3) \otimes k(v_1) \otimes k(v_2)$ $\qquad$ $k(e_4) \otimes k(v_1) \otimes k(v_2)$

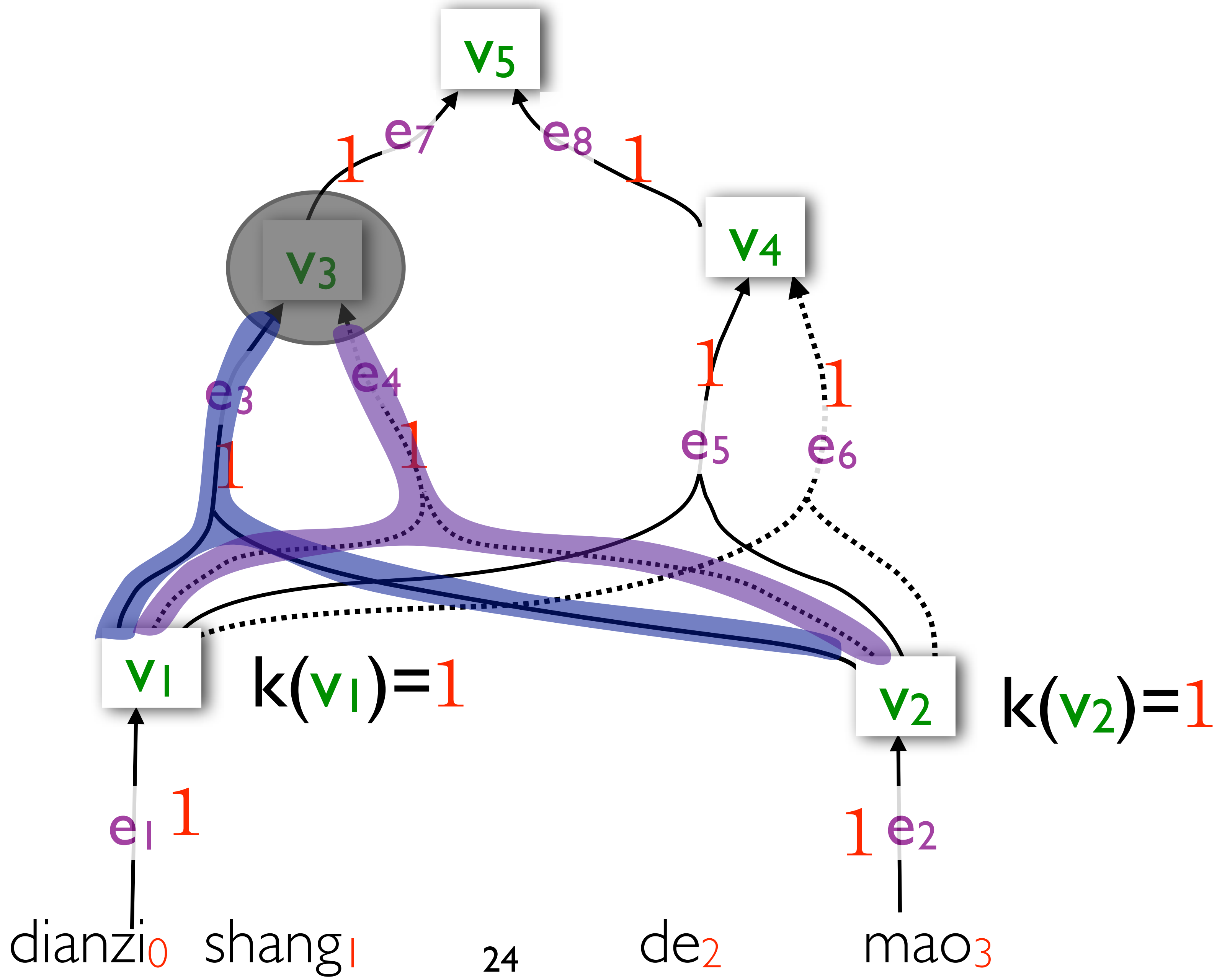
Bottom-up process in computing the number of trees



Compute $k(v_3)$: the weight at node v_3

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

Bottom-up process in computing the number of trees

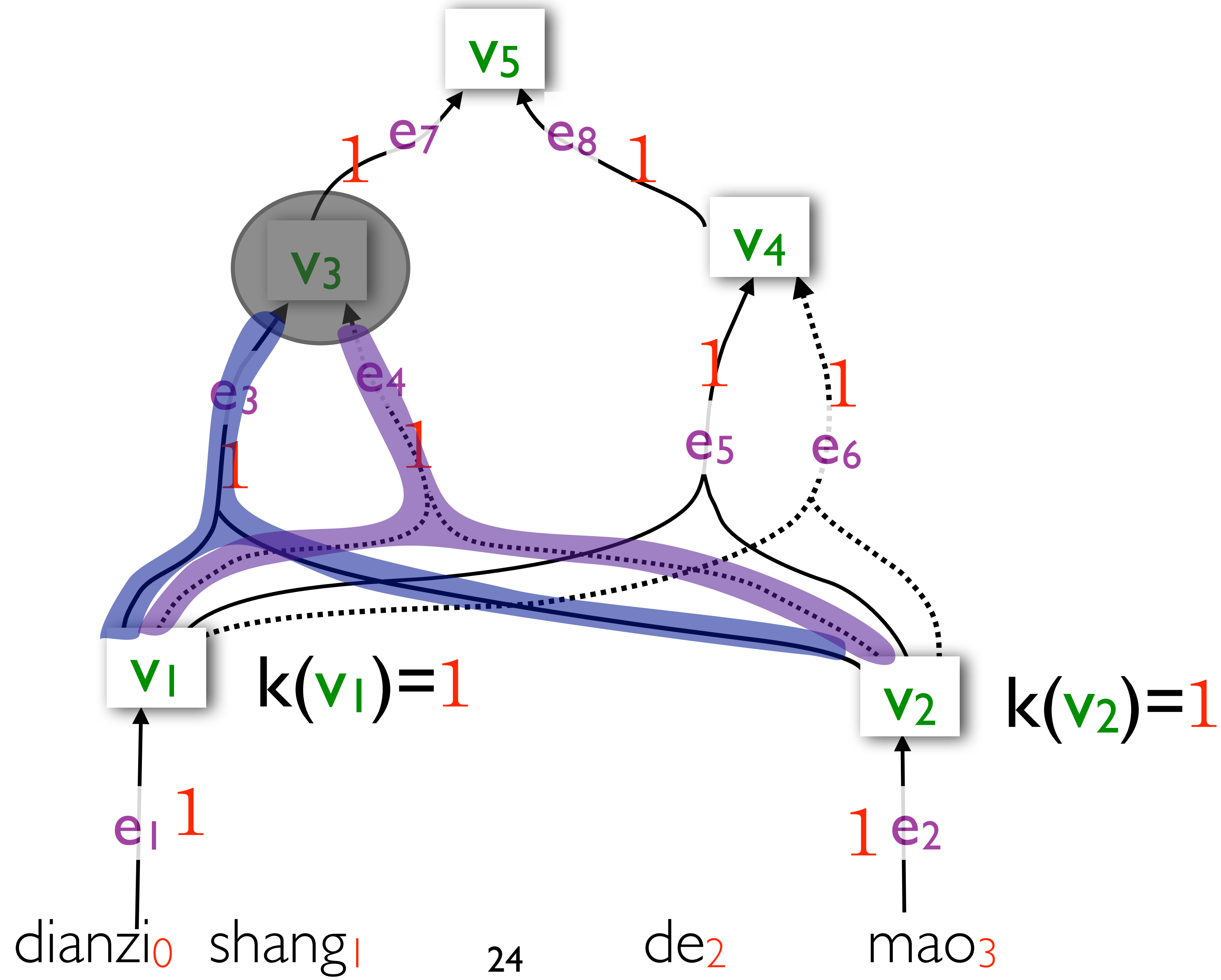


Compute $k(v_3)$: the weight at node v_3

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

$$1 \oplus 1 = 2$$

Bottom-up process in computing the number of trees

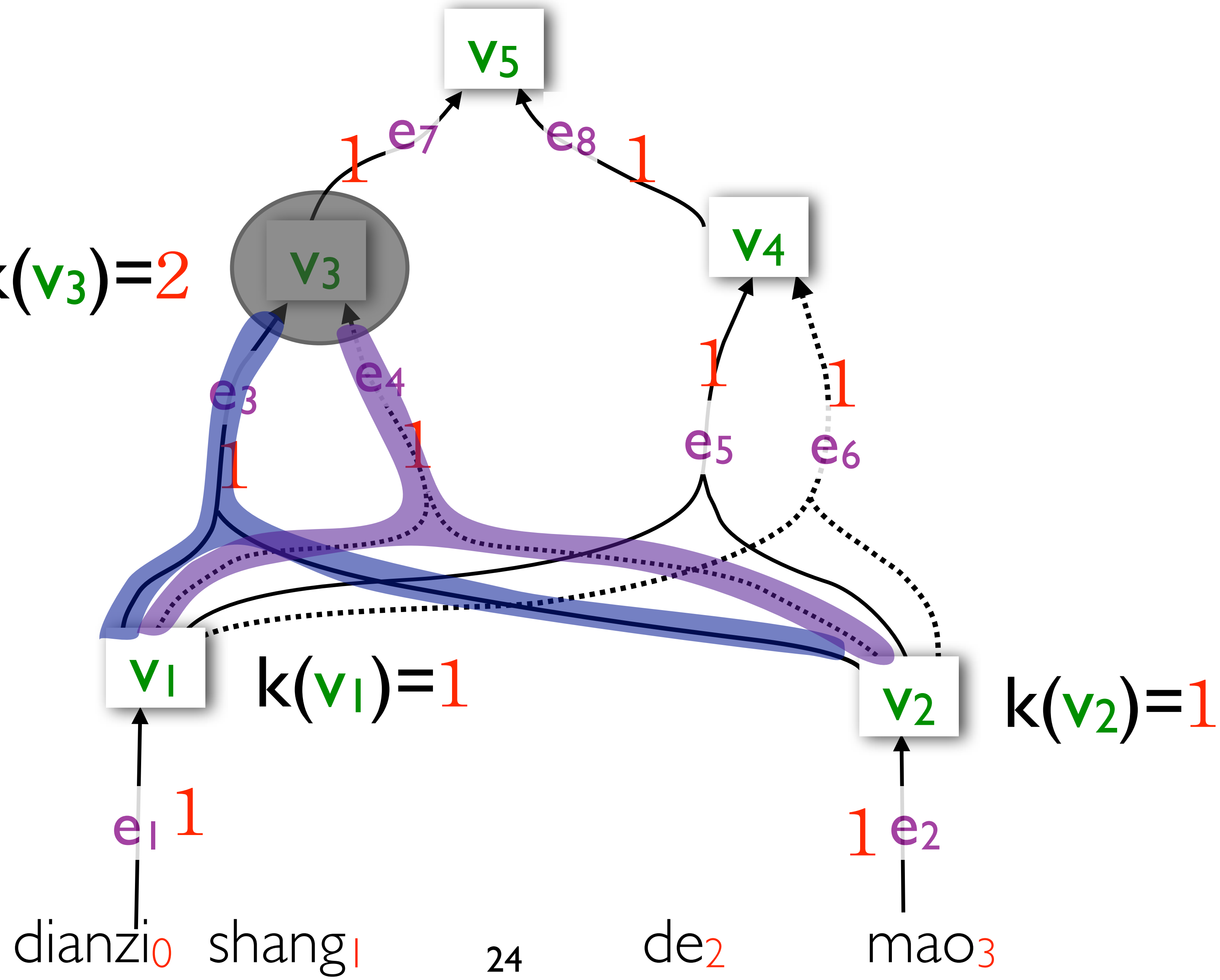


Compute k(v_3): the weight at node v_3

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

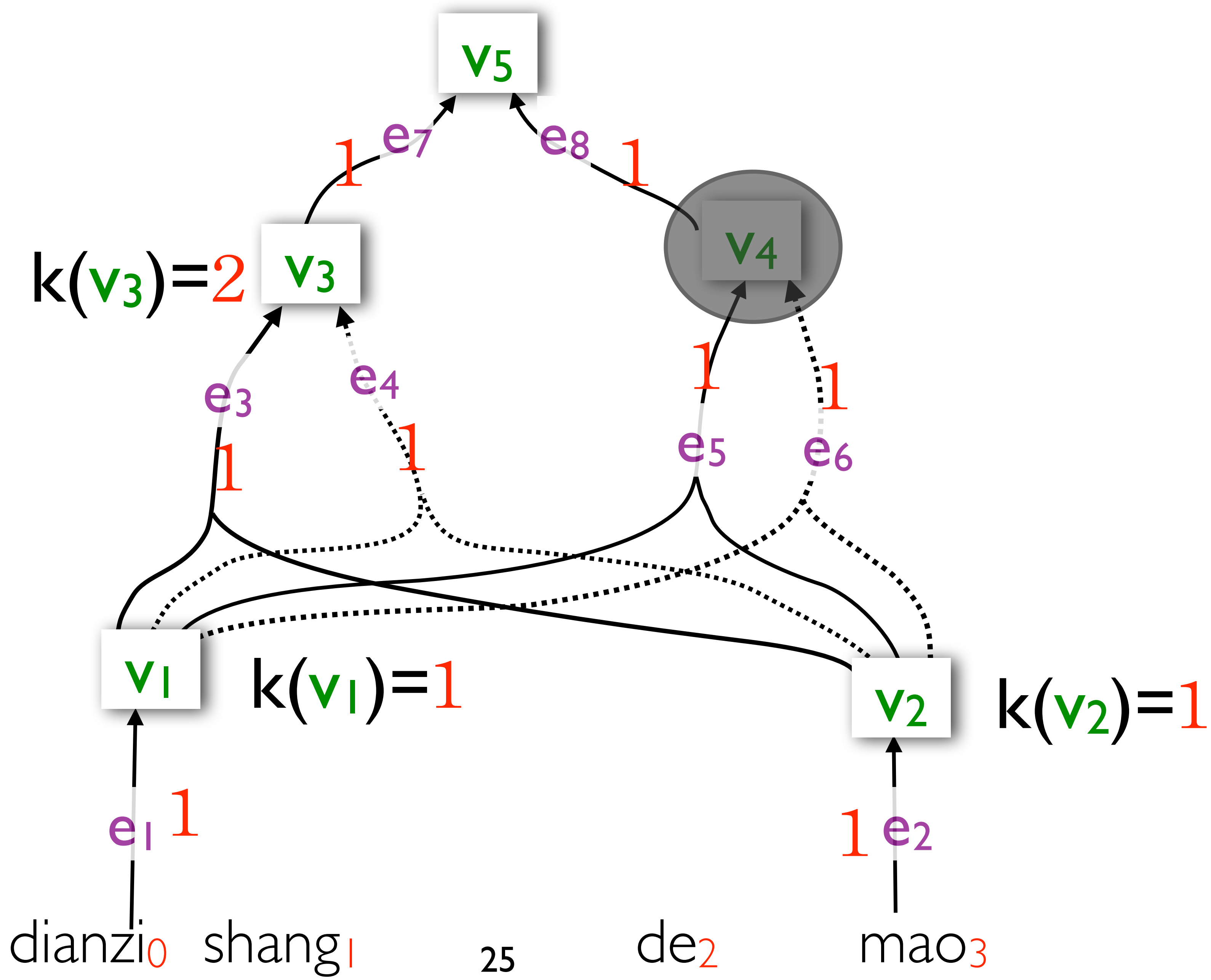
$$1 \oplus 1 = 2$$

Bottom-up process in computing the number of trees



$k(v_1) = k(e_1)$ $\quad$ $k(v_2) = k(e_2)$

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

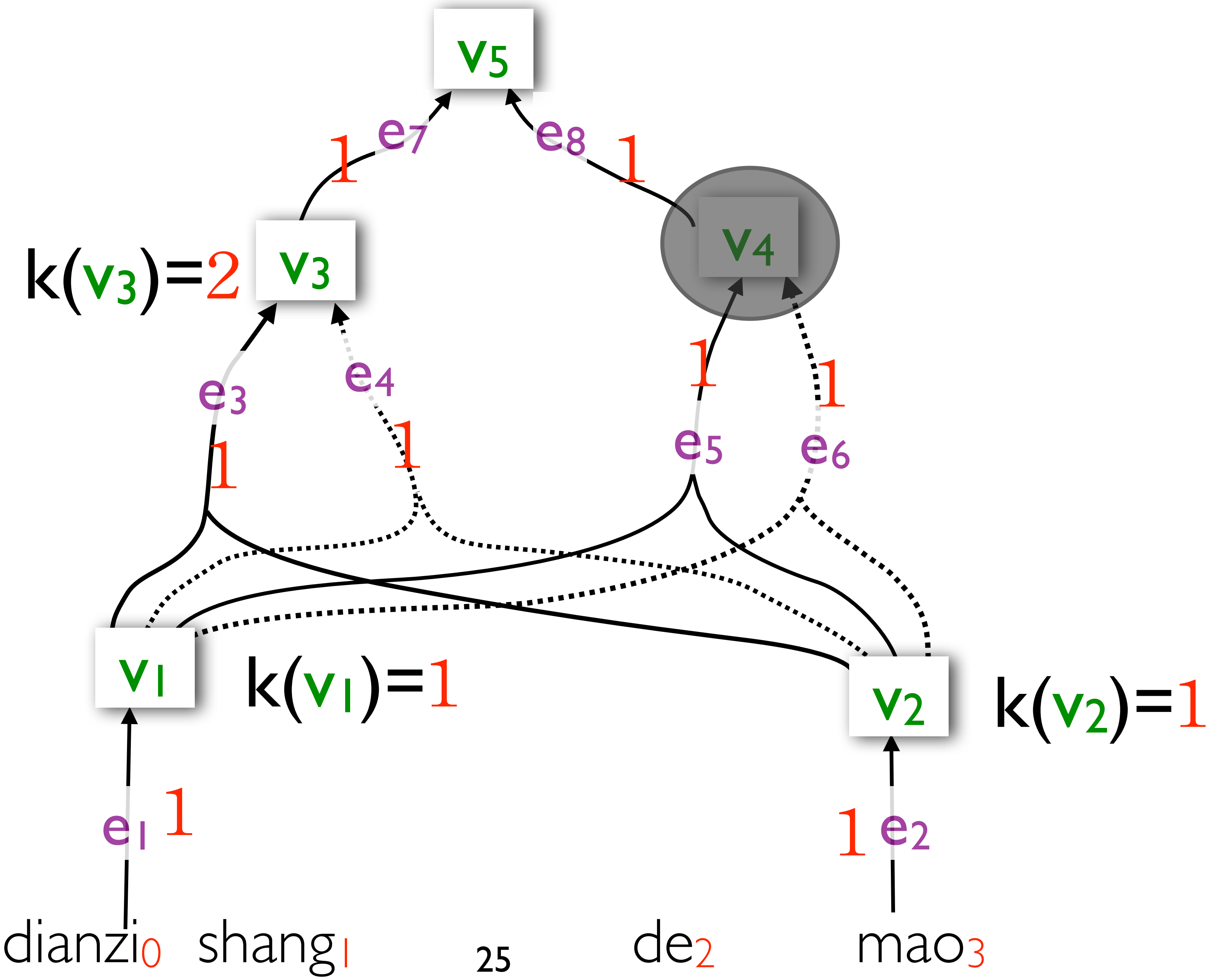


Bottom-up process in computing the number of trees

$k(v_1) = k(e_1)$ $\qquad$ $k(v_2) = k(e_2)$

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$$

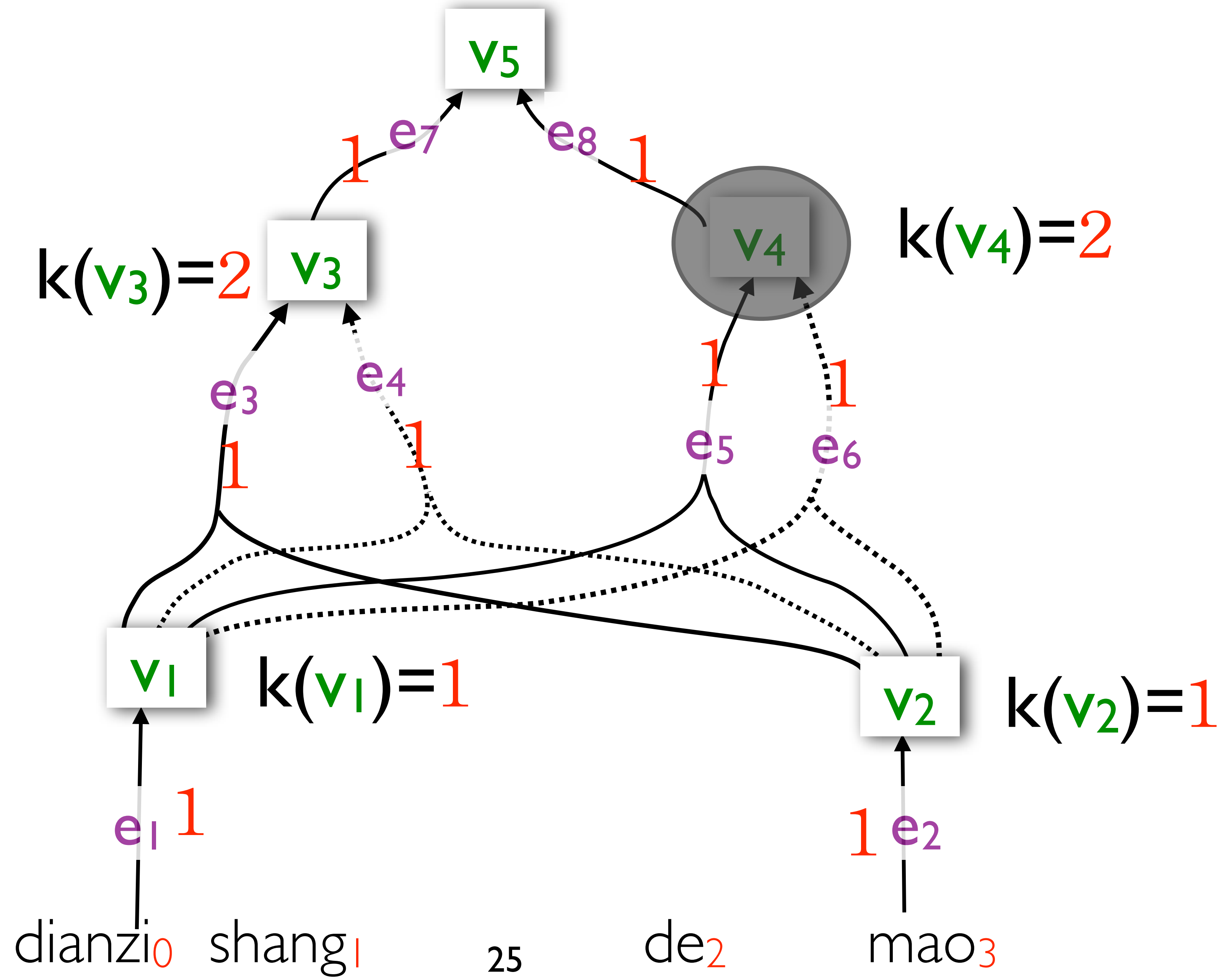


Bottom-up process in computing the number of trees

$k(v_1) = k(e_1) \qquad k(v_2) = k(e_2)$

$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$

$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$



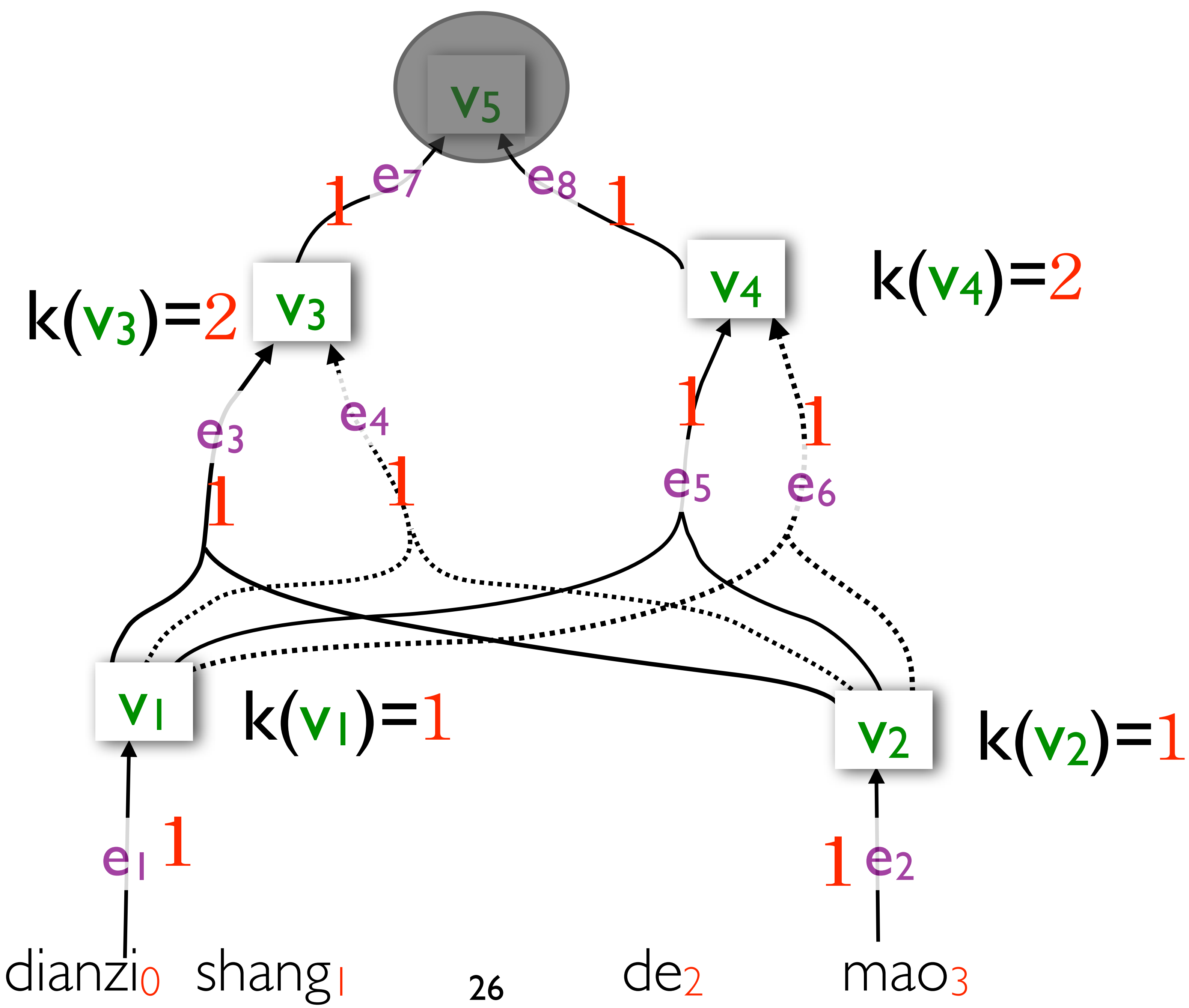
Bottom-up process in computing the number of trees

$k(v_1) = k(e_1)$ $\quad$ $k(v_2) = k(e_2)$

$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$

$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$

Bottom-up process in computing the number of trees

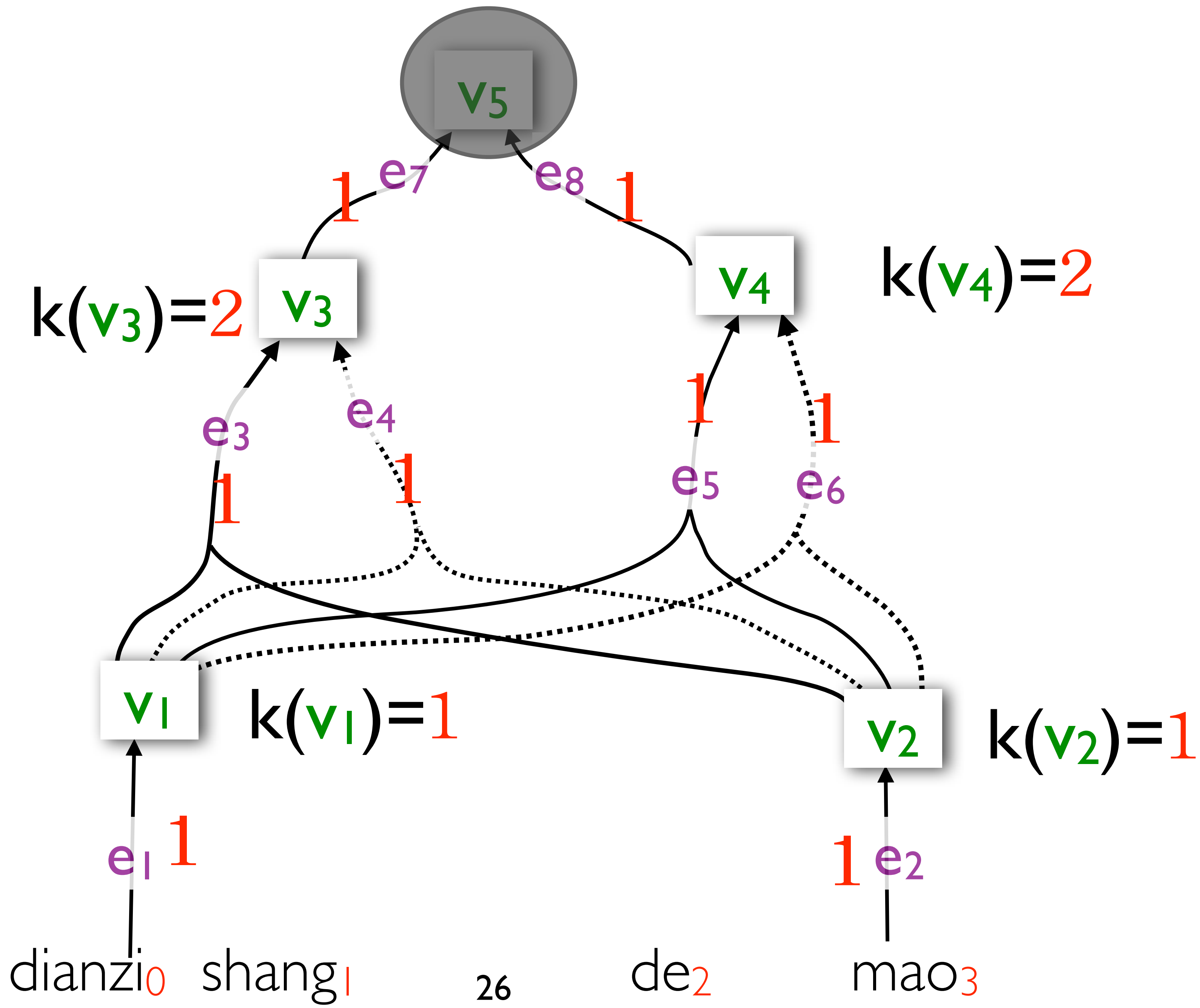


$$k(v_1) = k(e_1) \qquad k(v_2) = k(e_2)$$

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_5) = k(e_7) \otimes k(v_3) \oplus k(e_8) \otimes k(v_4)$$



Bottom-up process in computing the number of trees

$$k(v_1) = k(e_1) \qquad k(v_2) = k(e_2)$$

$$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$$

$$k(v_4) = k(e_5) \otimes k(v_1) \otimes k(v_2) \oplus k(e_6) \otimes k(v_1) \otimes k(v_2)$$

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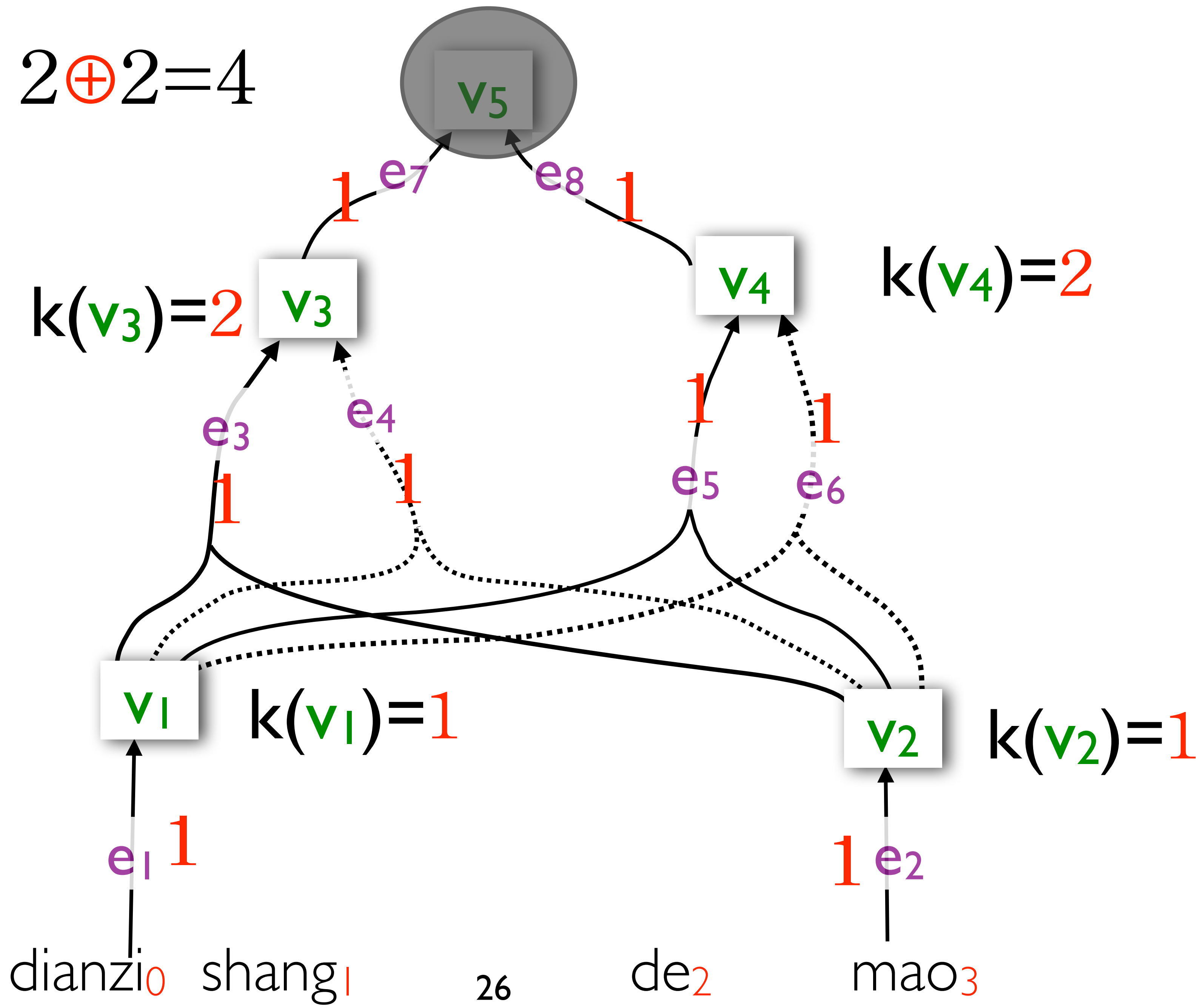

Bottom-up process in computing the number of trees

$k(v_1) = k(e_1)$ $\quad$ $k(v_2) = k(e_2)$

$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$

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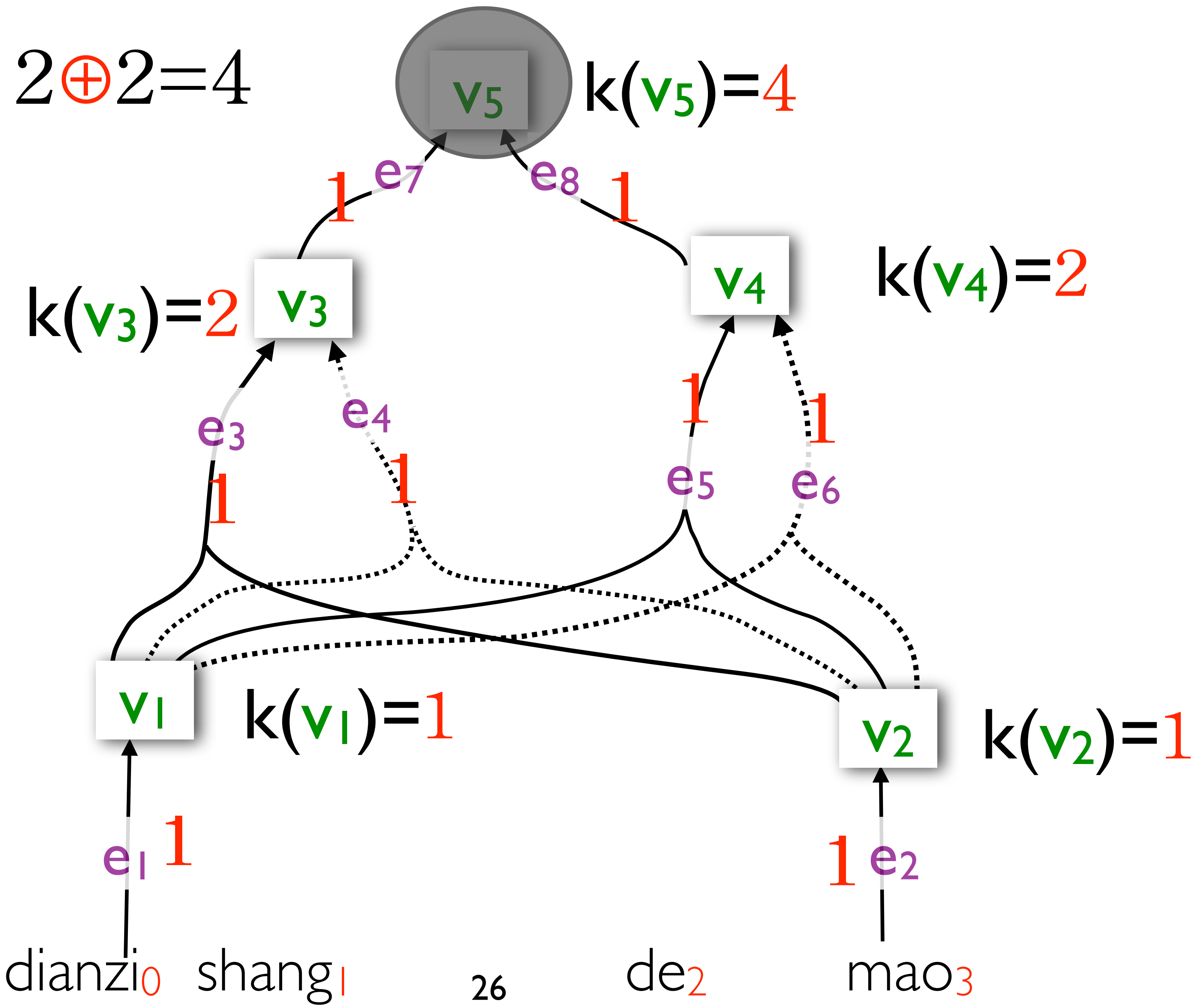

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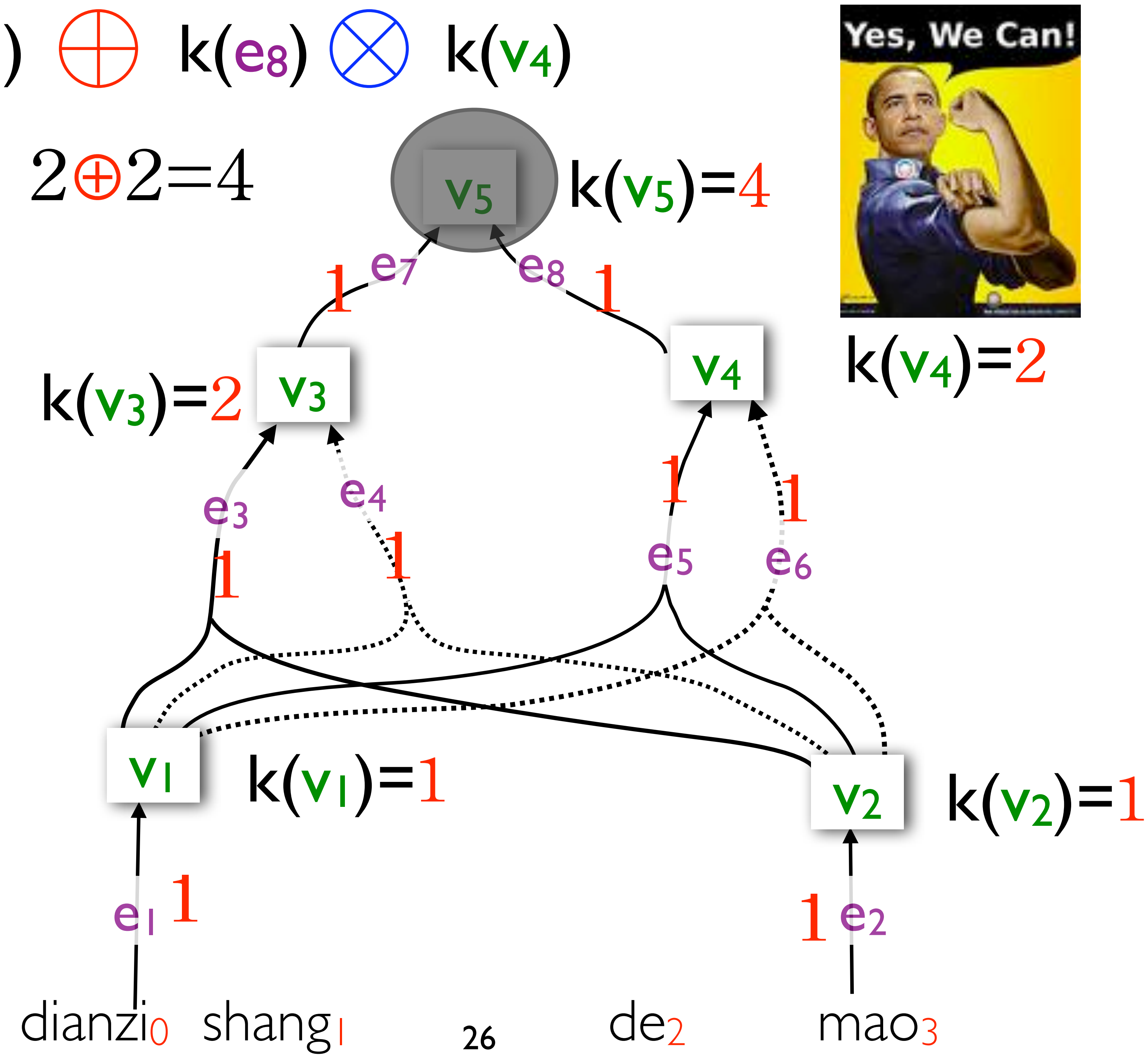

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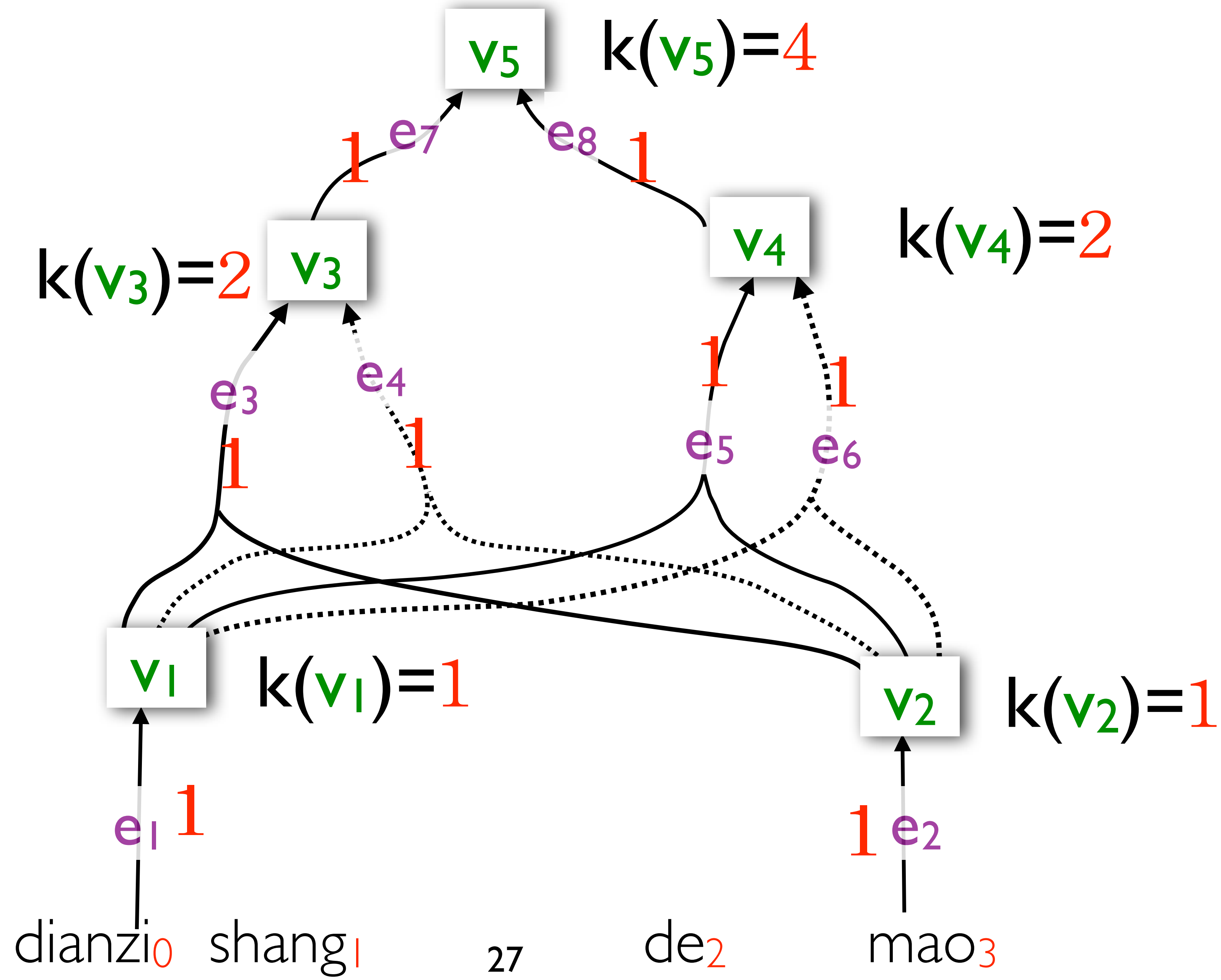

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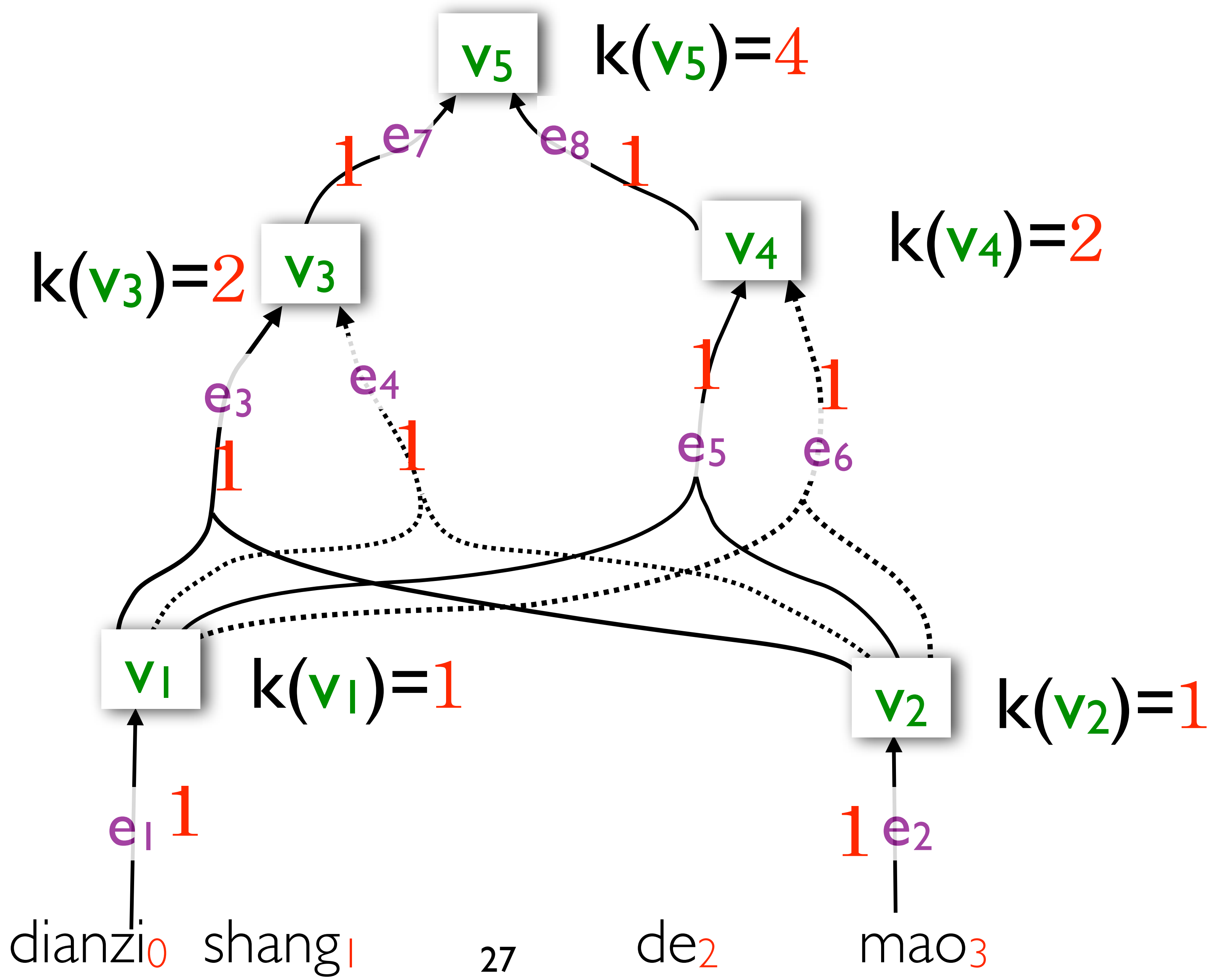
Summary:



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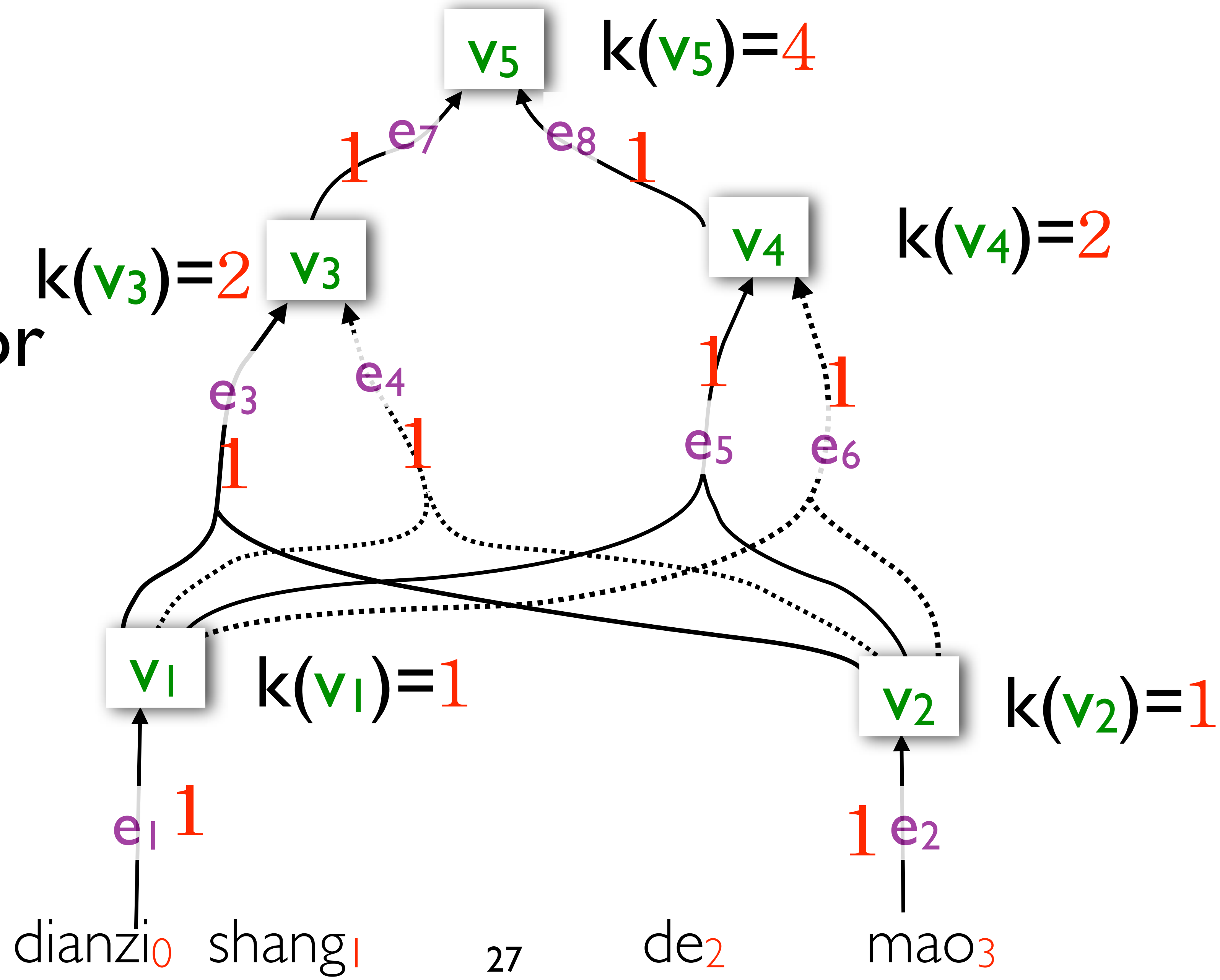
Summary:

- **input**: a weight for each edge



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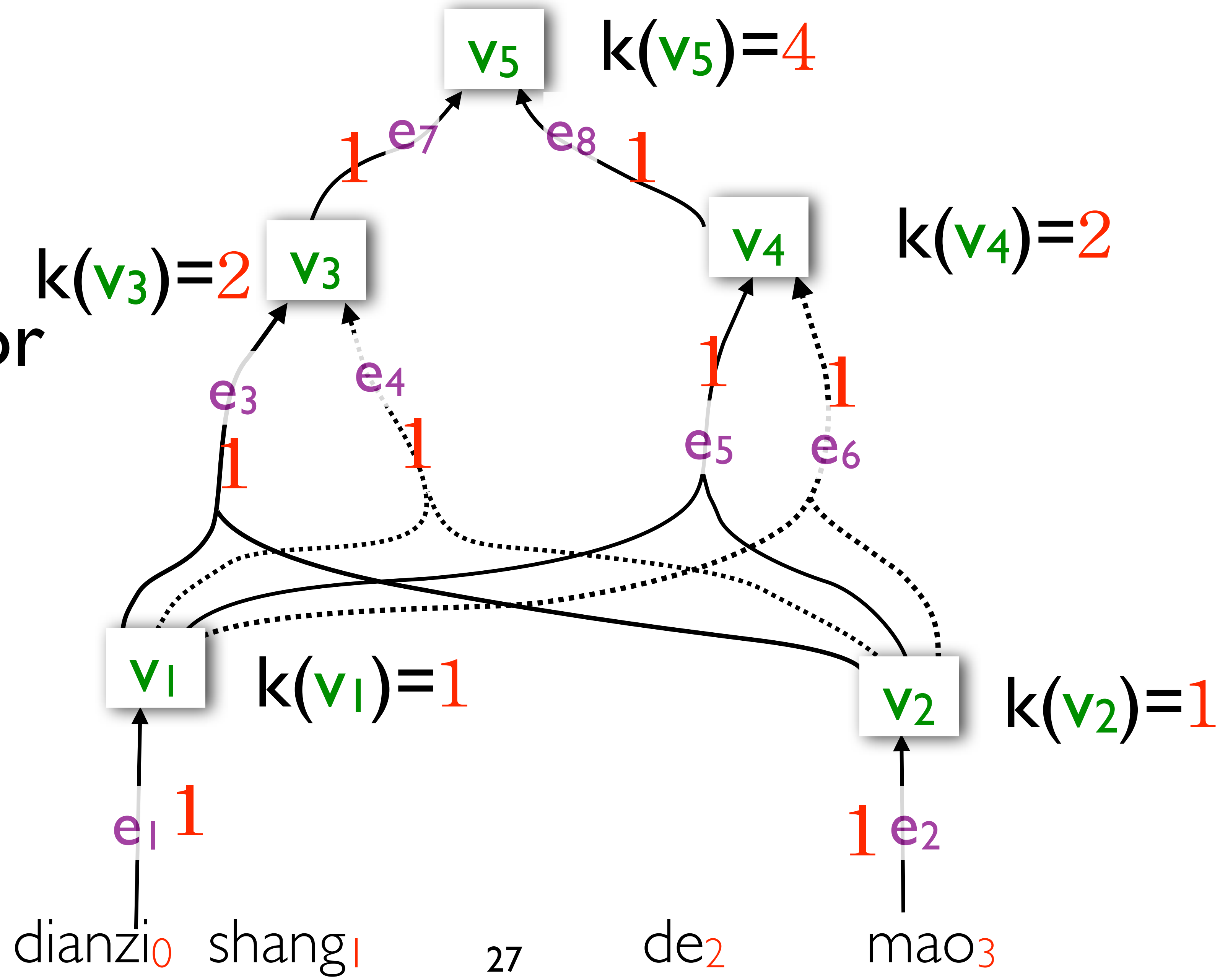
Summary:

- input: a weight for each edge
- output: a weight for each node



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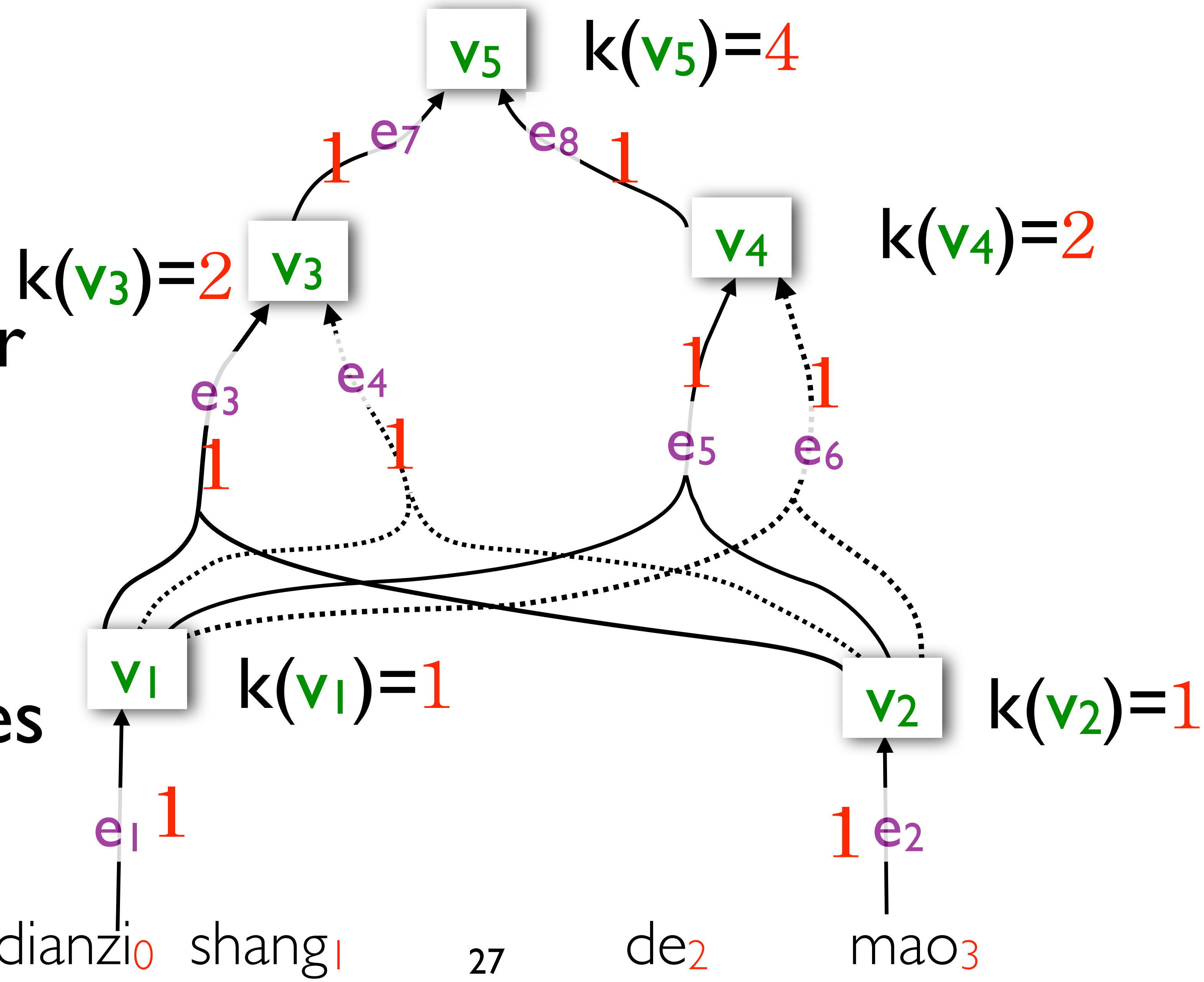
Summary:

- input: a weight for each edge
- output: a weight for each node
- $\oplus$ is used at nodes



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Summary:

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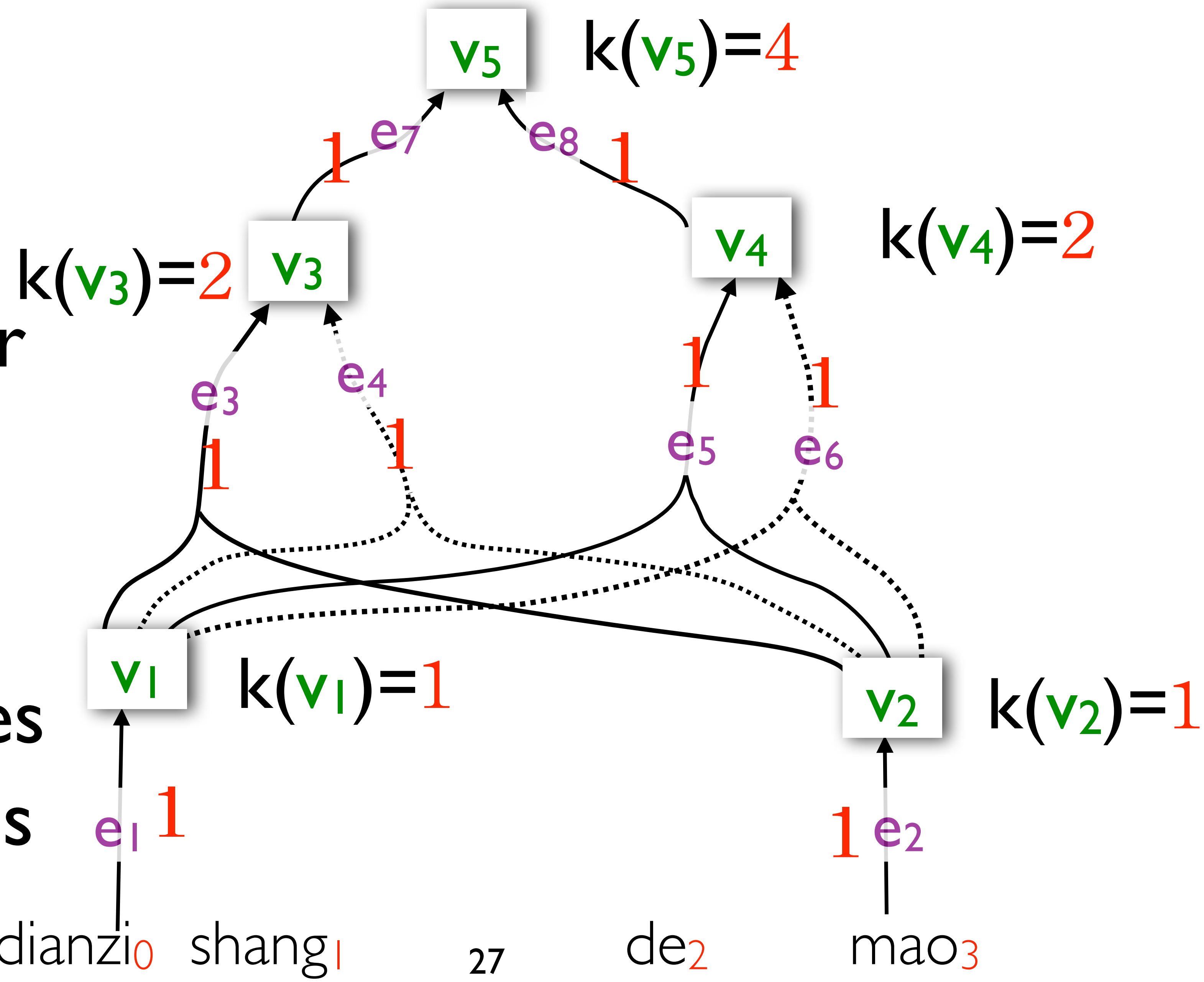
- output: a weight for each node
- $\oplus$ is used at nodes
- $\otimes$ is used at edges



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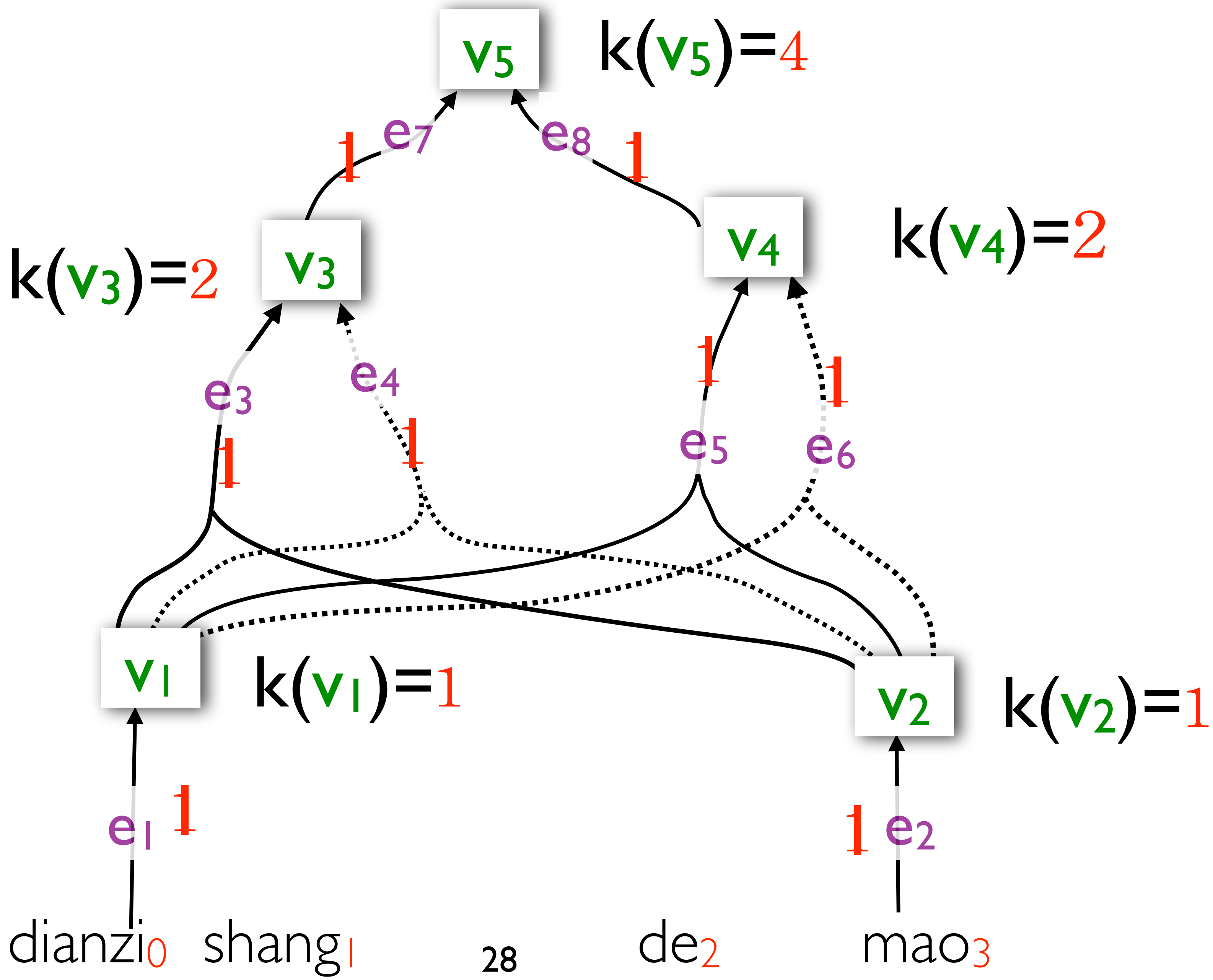

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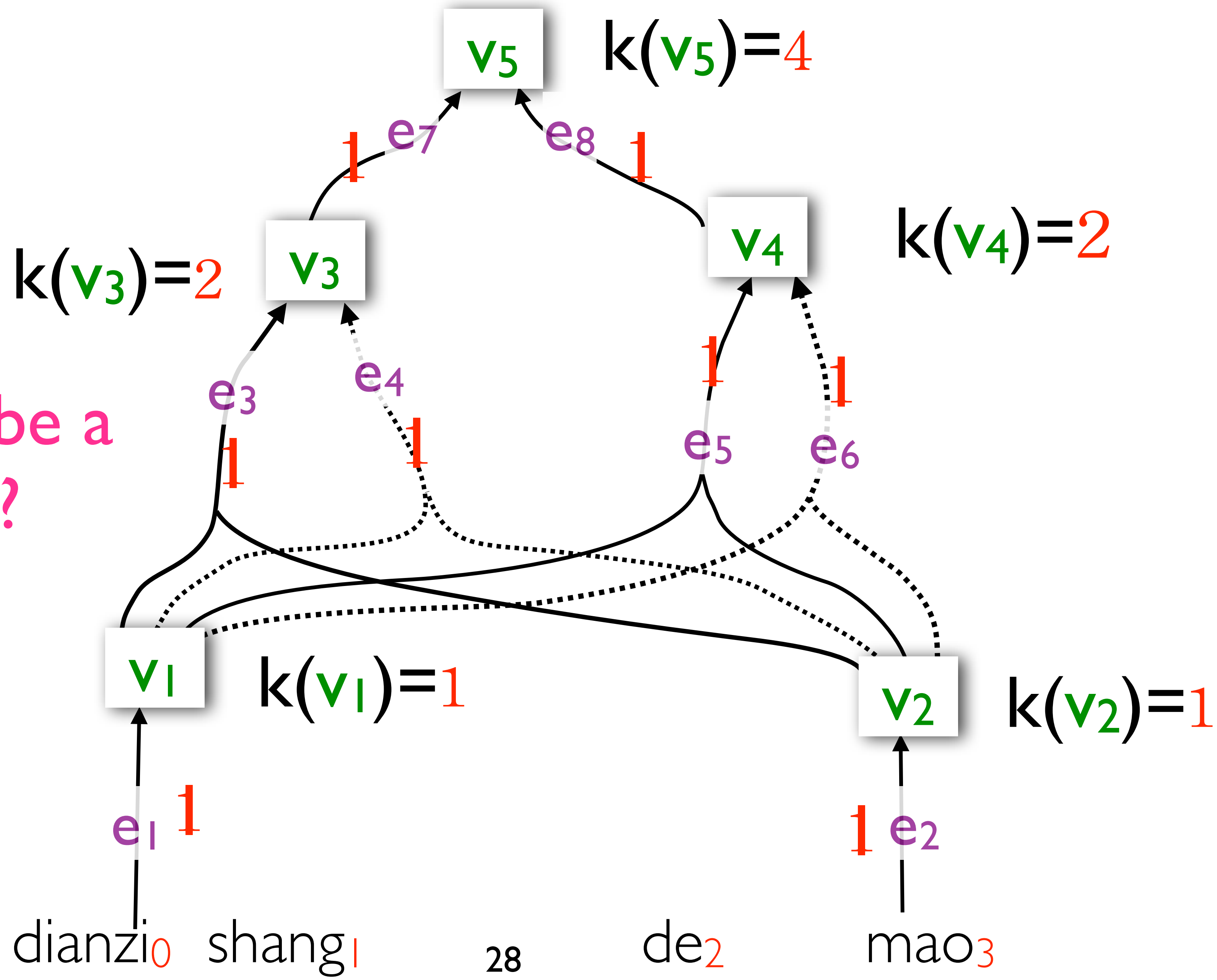

Does it have to be a single integer?

$k(v_1) = k(e_1)$ $\quad$ $k(v_2) = k(e_2)$

$k(v_3) = k(e_3) \otimes k(v_1) \otimes k(v_2) \oplus k(e_4) \otimes k(v_1) \otimes k(v_2)$

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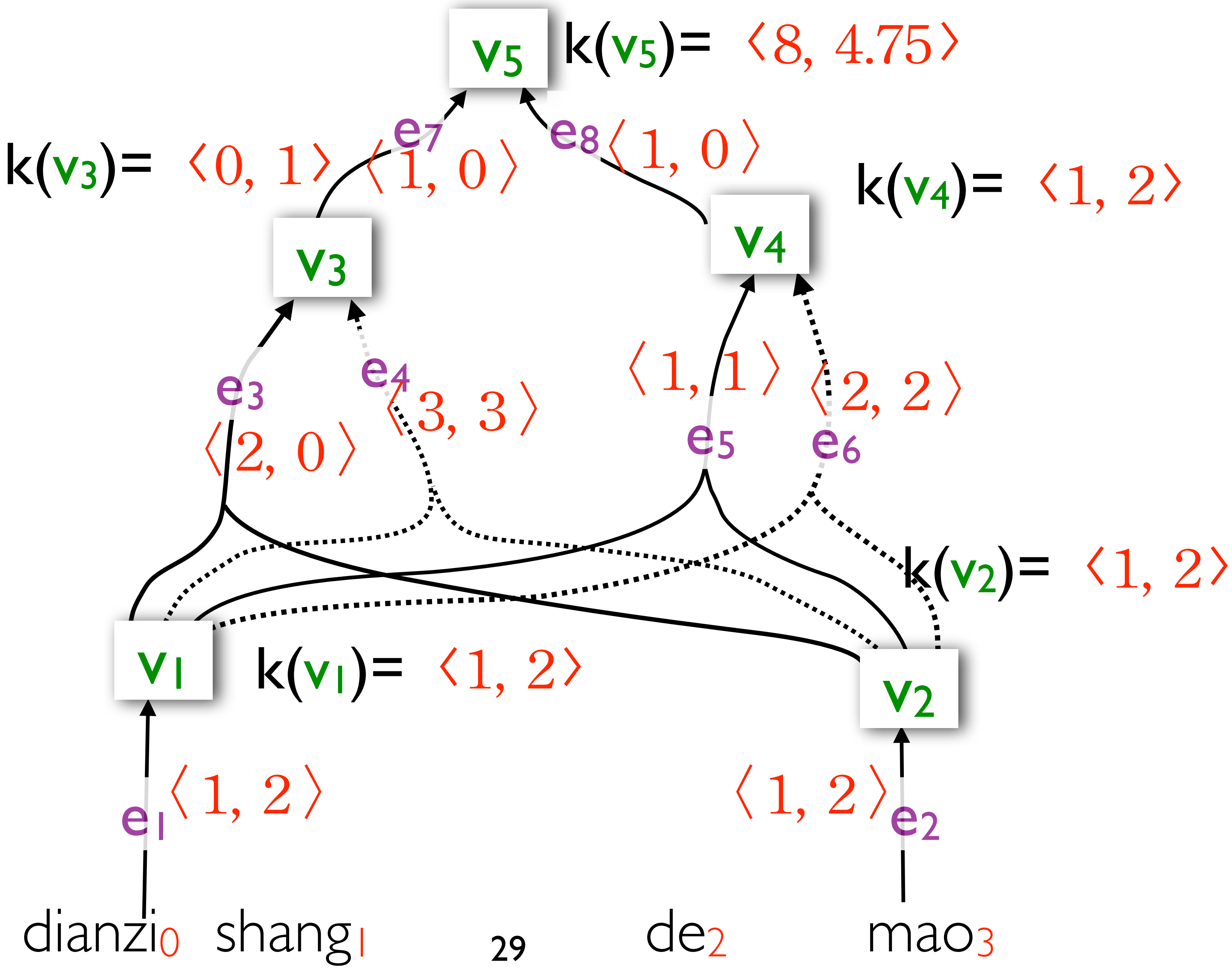

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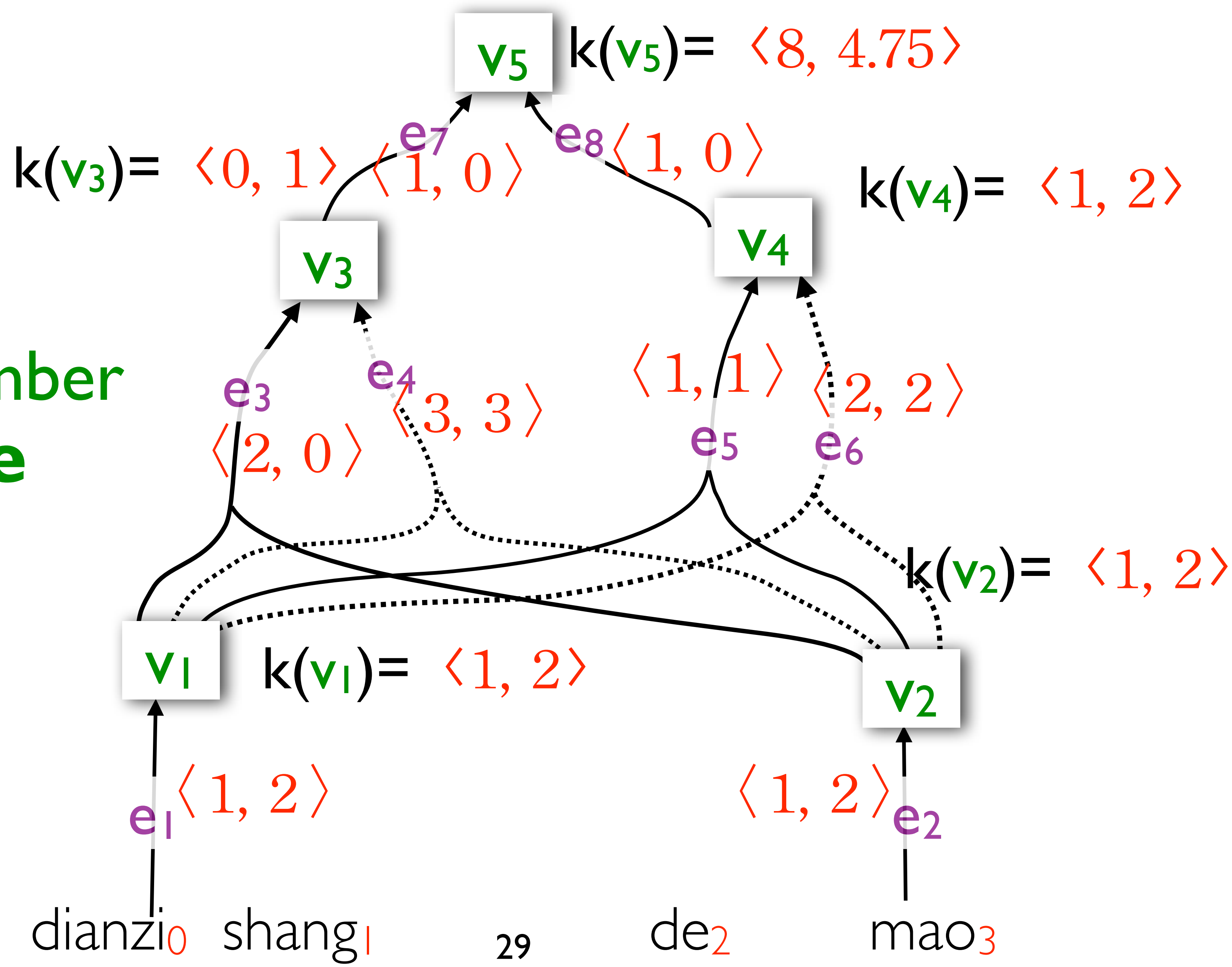

A semiring member is a **2-tuple**

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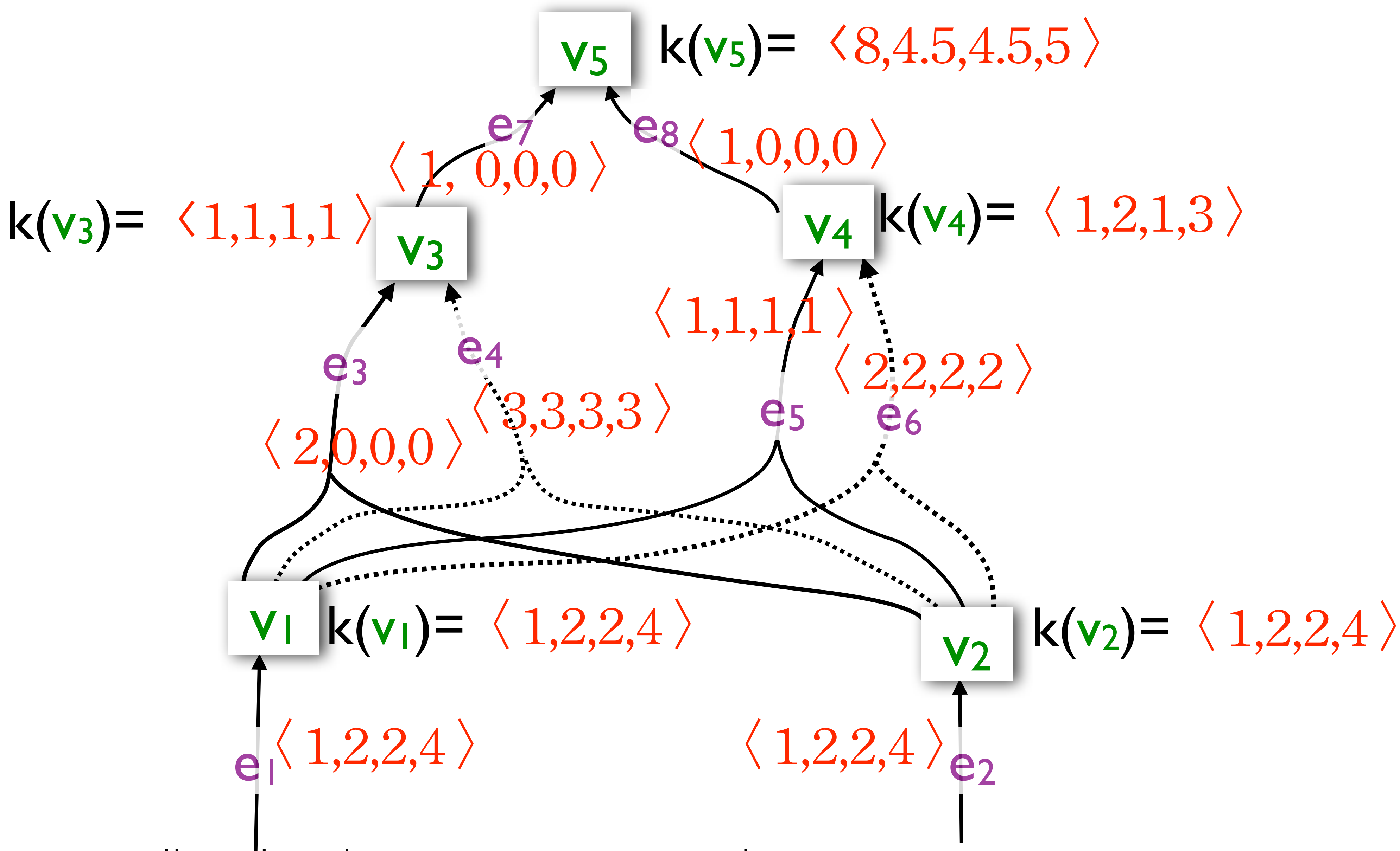

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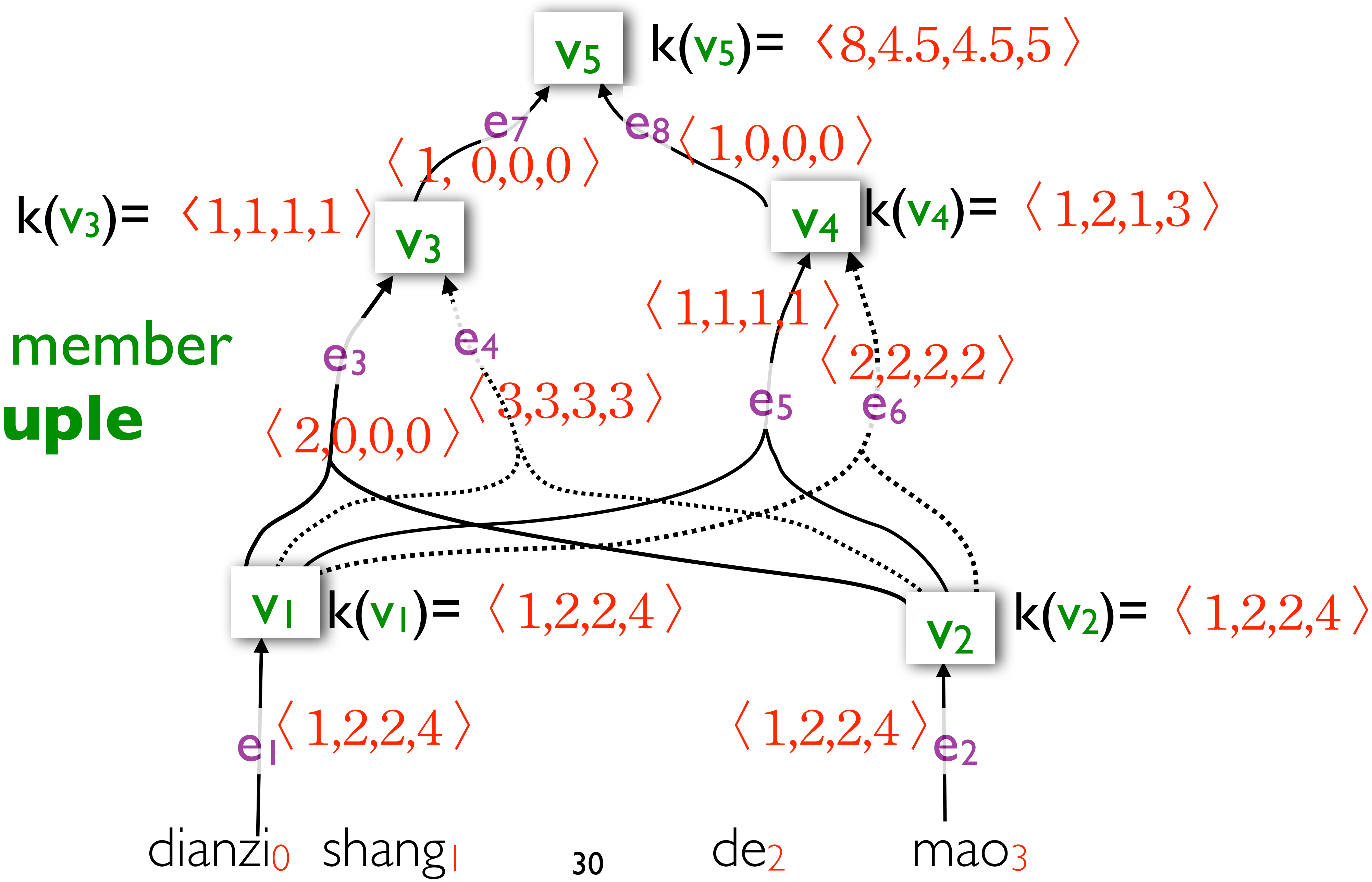
A semiring member is a **4-tuple**



Why do we want to work on **tuples**?

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expectation semiring

second-order expectation semiring

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useful for
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Goodman (1999) defines many other semirings useful at "**testing**" time

First- and Second-order Expectation Semirings

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First-order:

(Eisner, 2002)

First- and Second-order Expectation Semirings

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$\langle p_1, r_1 \rangle \oplus \langle p_2, r_2 \rangle$	$\langle p_1 + p_2,\ r_1 + r_2 \rangle$

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What does p, r, s, or t mean?

Second-order:

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What does p, r, s, or t mean?

Second-order:

- ea

application-dependent

$\langle p_1, r_1, s_1, t_1 \rangle \otimes \langle p_2, r_2, s_2, t_2 \rangle$	$\langle p_1 p_2,\ p_1 r_2 + p_2 r_1,\ p_1 s_2 + p_2 s_1,$ $p_1 t_2 + p_2 t_1 + r_1 s_2 + r_2 s_1 \rangle$
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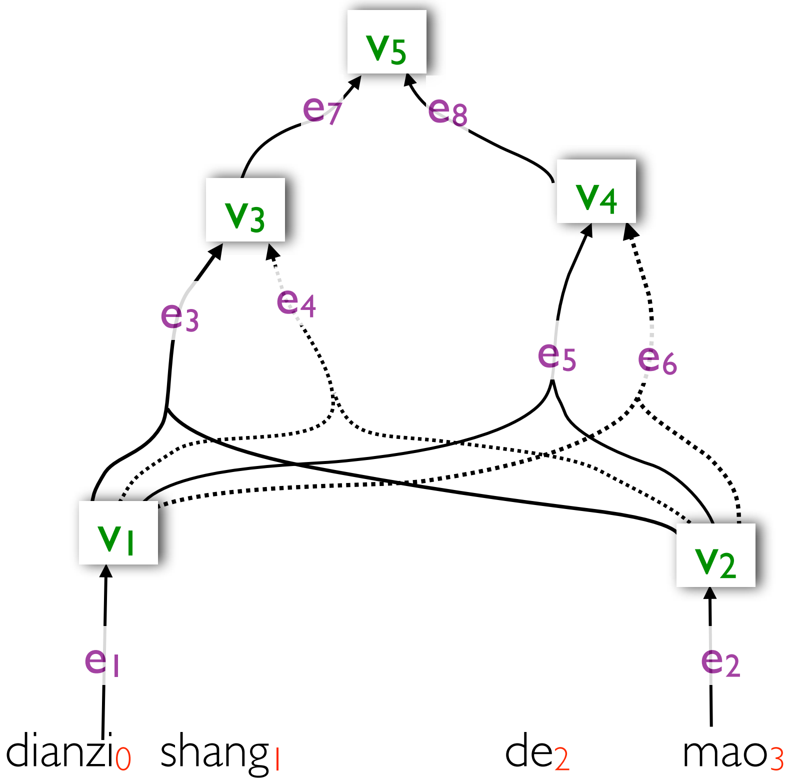
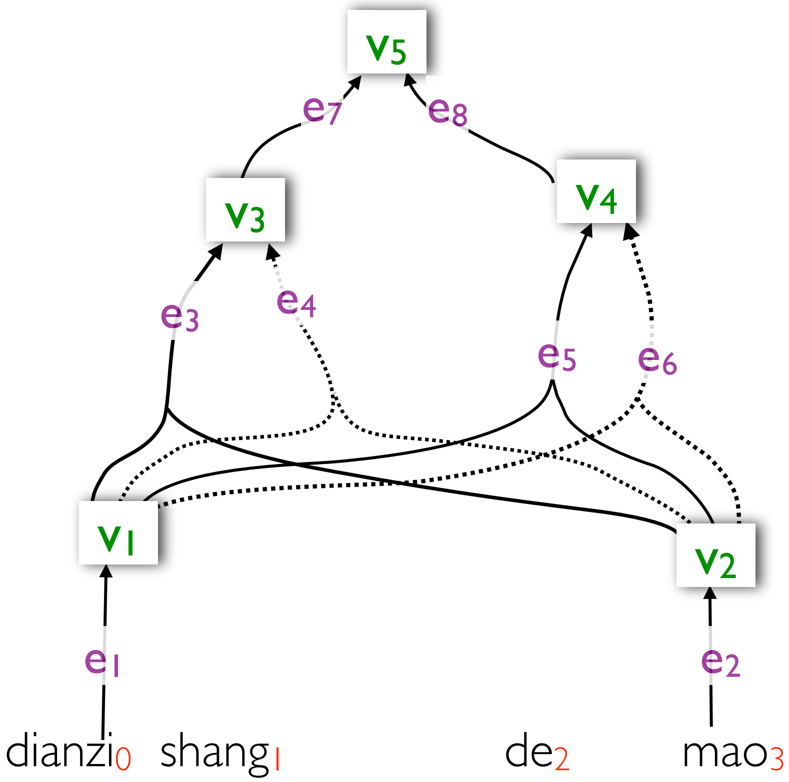
Applications

Compute Quantities on a Hypergraph: a Recipe

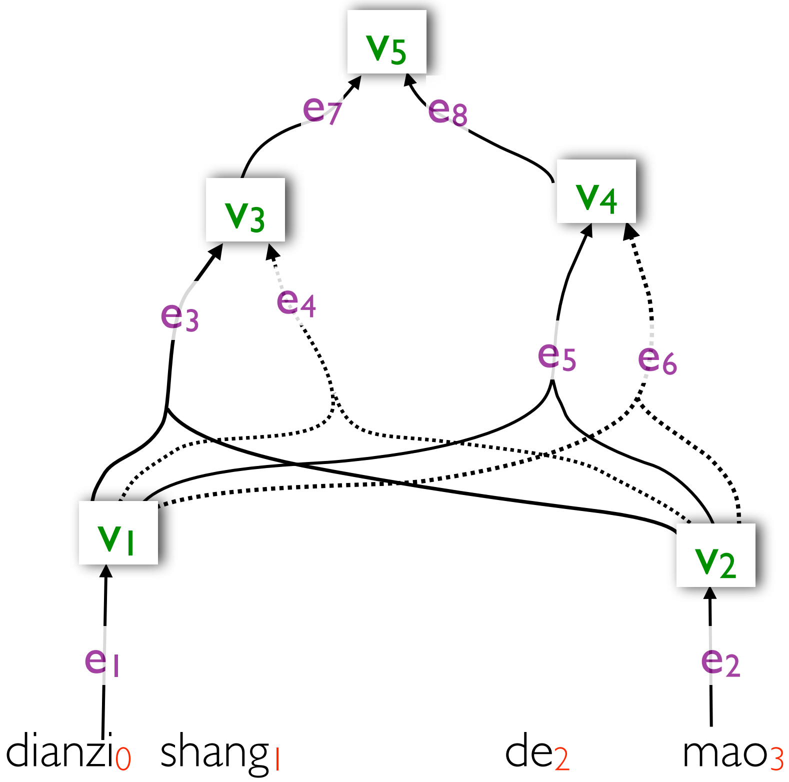


Compute Quantities on a Hypergraph: a Recipe



Three steps:

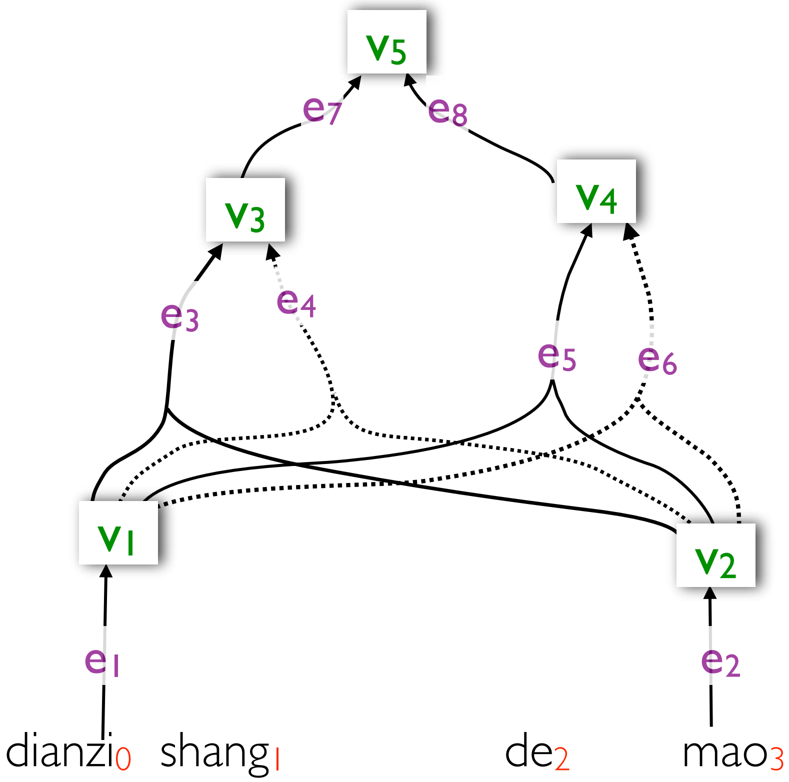
Compute Quantities on a Hypergraph: a Recipe



Three steps:

- choose a semiring

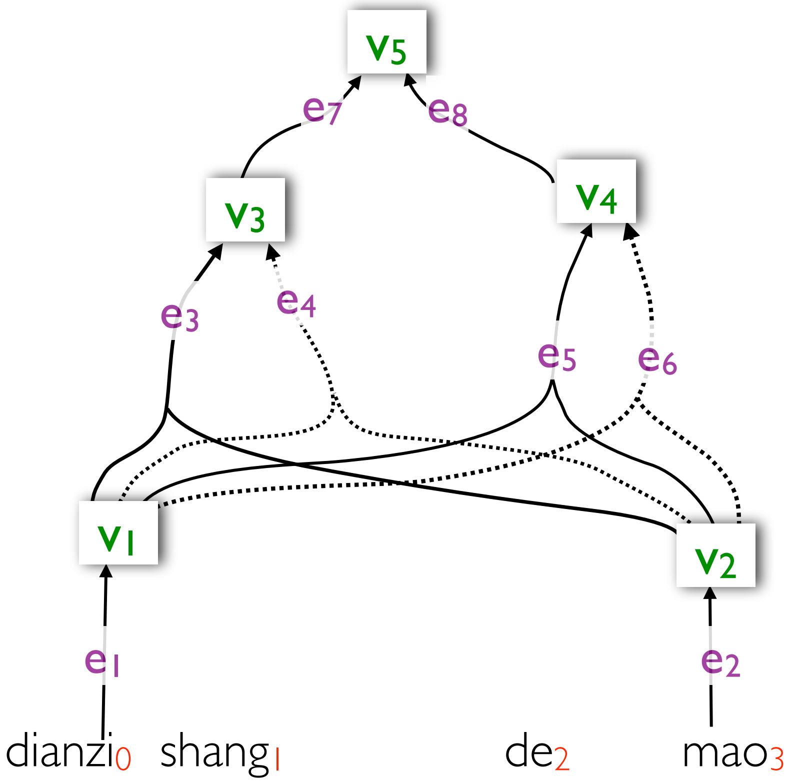
Compute Quantities on a Hypergraph: a Recipe



Three steps:

- choose a semiring
- specify a weight for each edge

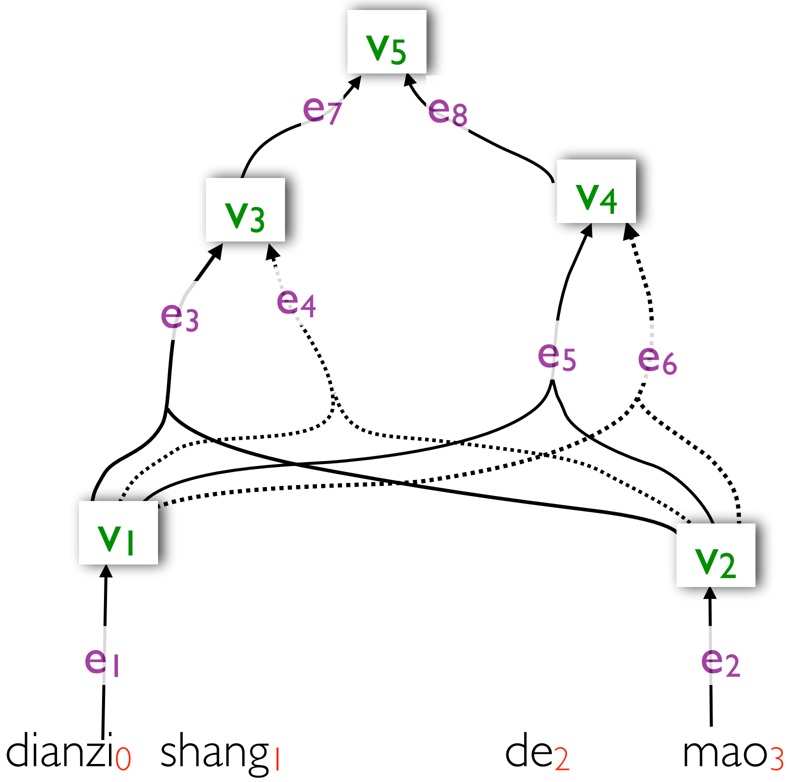
Compute Quantities on a Hypergraph: a Recipe



Three steps:

- choose a semiring
- specify a weight for each edge
- run the inside algorithm

Compute Quantities on a Hypergraph: a Recipe



Three steps:

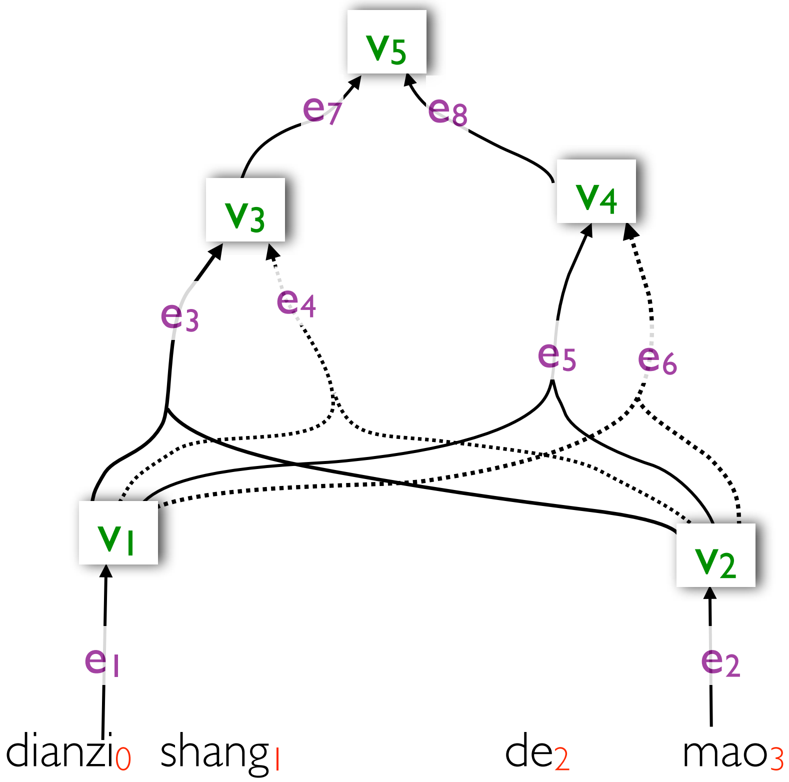
- choose a semiring

  first- or second-order expectation semiring

- specify a weight for each edge

- run the inside algorithm

Compute Quantities on a Hypergraph: a Recipe



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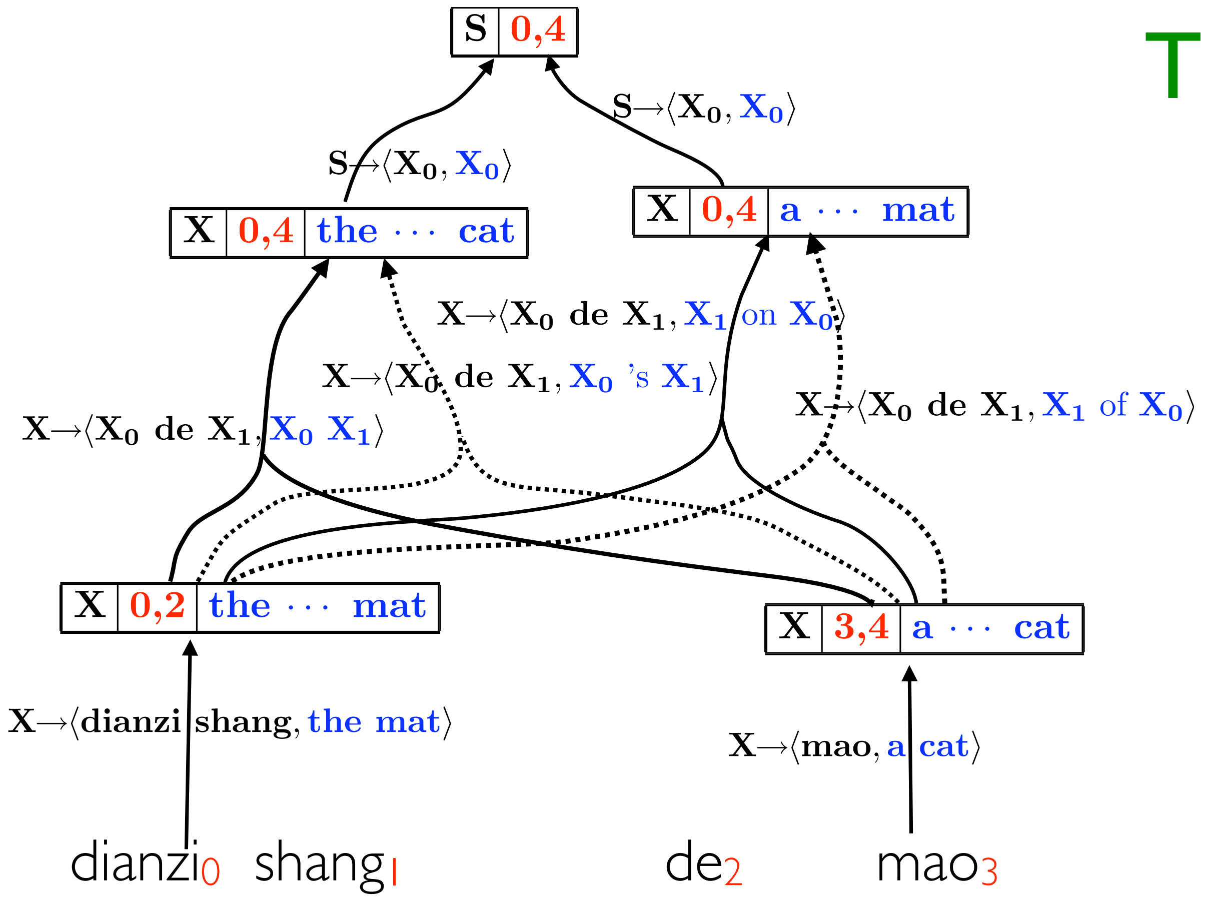
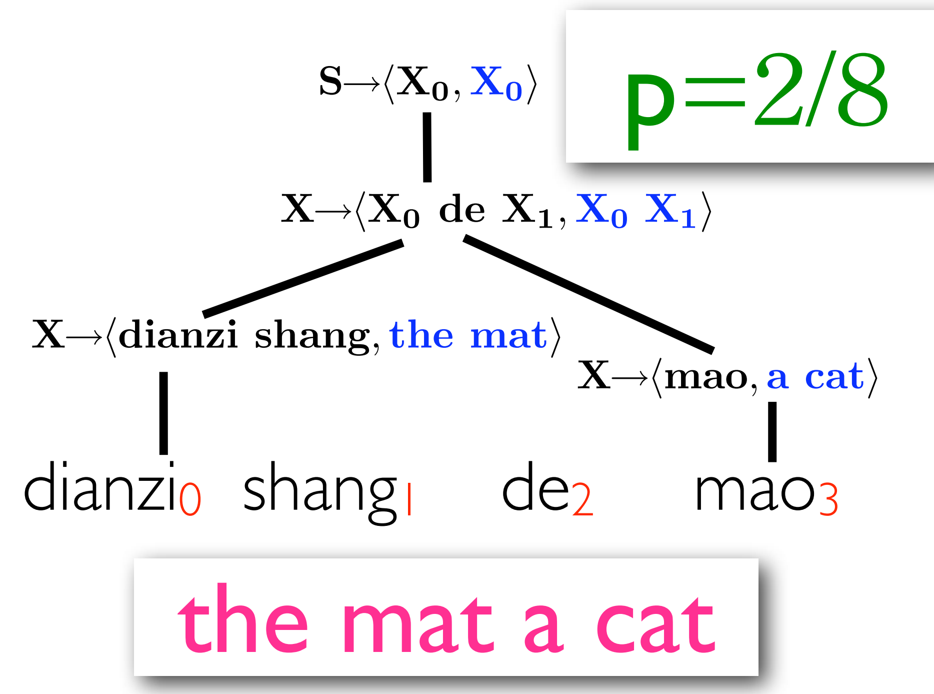
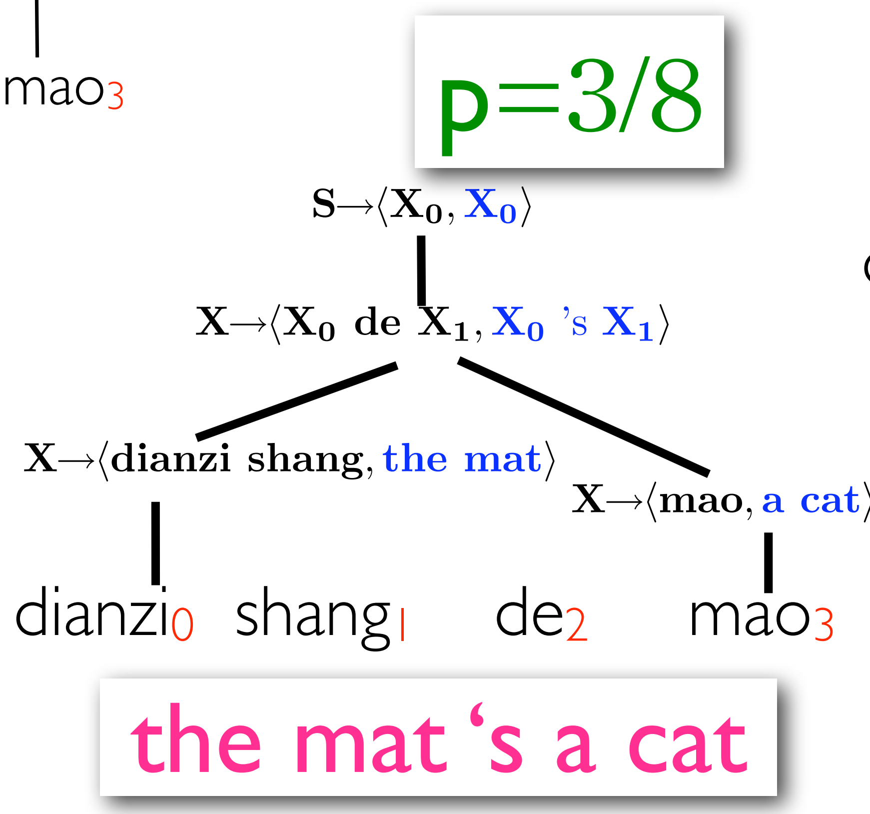
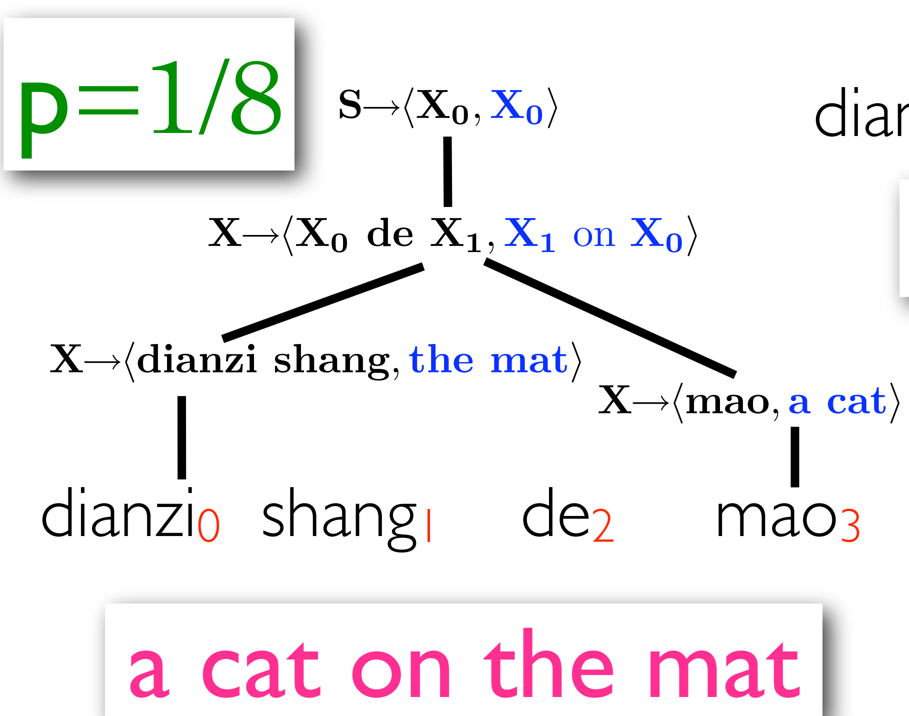
Compute First-order Expectations

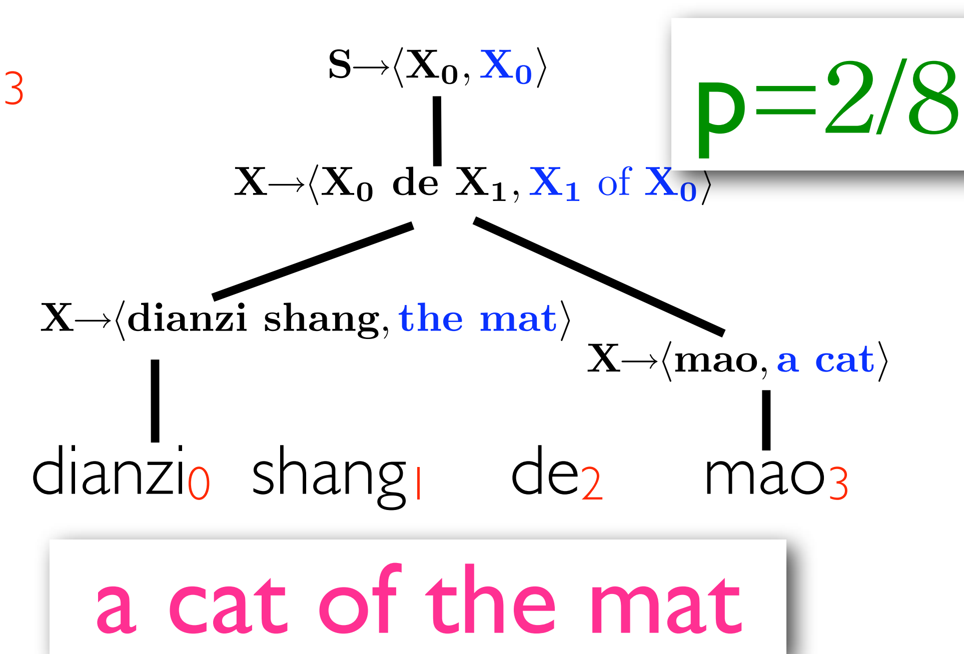
The hypergraph defines a probability distribution over trees!





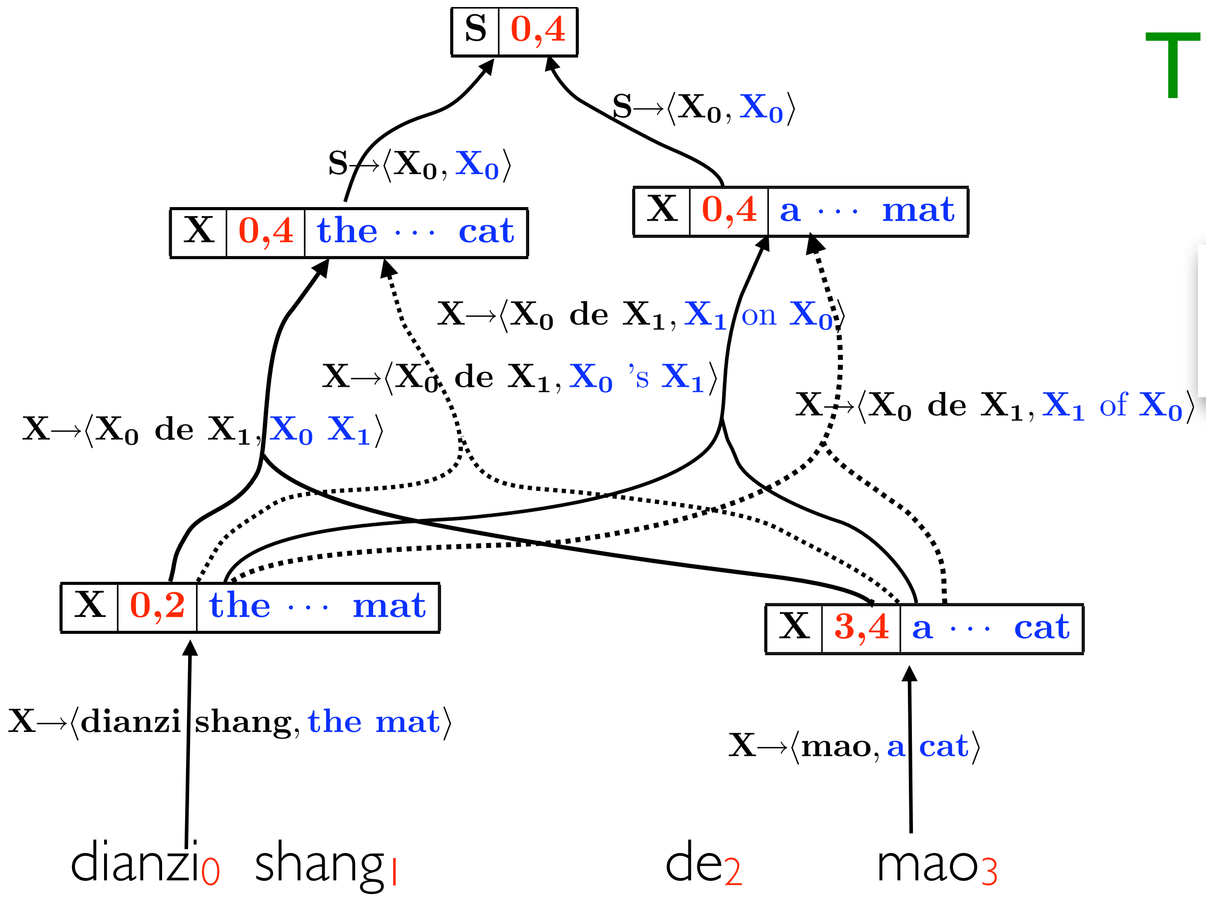




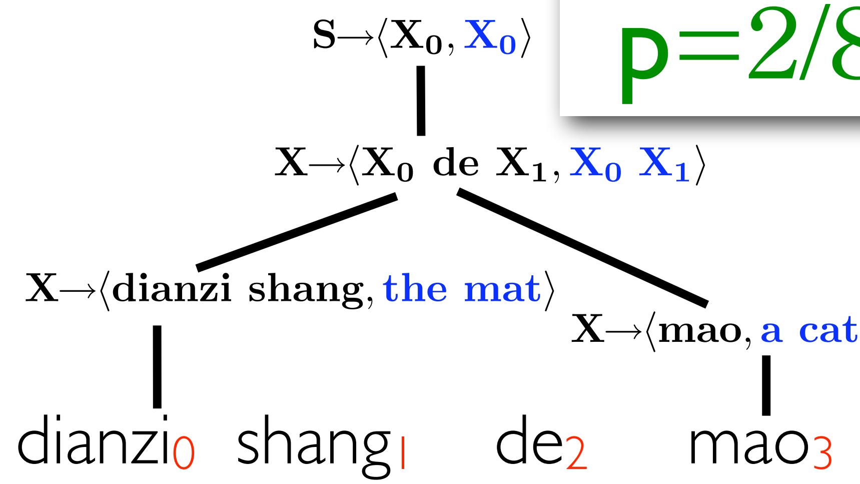


The hypergraph defines a probability distribution over trees!

expected translation length?

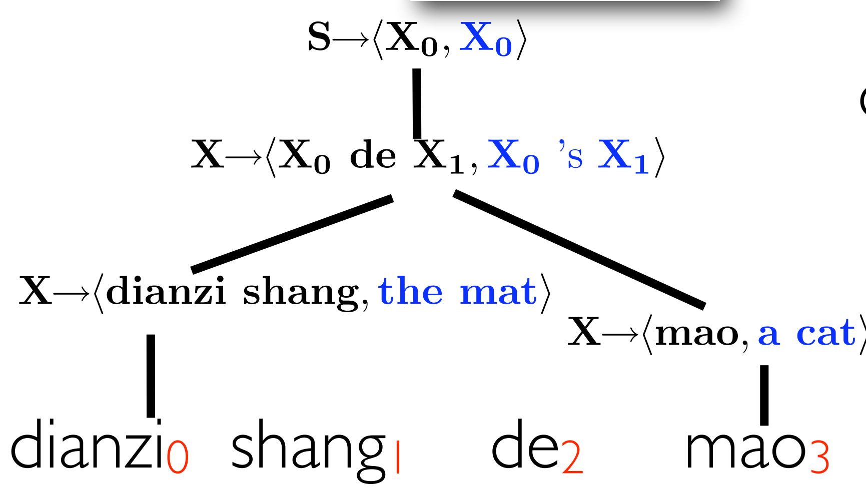


p=2/8



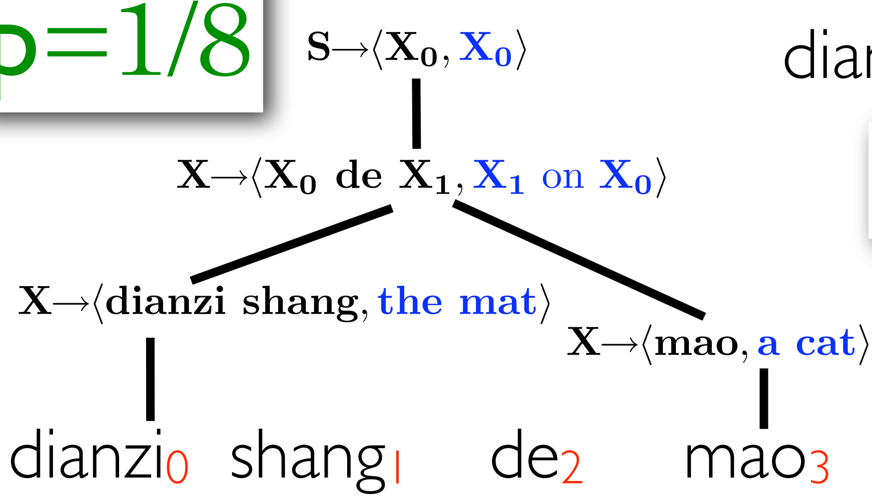
the mat a cat

p=3/8



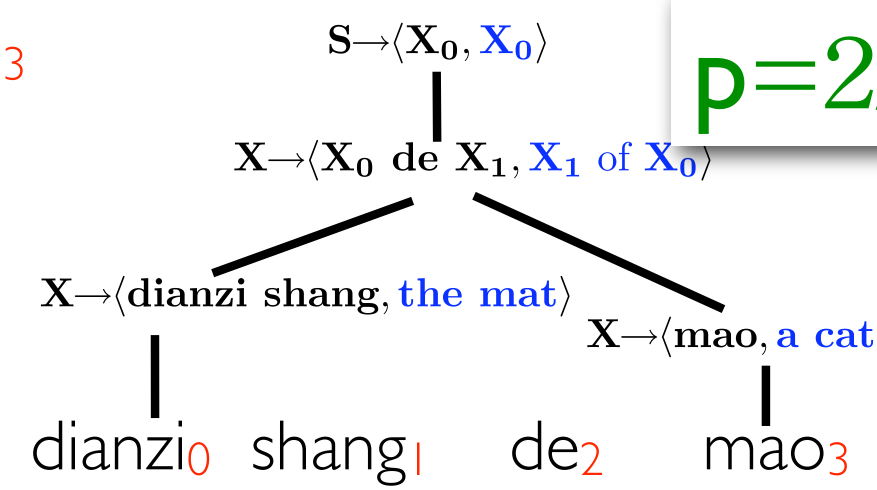
the mat 's a cat

p=1/8



a cat on the mat

p=2/8



a cat of the mat

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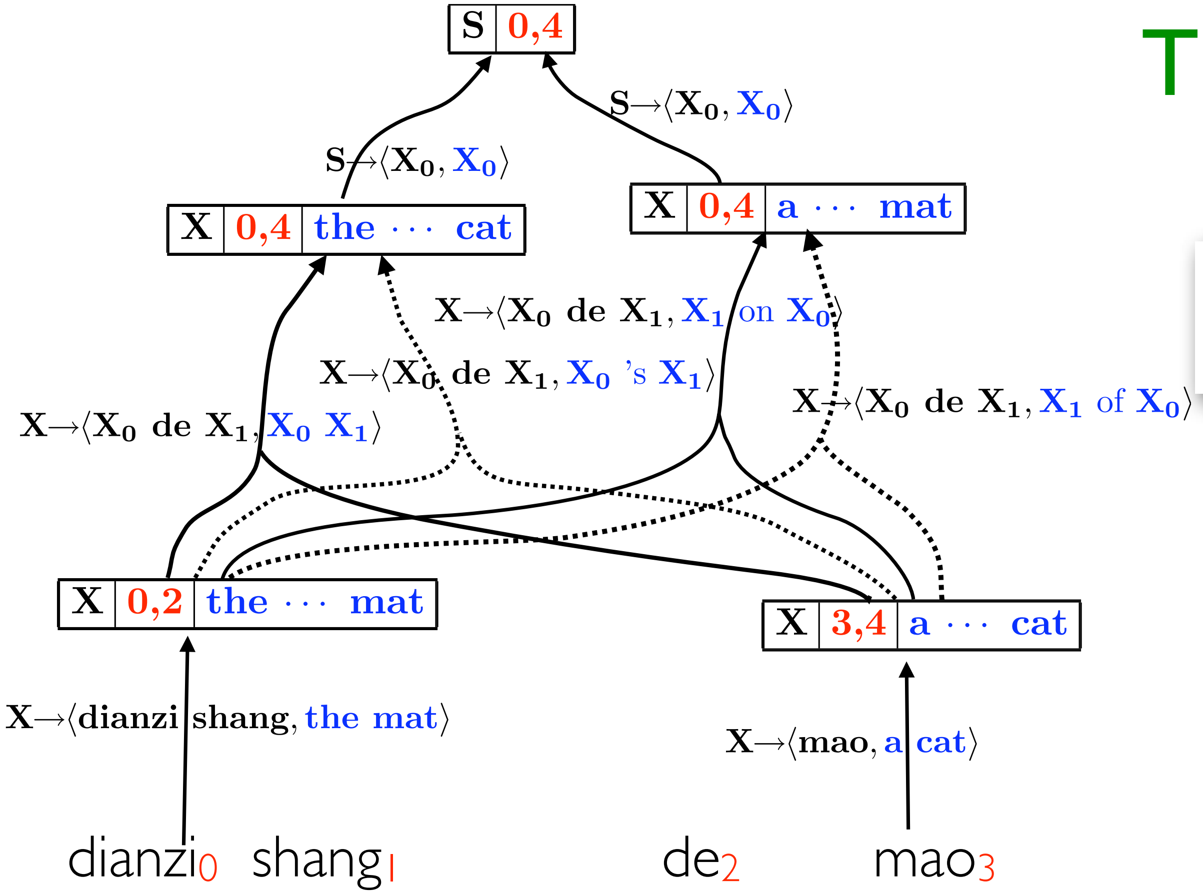
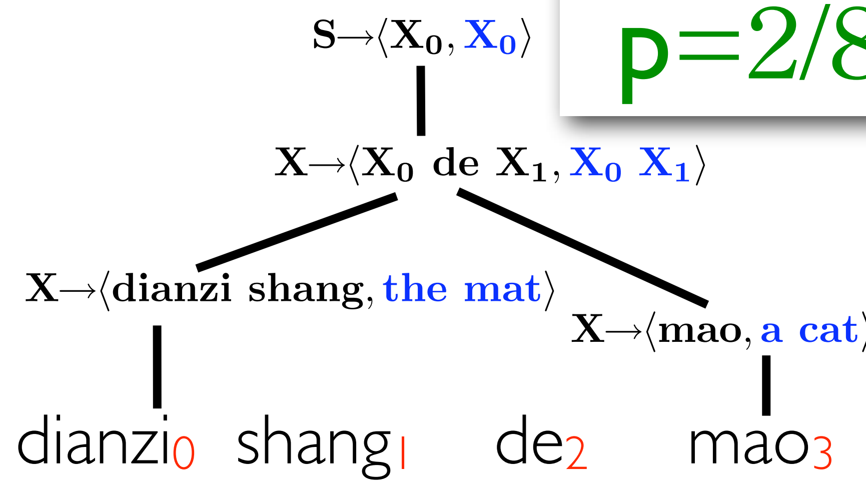
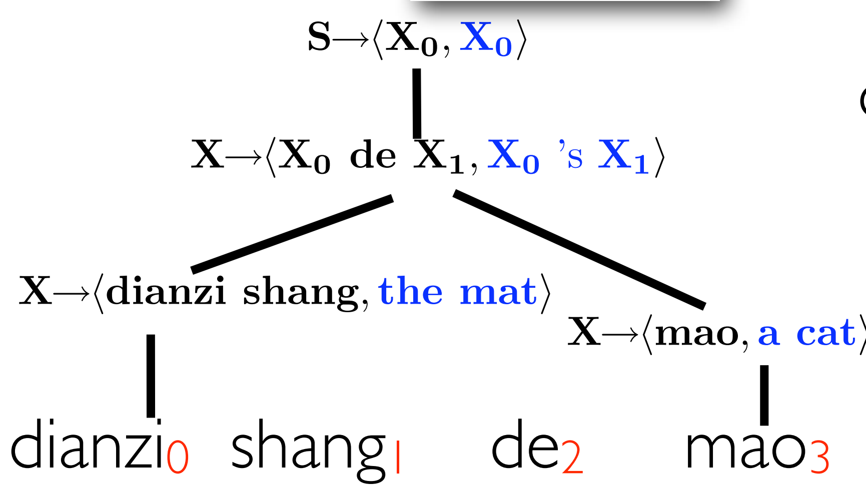


p=2/8



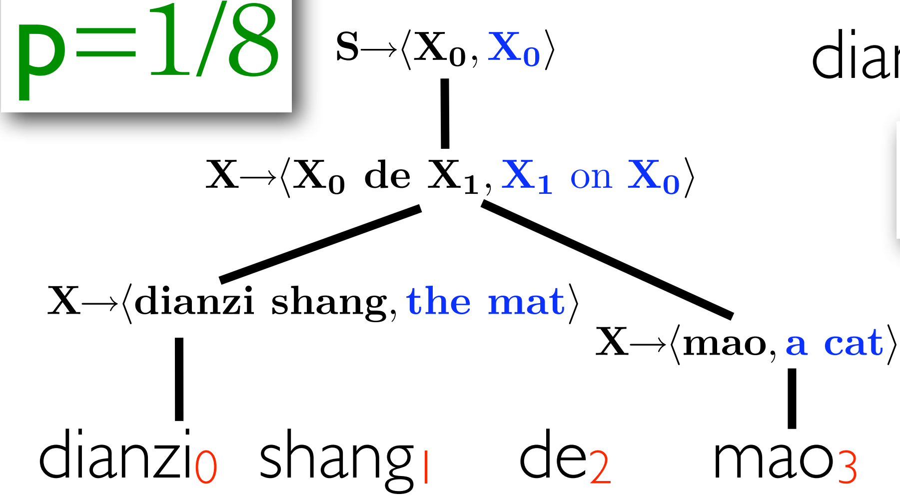
the mat a cat 4

p=3/8



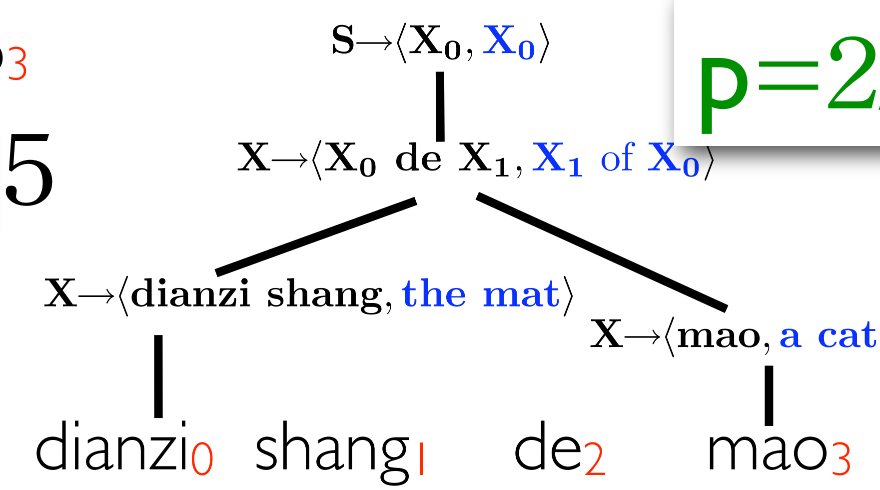
the mat 's a cat 5

p=1/8

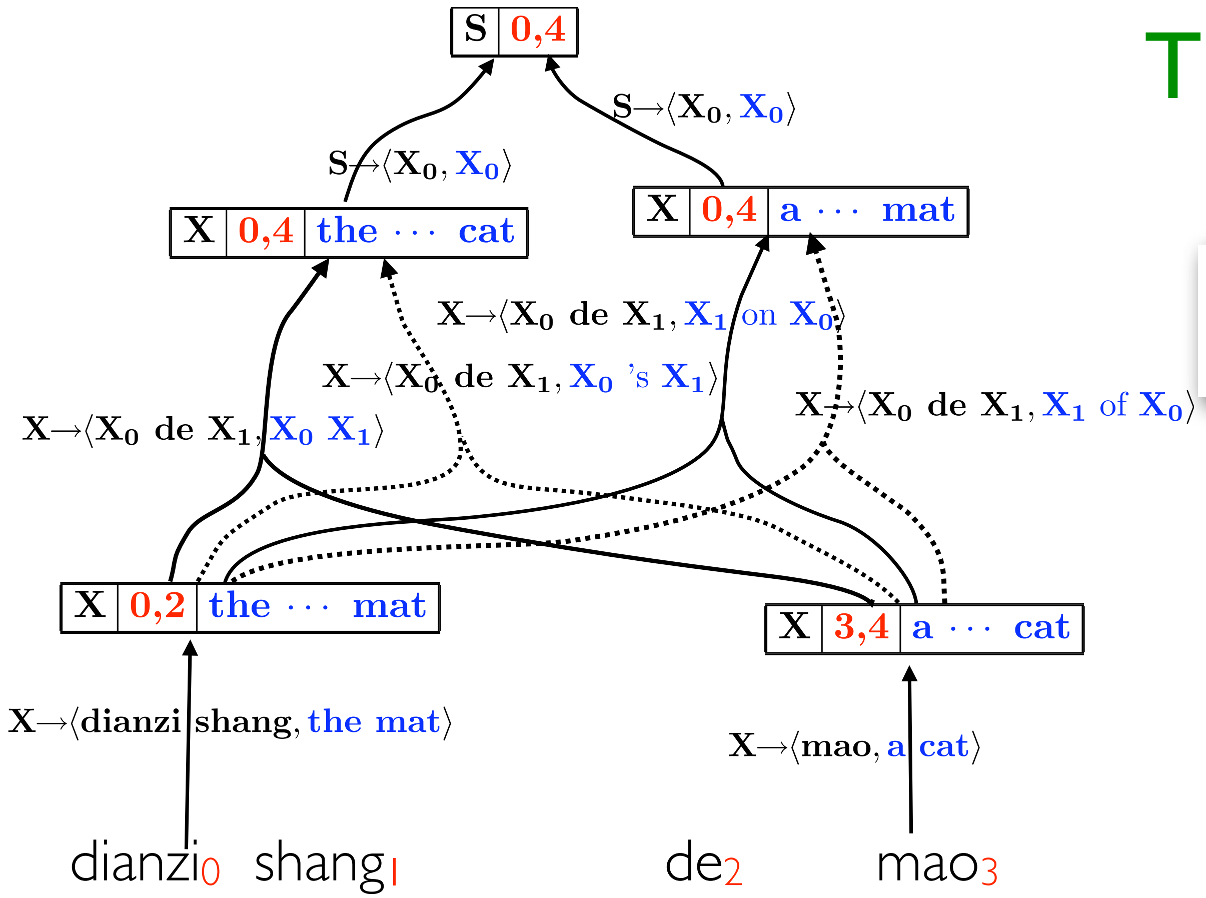


a cat on the mat 5

p=2/8



a cat of the mat 5

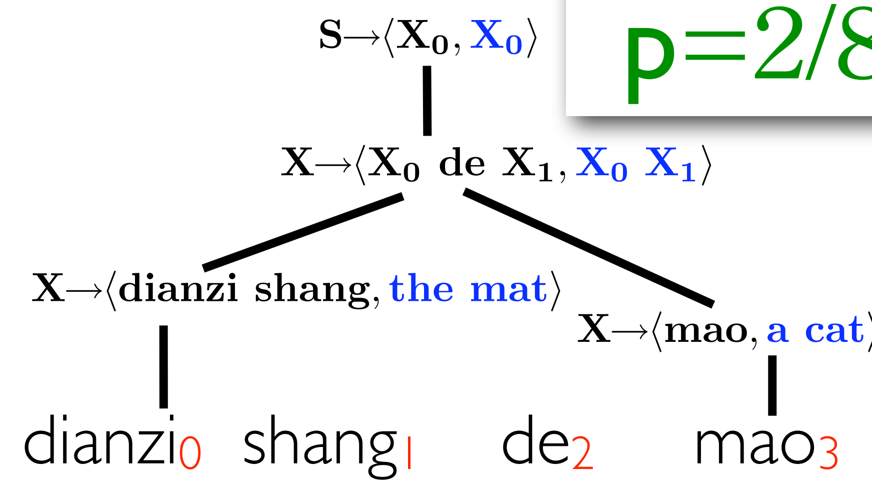


The hypergraph defines a probability distribution over trees!

expected translation length?

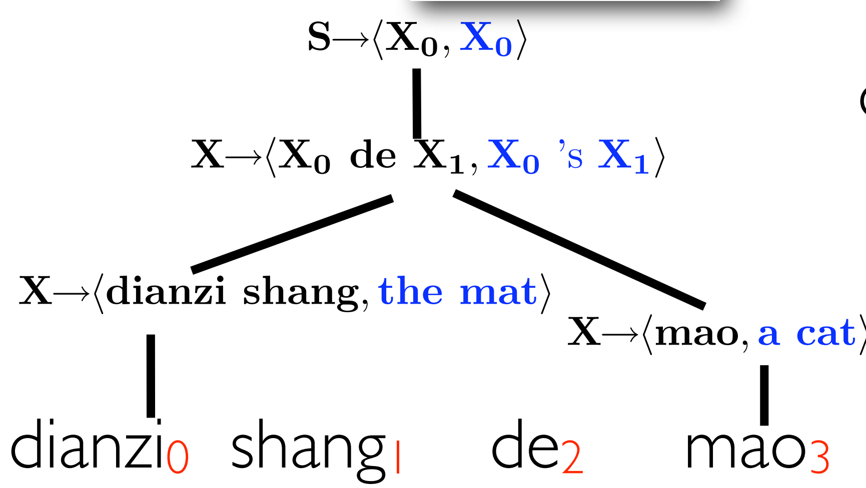
4*2/8+5*6/8=4.75

p=2/8



the mat a cat 4

p=3/8



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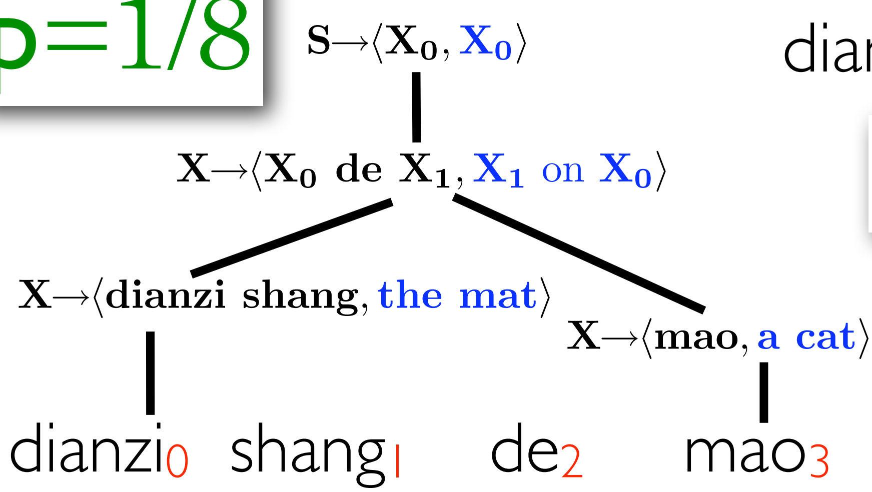

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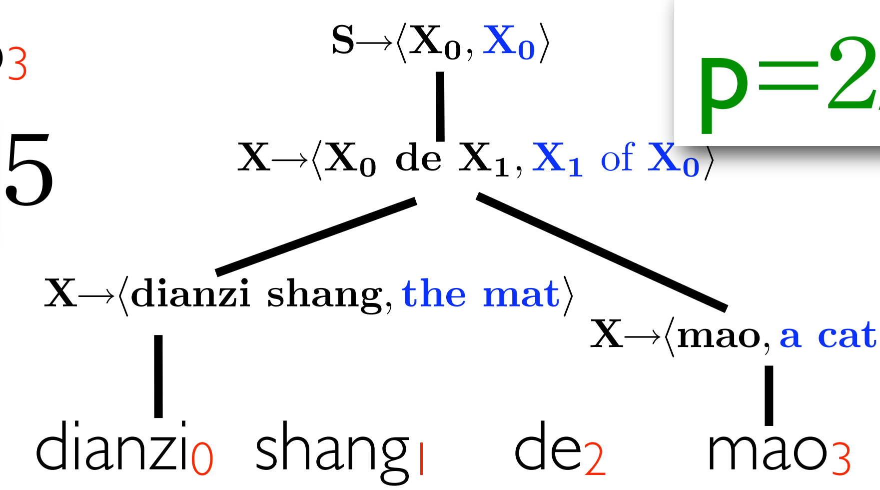

a cat of the mat 5

Compute the expected translation length

using an expectation semiring

Compute Expected Translation Length

- choose a semiring

- specify a weight for each edge

- run the inside algorithm

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  expectation semiring

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$$k_e \stackrel{\text{def}}{=} \langle p_e, p_e r_e \rangle$$

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p_e: transition probability or log-linear score at edge e

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p_e: transition probability or log-linear score at edge e

r_e: number of English words generated at edge e

- run the inside algorithm

Expectations on Hypergraphs

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- Expectation over a hypergraph

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$$\overline{r} \stackrel{\mathrm{def}}{=} \mathbb{E}_p[r] = \sum_{d \in \mathrm{HG}} p(d) r(d)$$

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  - *r*(*d*) is a function over a derivation *d*

    e.g., the length of the translation yielded by *d*

  - *r*(*d*) is additively decomposed

$$r(d) \stackrel{\text{def}}{=} \sum_{e \in d} r_e$$

e.g., translation length is additively decomposed!

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log $p(d)$ is additively decomposed!

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Why? cross-entropy is an **expectation**

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(Tromble et al. 2008)

Why?

Bayes risk is an **expectation**

$$\text{Risk} = \mathbb{E}_p(L) = -\sum_{d \in \text{HG}} p(d) \cdot L(Y(d))$$

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Why?

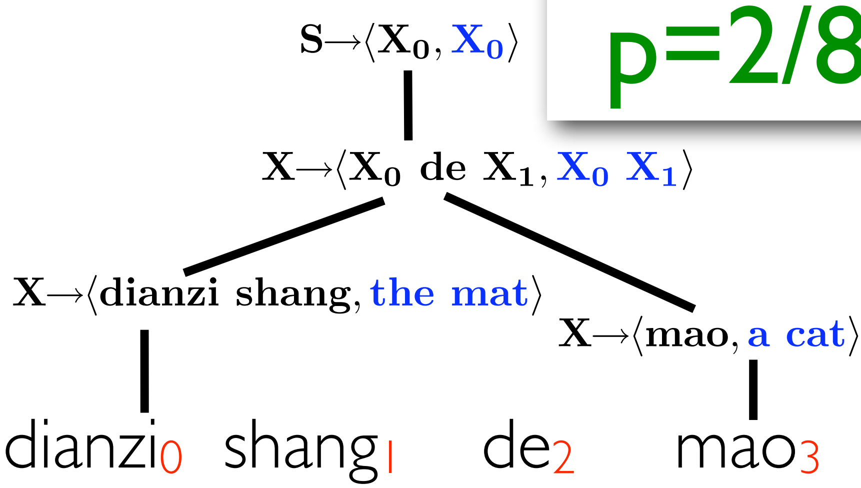
Bayes risk is an **expectation**

$$\text{Risk} = \mathbb{E}_p(L) = -\sum_{d \in \text{HG}} p(d) \cdot L(Y(d))$$

L(*Y*(*d*)) is additively decomposed!

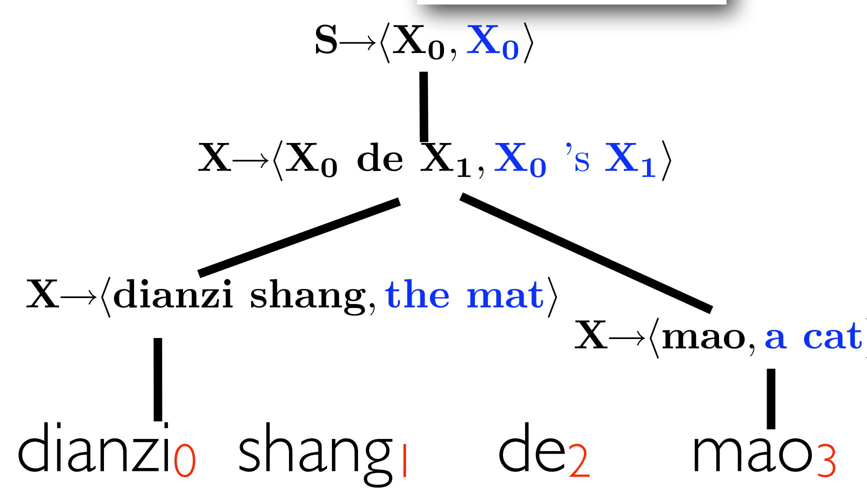
Compute Expectations over Products

p=2/8



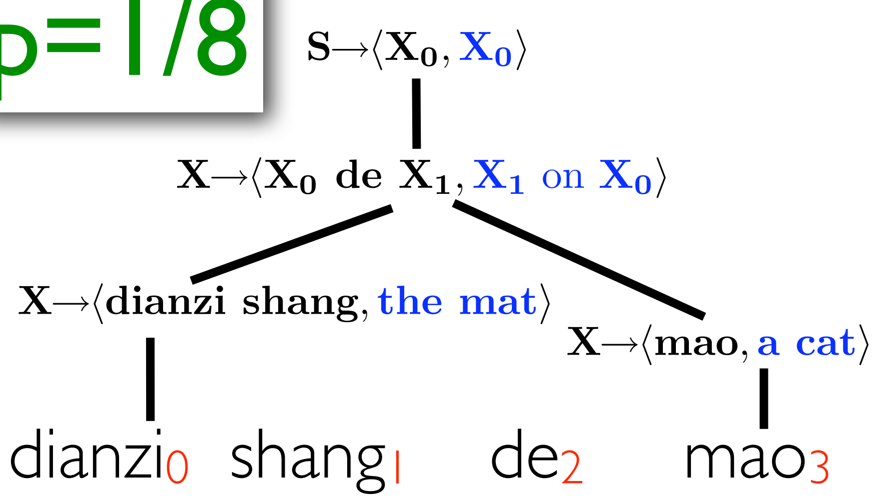
the mat a cat

p=3/8



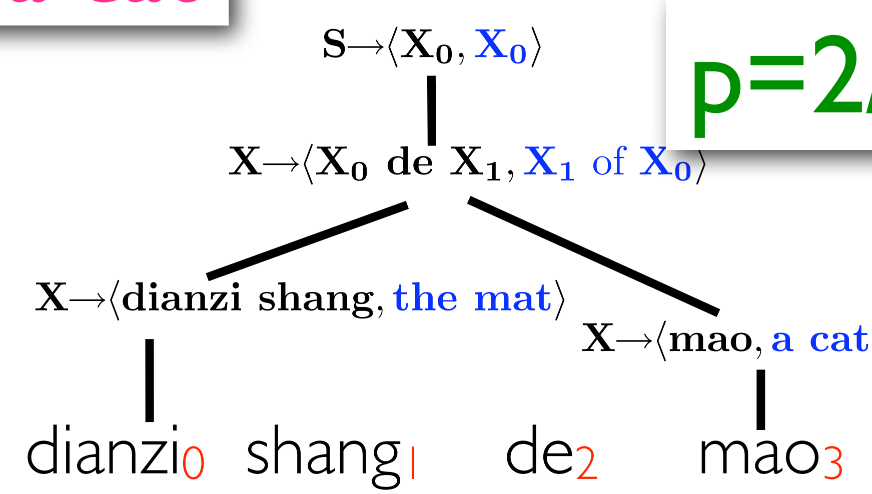
the mat 's a cat

p=1/8



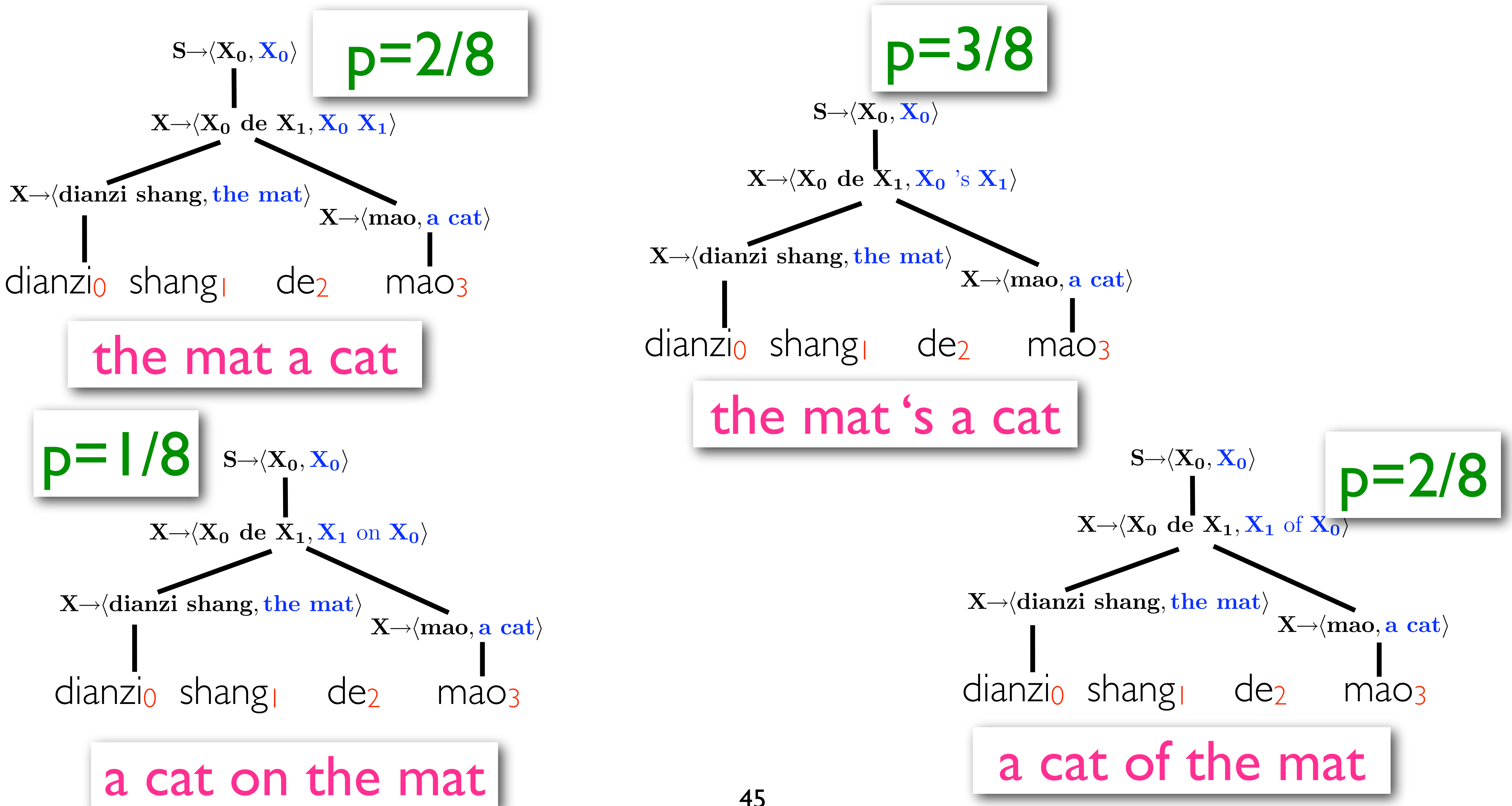
a cat on the mat

p=2/8

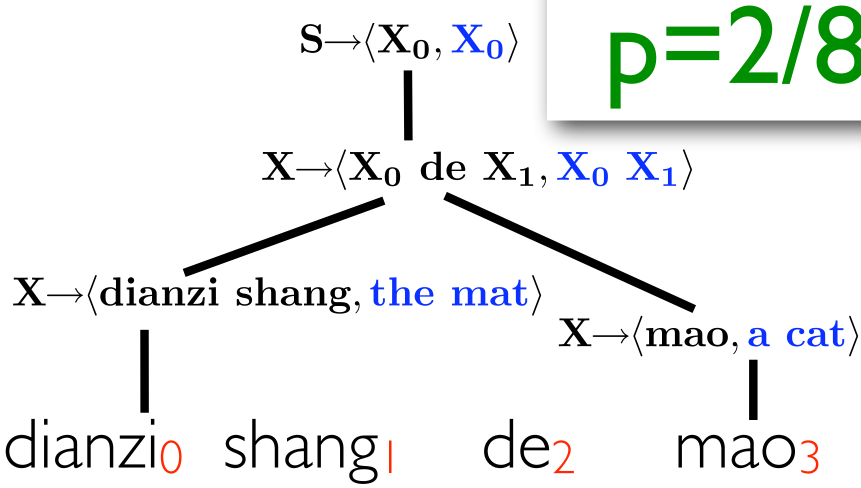


a cat of the mat

expected translation length:



expected translation length: 4.75



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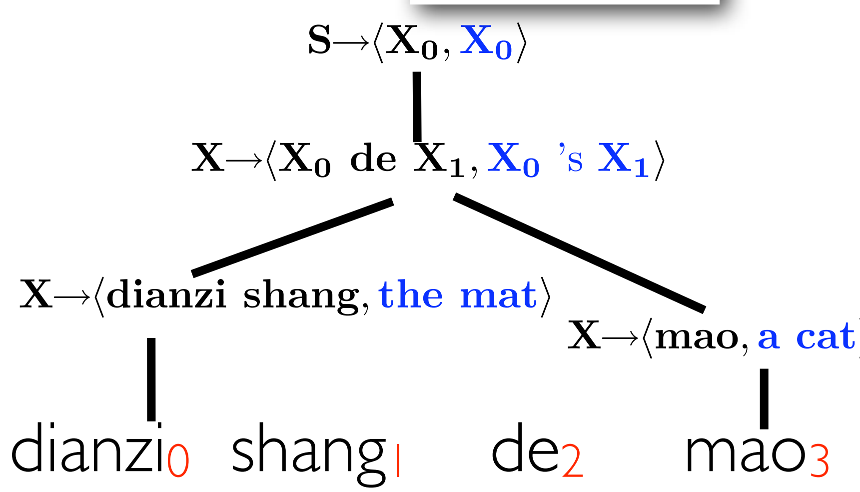

p=3/8

the mat 's a cat

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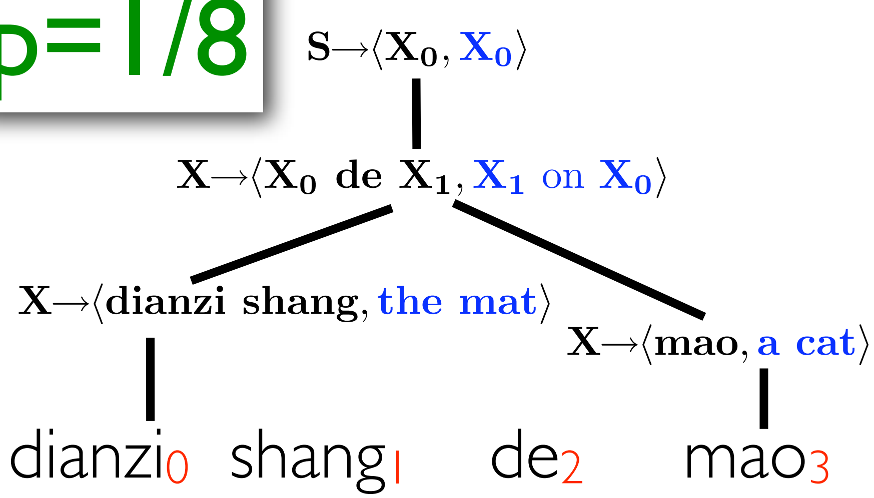

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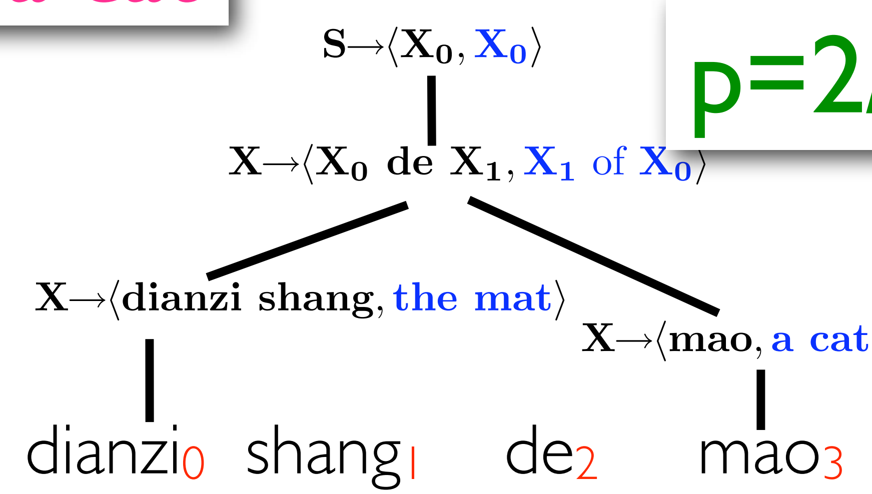


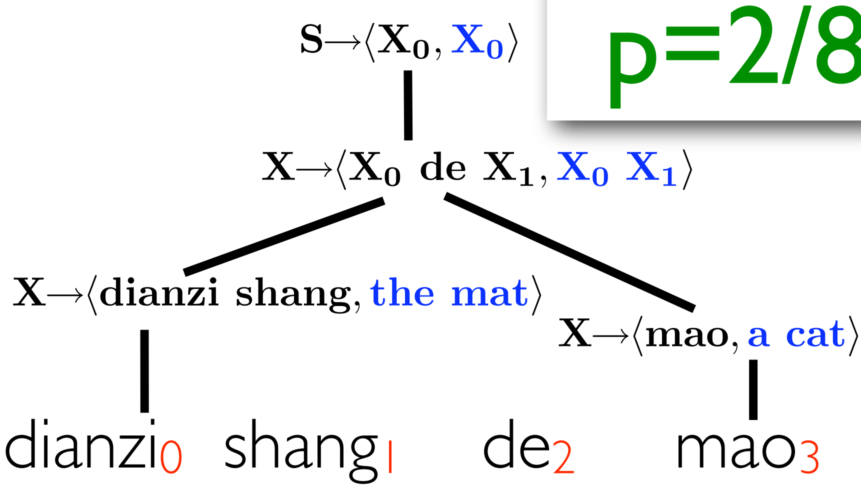
p=2/8

a cat of the mat

expected translation length: 4.75

variance of translation length?

p=2/8



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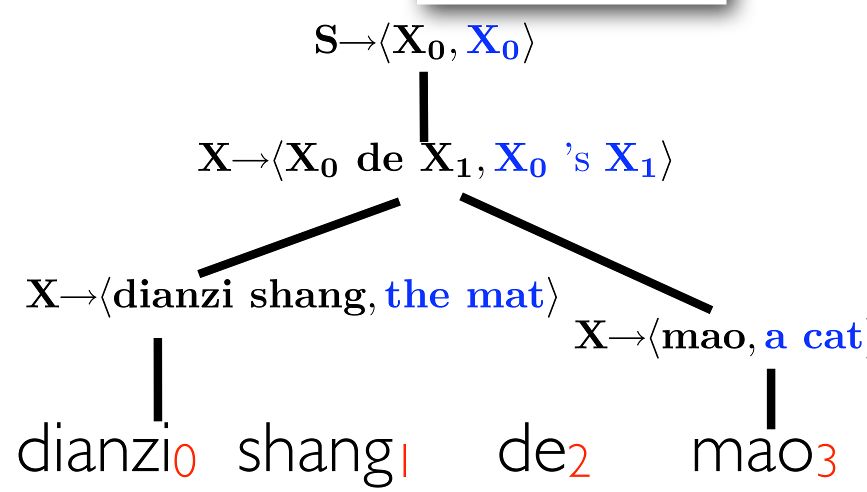

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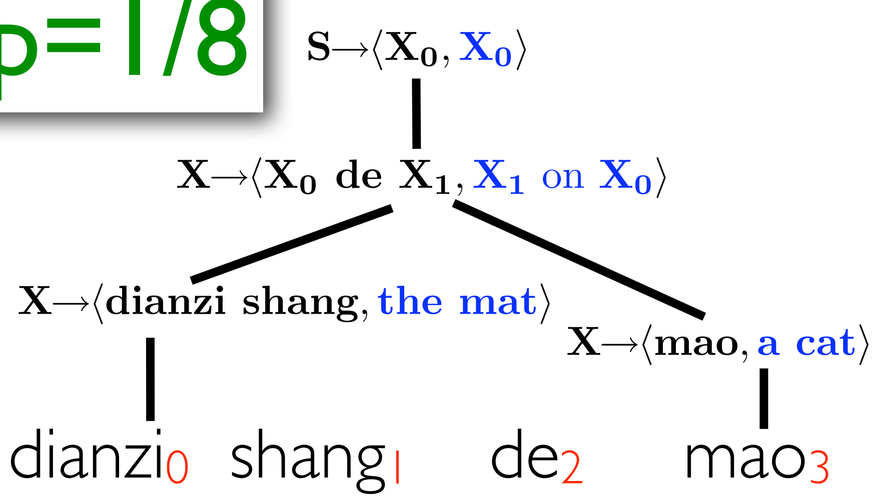

a cat on the mat

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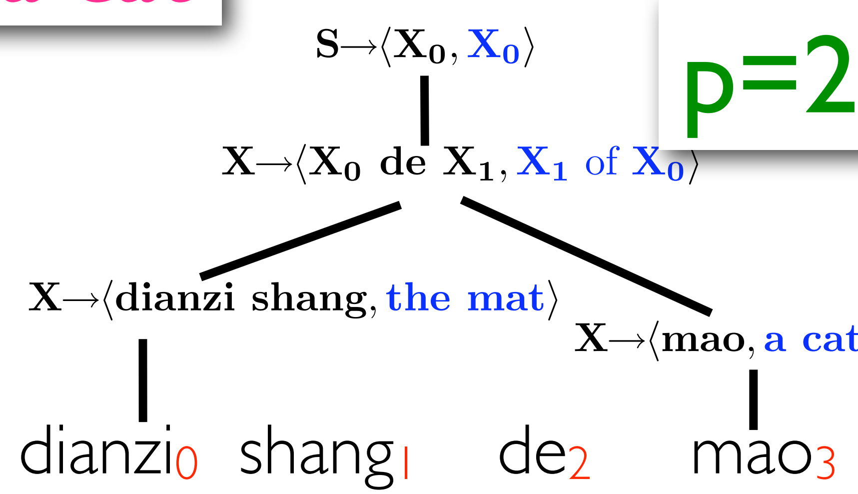


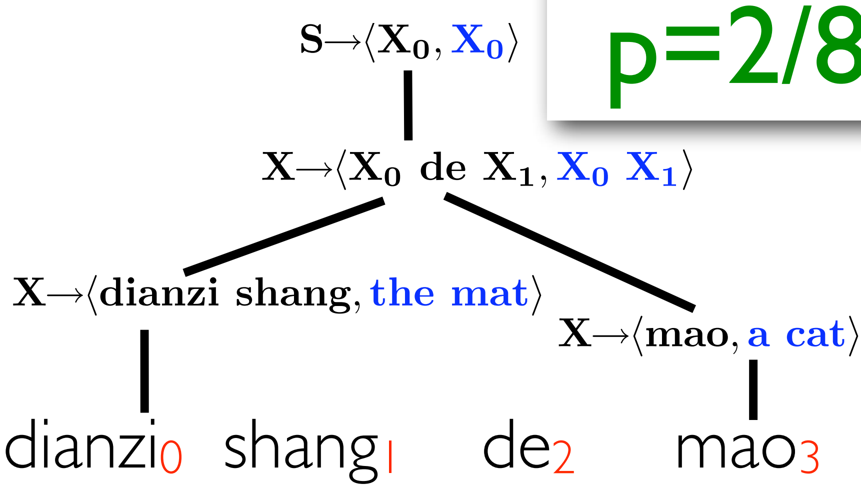
a cat of the mat

expected translation length: 4.75

variance of translation length?

2/8×(4-4.75)^2 + 6/8×(5-5.75)^2 ≈ 0.56

p=2/8



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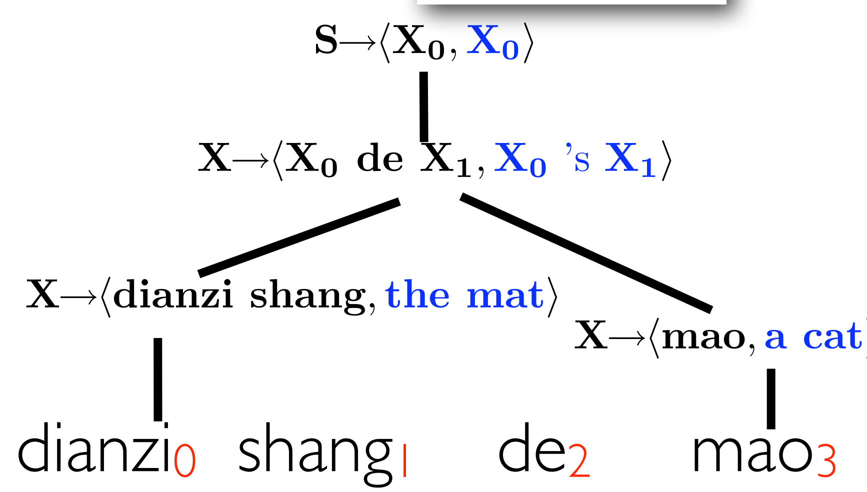

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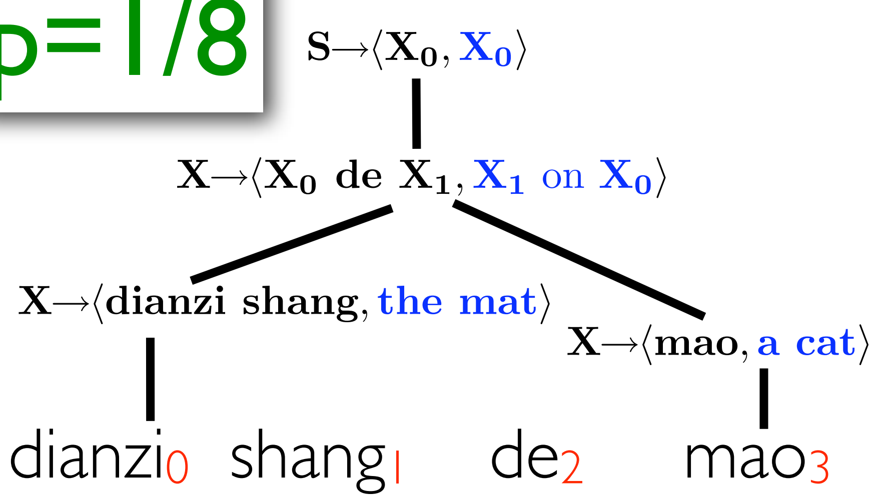

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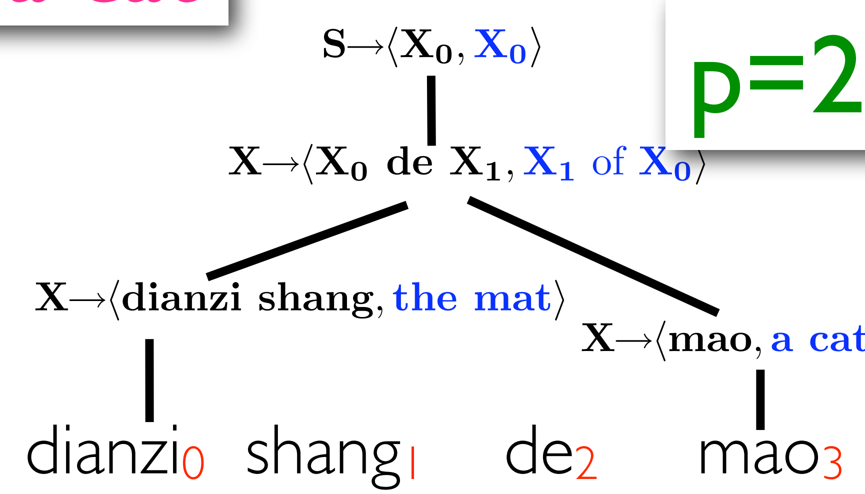

a cat of the mat

Compute the Variance in Translation Length

- choose a semiring
- specify a weight for each edge
- run the inside algorithm

Compute the Variance in Translation Length

- choose a semiring

  second-order expectation semiring

- specify a weight for each edge

- run the inside algorithm

Compute the Variance in Translation Length

- choose a semiring

  second-order expectation semiring

- specify a weight for each edge

$$k_e \stackrel{\text{def}}{=} \langle p_e, p_e r_e, p_e s_e, p_e r_e s_e \rangle$$

p_e: transition probability or log-linear score at edge e

$r_e = s_e$: number of English words generated at edge e

- run the inside algorithm

Second-order Expectations on Hypergraphs

- Expectation of **products** over a hypergraph

$$\bar{t} \stackrel{\text{def}}{=} \mathbb{E}_p[r \cdot s] = \sum_{d \in \mathrm{HG}} p(d) r(d) s(d)$$

- r and s are additively decomposed

$$r(d) \stackrel{\text{def}}{=} \sum_{e \in d} r_e$$

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r and s can be identical or different functions.

Applications of expectation semirings: a summary

First-order:

Quantity	Weight for edge e	Value at root
Expectation	$\langle p_e, p_e r_e \rangle$	$\langle Z, \overline{r} \rangle$
First-order gradient	$\langle p_e, \nabla p_e \rangle$	$\langle Z, \nabla Z \rangle$

Second-order:

Covariance matrix	$\langle p_e, p_e r_e, p_e s_e, p_e r_e s_e \rangle$	$\langle Z, \overline{r}, \overline{s}, \overline{t} \rangle$
Hessian matrix	$\langle p_e, \nabla p_e, \nabla p_e, \nabla^2 p_e \rangle$	$\langle Z, \nabla Z, \nabla Z, \nabla^2 Z \rangle$
Gradient of expectation	$\langle p_e, p_e r_e, \nabla p_e, (\nabla p_e) r_e + p_e (\nabla r_e) \rangle$	$\langle Z, \overline{r}, \nabla Z, \nabla \overline{r} \rangle$

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- define a weight for each edge
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$$p_e = \exp(\Phi_e \cdot \theta)$$

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$$p_e = \exp(\Phi_e \cdot \theta) \qquad \nabla p_e = p_e \Phi_e$$

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Entropy is an expectation!

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Bayes risk is an expectation!

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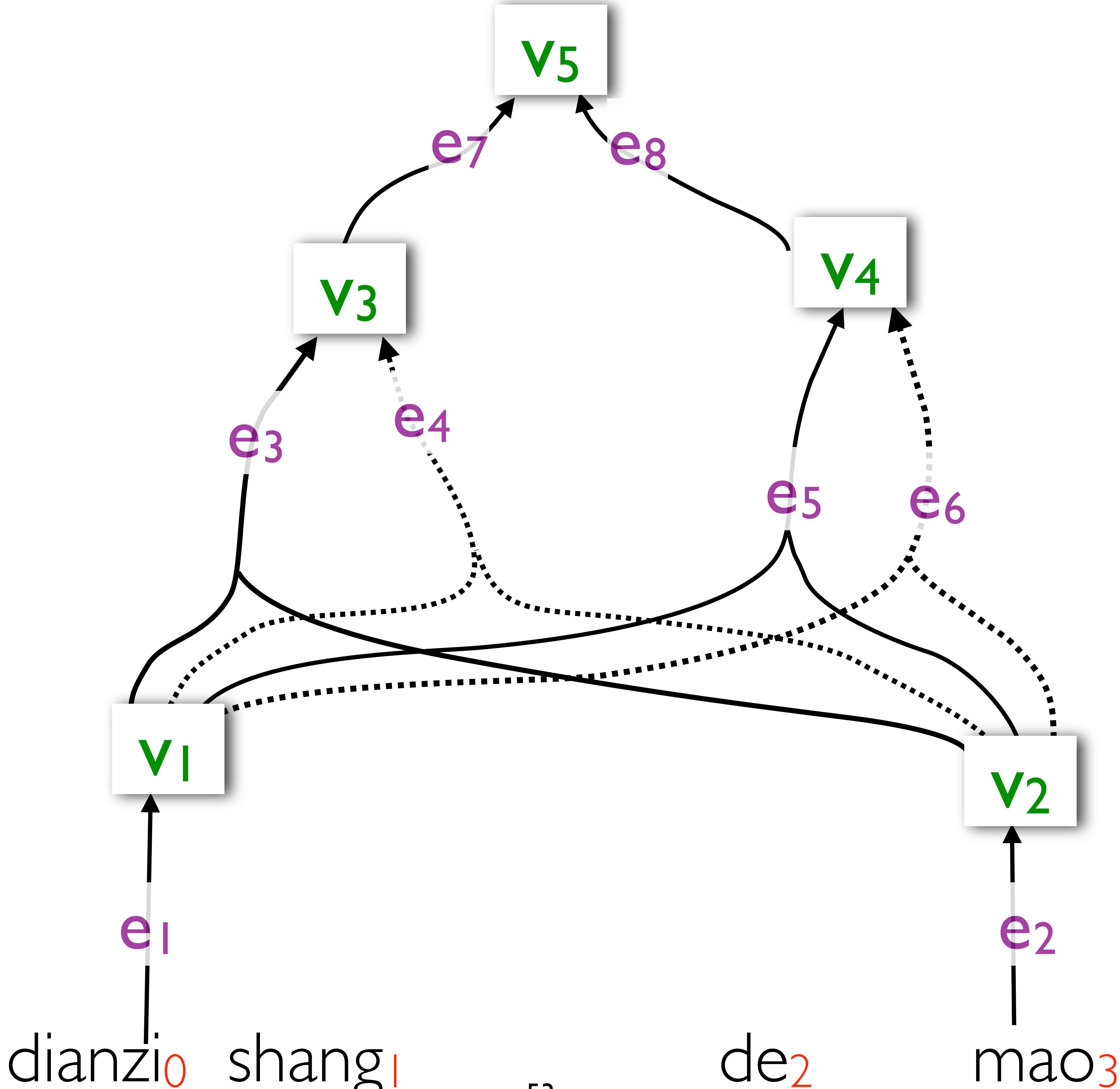
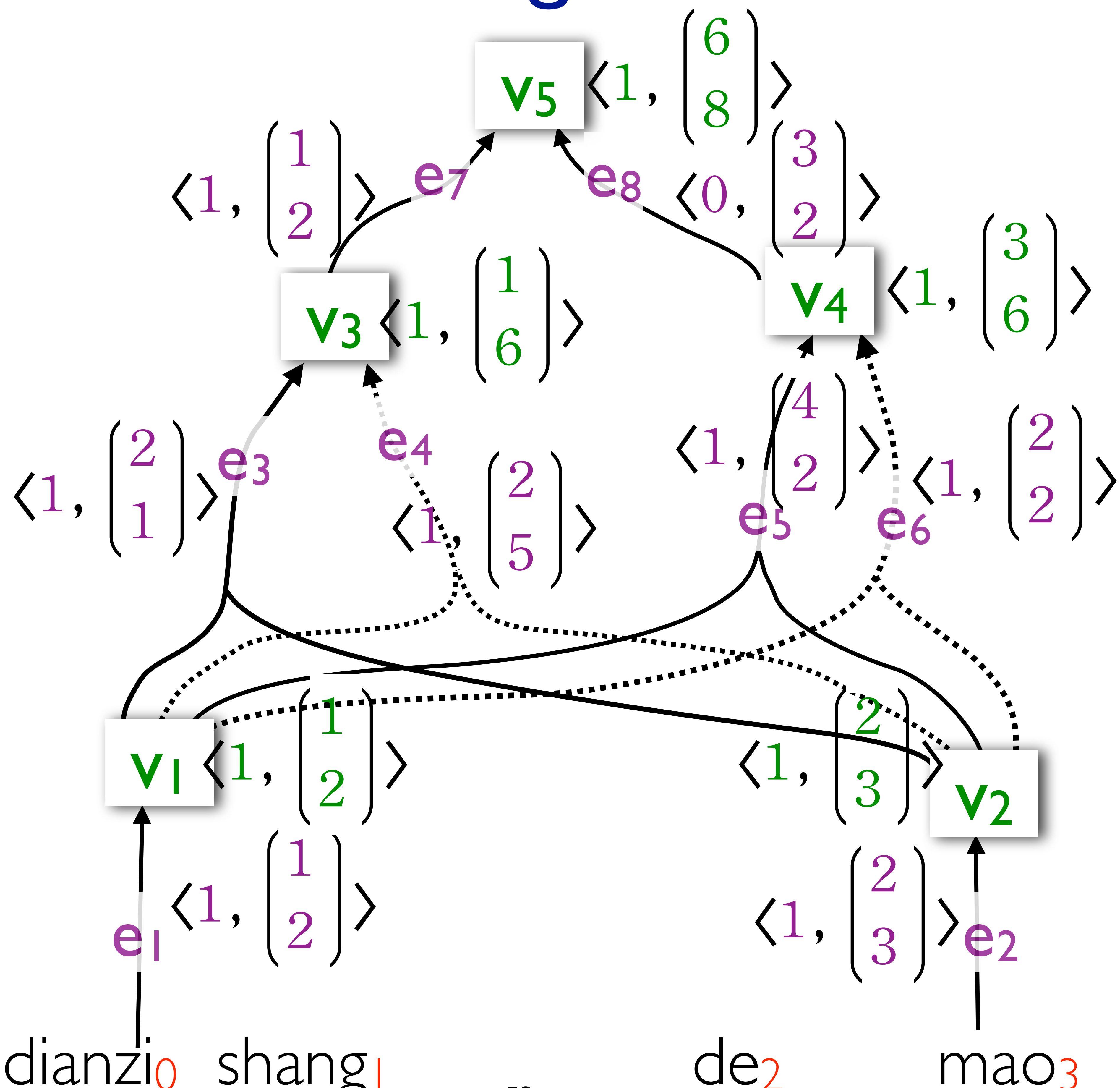
Inside-Outside Speedup

Why is it slow?

Expectation Semirings with Vectors

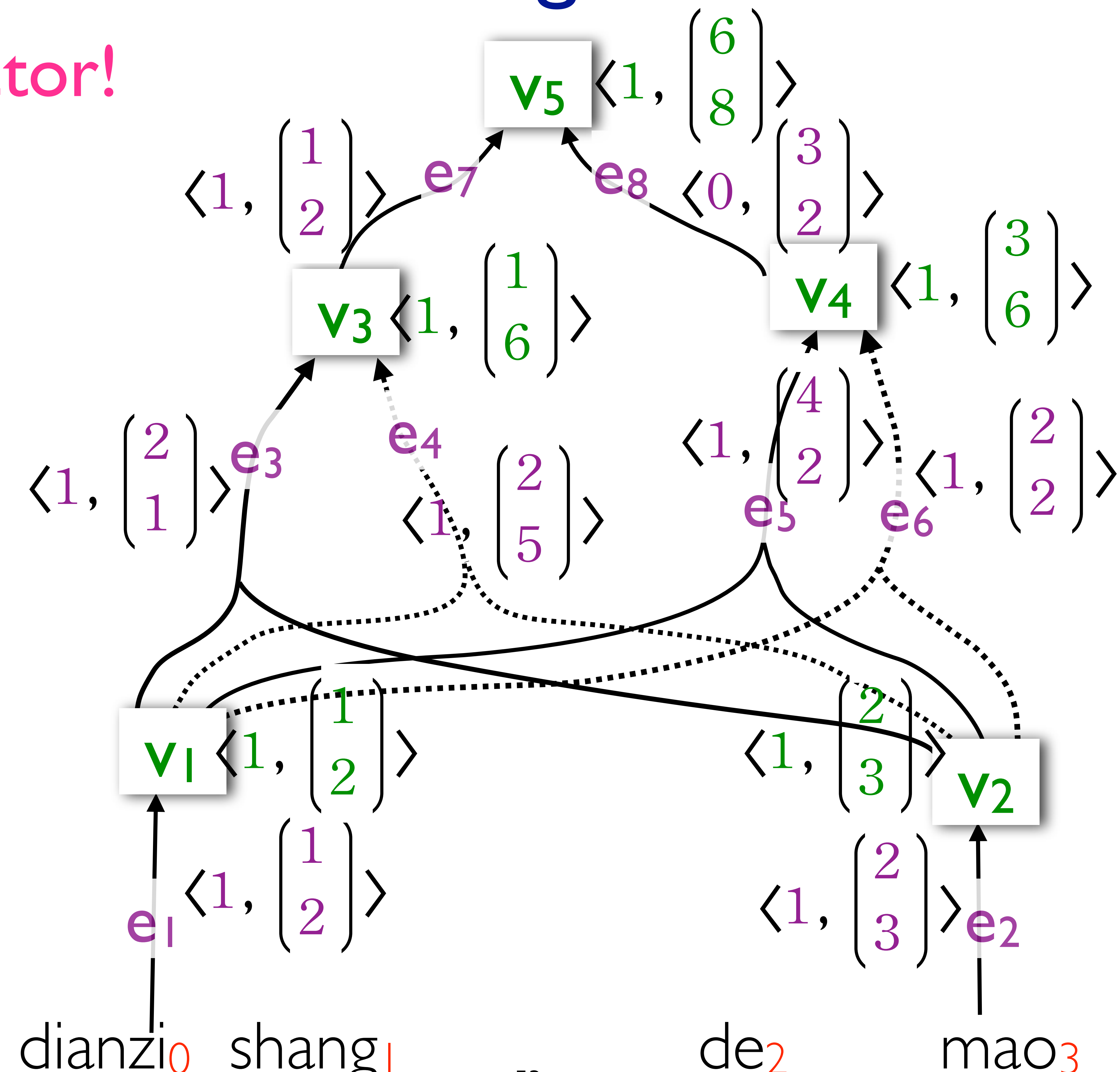


Expectation Semirings with Vectors



Expectation Semirings with Vectors

r is a vector!



Wednesday, August 19, 2009

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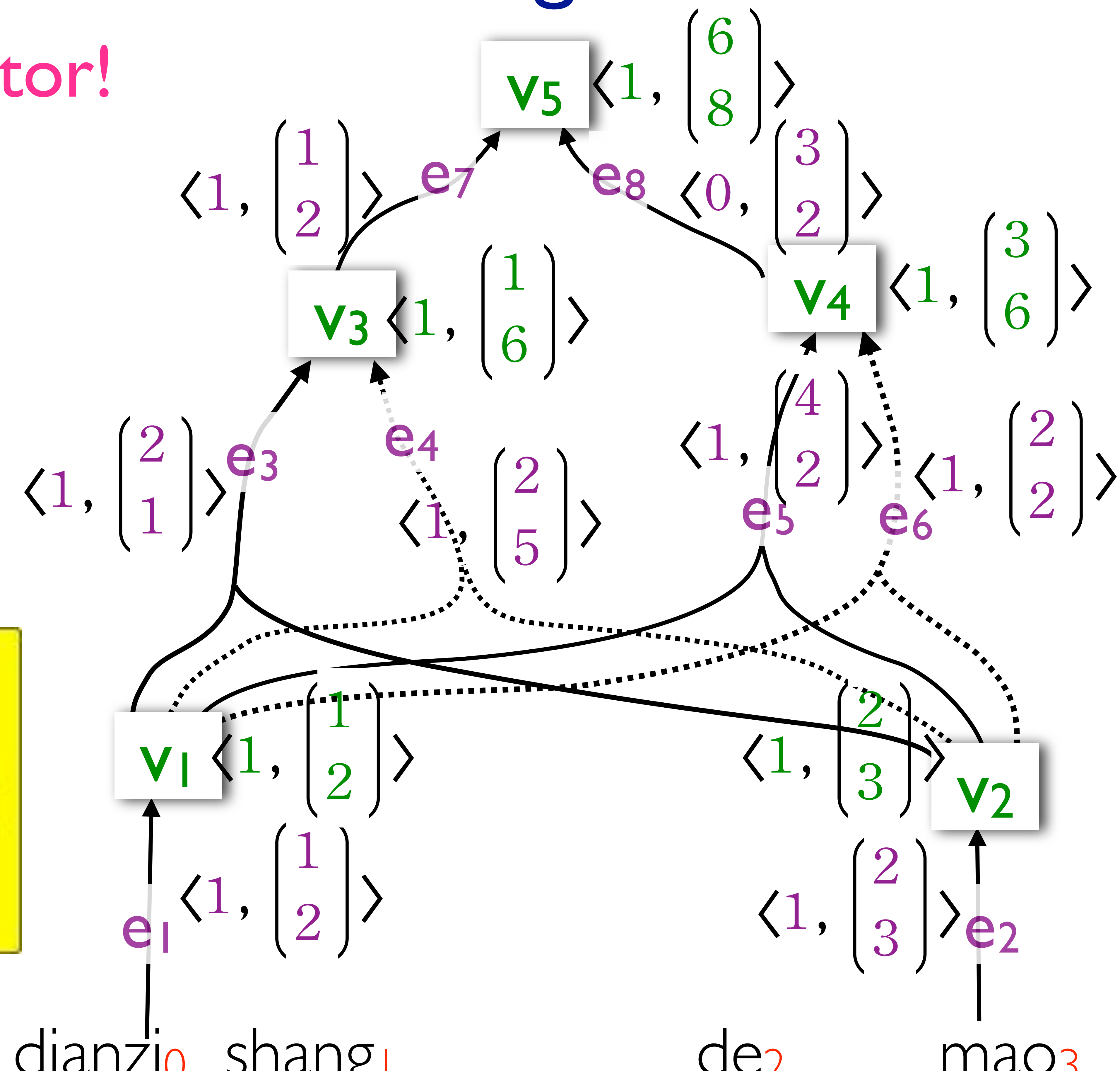
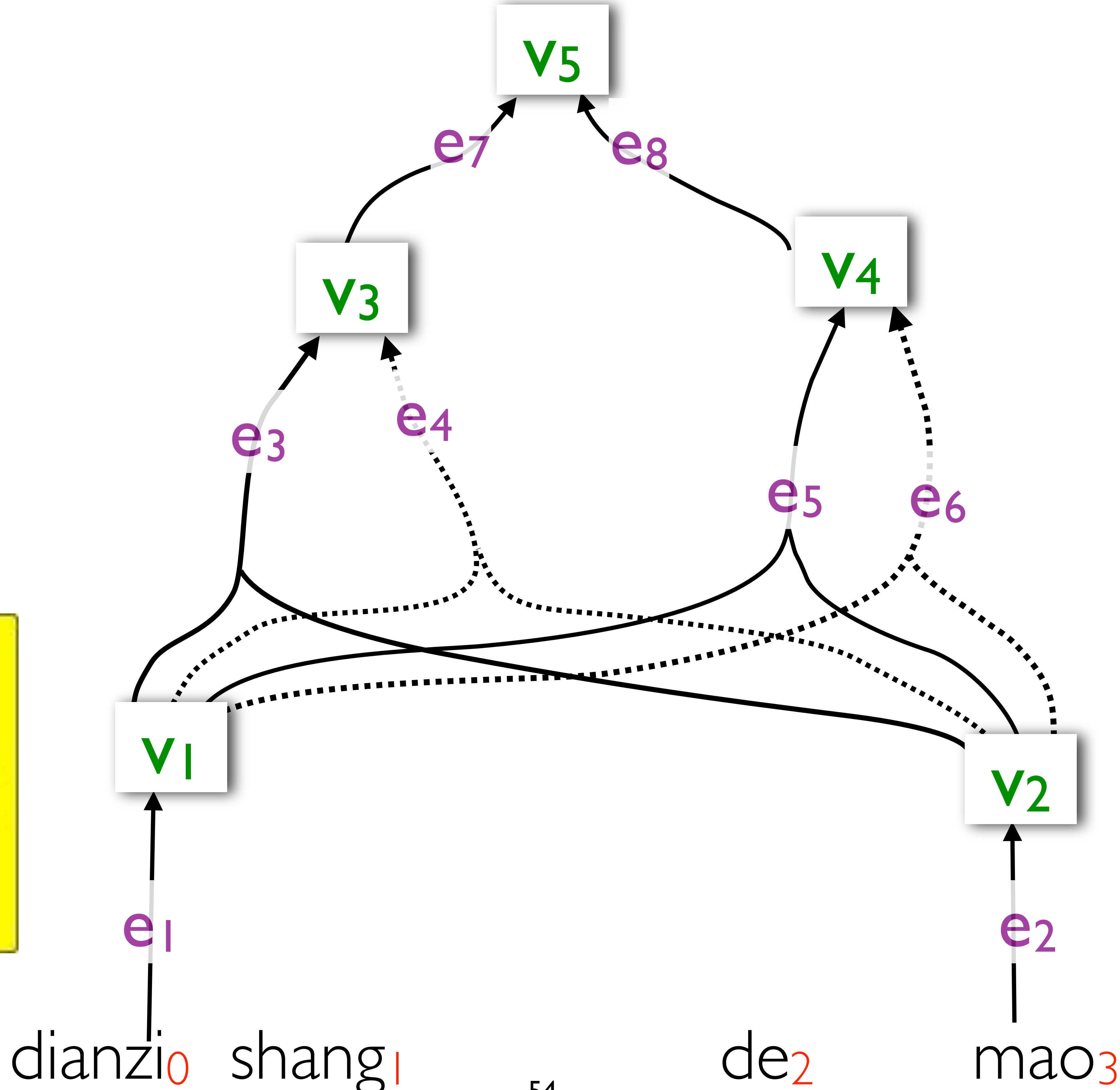


Second-order Expectation Semirings



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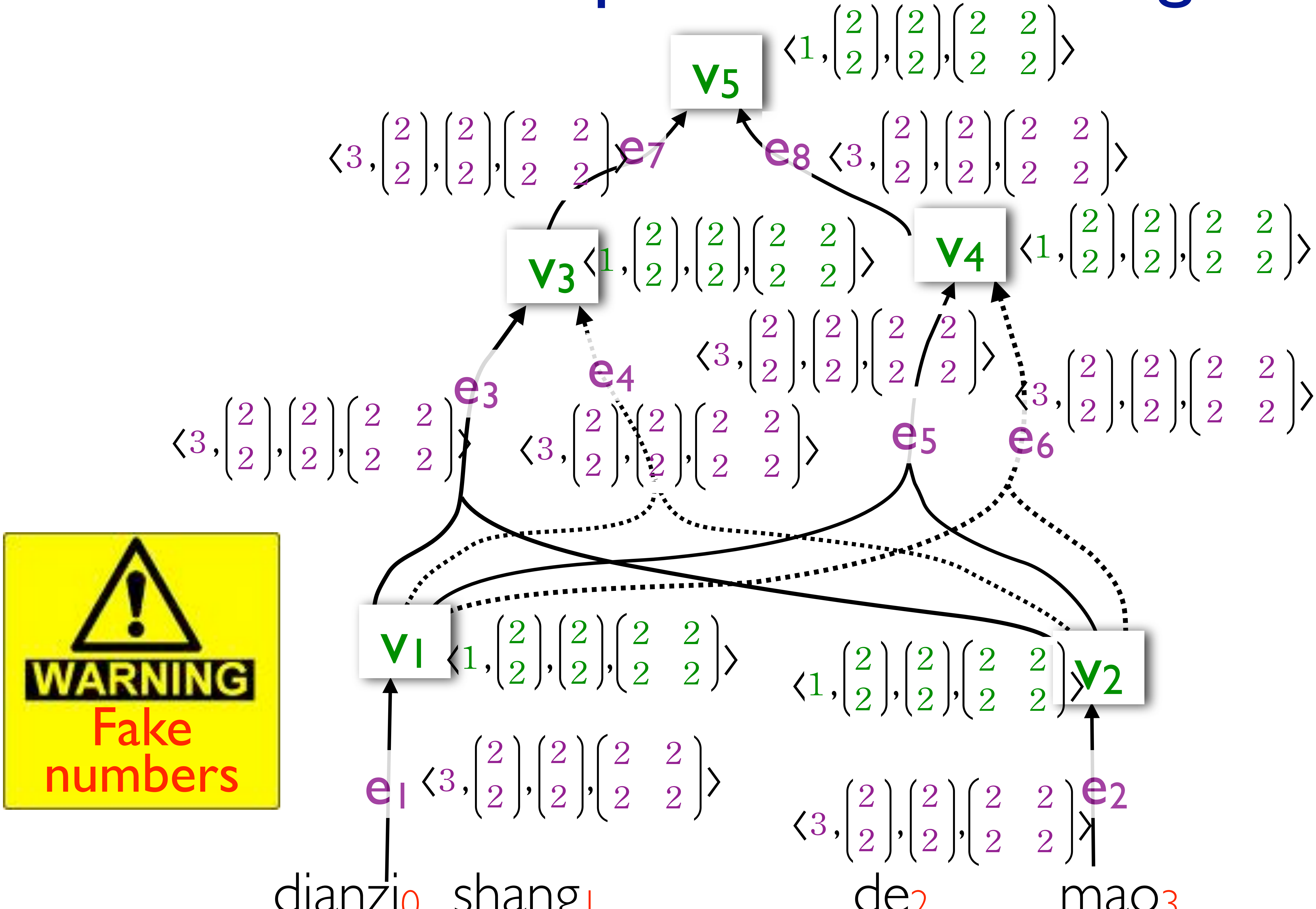
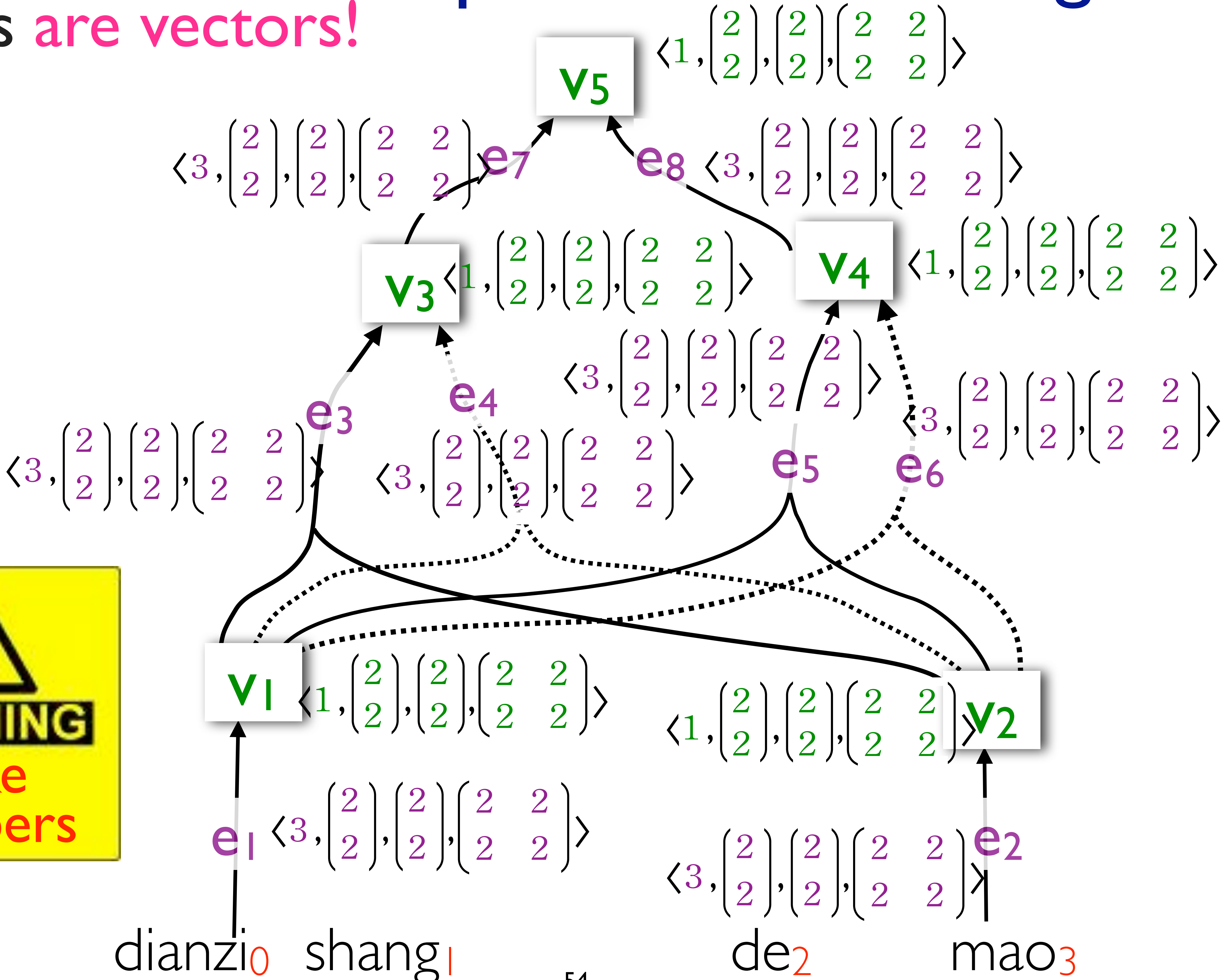


Second-order Expectation Semirings

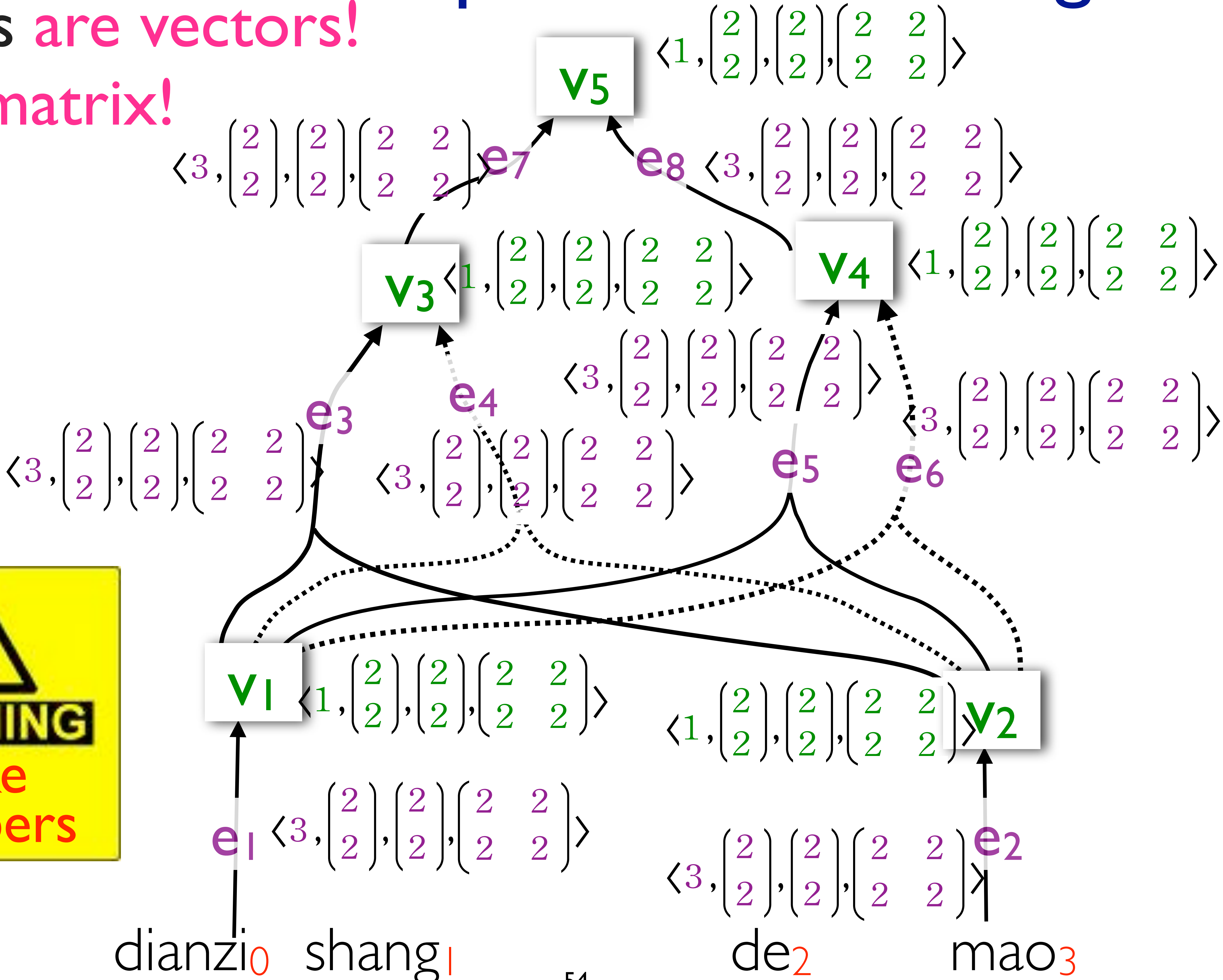
r and s are vectors!



Second-order Expectation Semirings

r and s are vectors!

t is a matrix!



Inside-Outside Speedup

Inside-Outside Speedup

See the paper for details!

Minimum Risk Training over Translation Forests

Minimum Risk Training

Minimum Risk Training

Finding optimal parameters:

Minimum Risk Training

Finding optimal parameters:

$$\theta^* = \textit{argmin}\ \mathrm{Risk}(P_\theta) - \mathrm{Temperature} \times \mathrm{Entropy}(P_\theta)$$

Minimum Risk Training

Finding optimal parameters:

$\theta^* = \textit{argmin}\ \mathrm{Risk}(P_\theta) - \mathrm{Temperature} \times \mathrm{Entropy}(P_\theta)$

- P_θ is a probability distribution over trees, parameterized by the parameters θ

Why Minimum Risk Training?

	Min-Risk	MERT	CRF	Perceptron	MIRA
Gradient descent	☺	☹	☺	?	?
BLEU	☺	☺	☹	☹	☺
Latent variable	☺	☹	☺	☹	☹
Oracle translation	☺	☺	☹	☹	☹
Model regularization	☺	☹	☺	☺	☺

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Oracle translation	☺	☺	☹	☹	☹
Model regularization	☺	☹	☺	☺	☺

Experimental Results

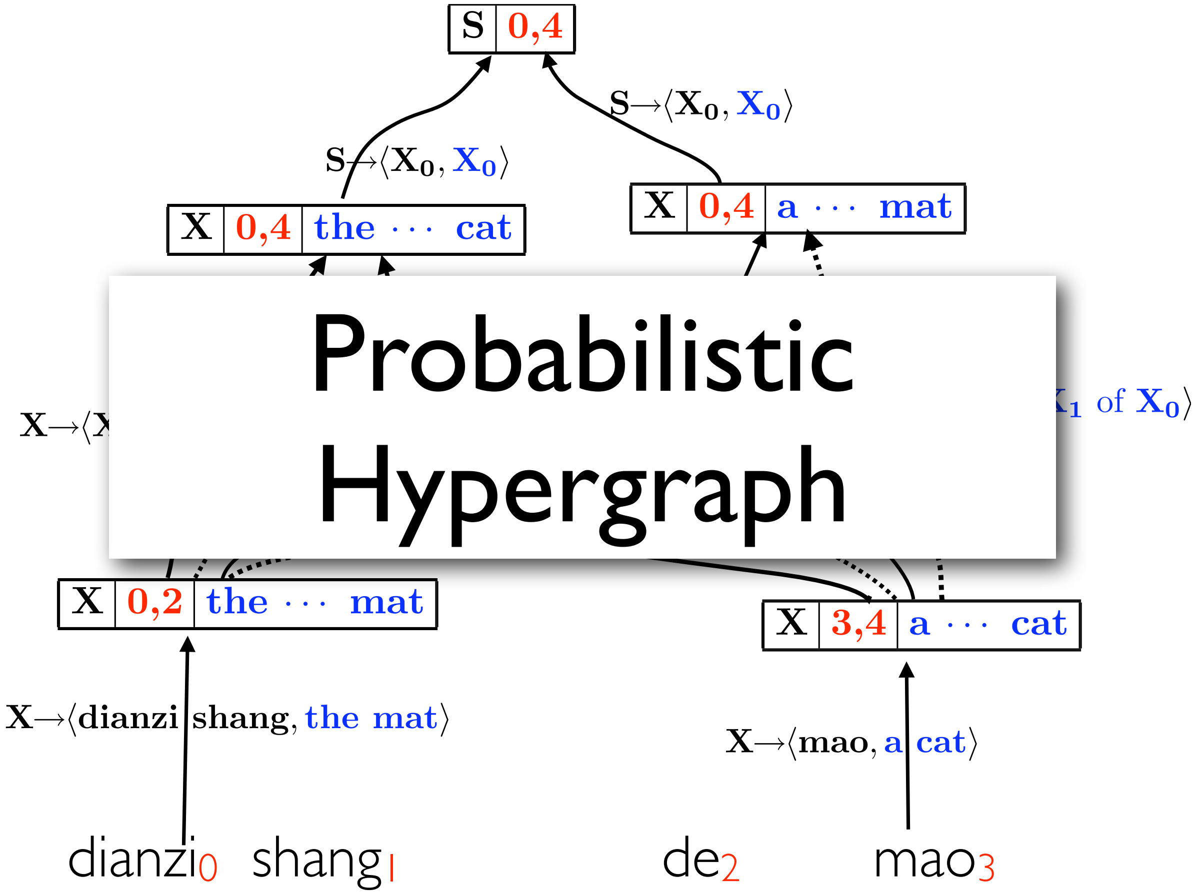
- Data set: IWSLT CN-EN 2005

	Training scheme	test
(Och, 2002)	MERT (Nbest, small)	47.7
(Smith and Eisner, 2006)	MR (Nbest, small)	47.7
NEW!	MR (hypergraph, small)	48.4
NEW!	MR (hypergraph, large)	**48.7**

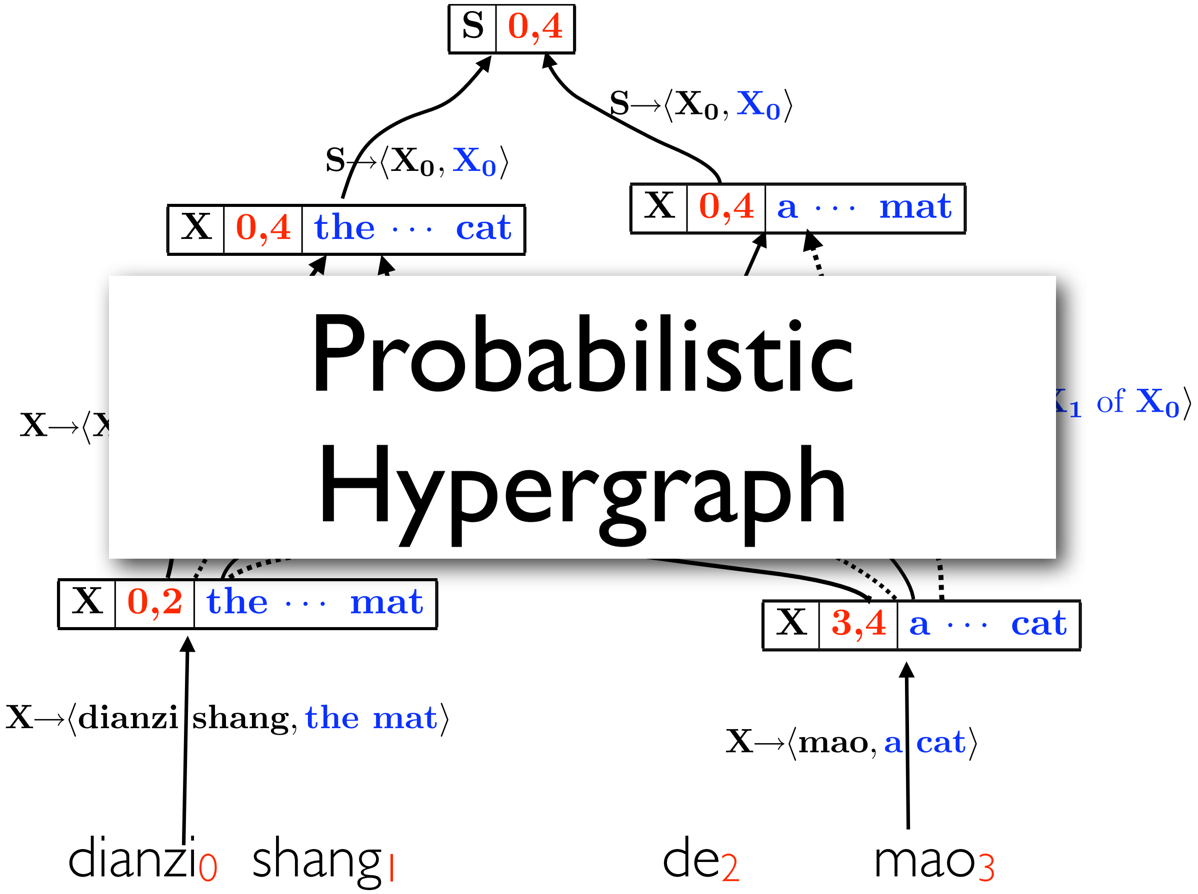
Small: five features as in regular MERT

Large: two-stage forest-reranking with **21k** additional target-side ngram features

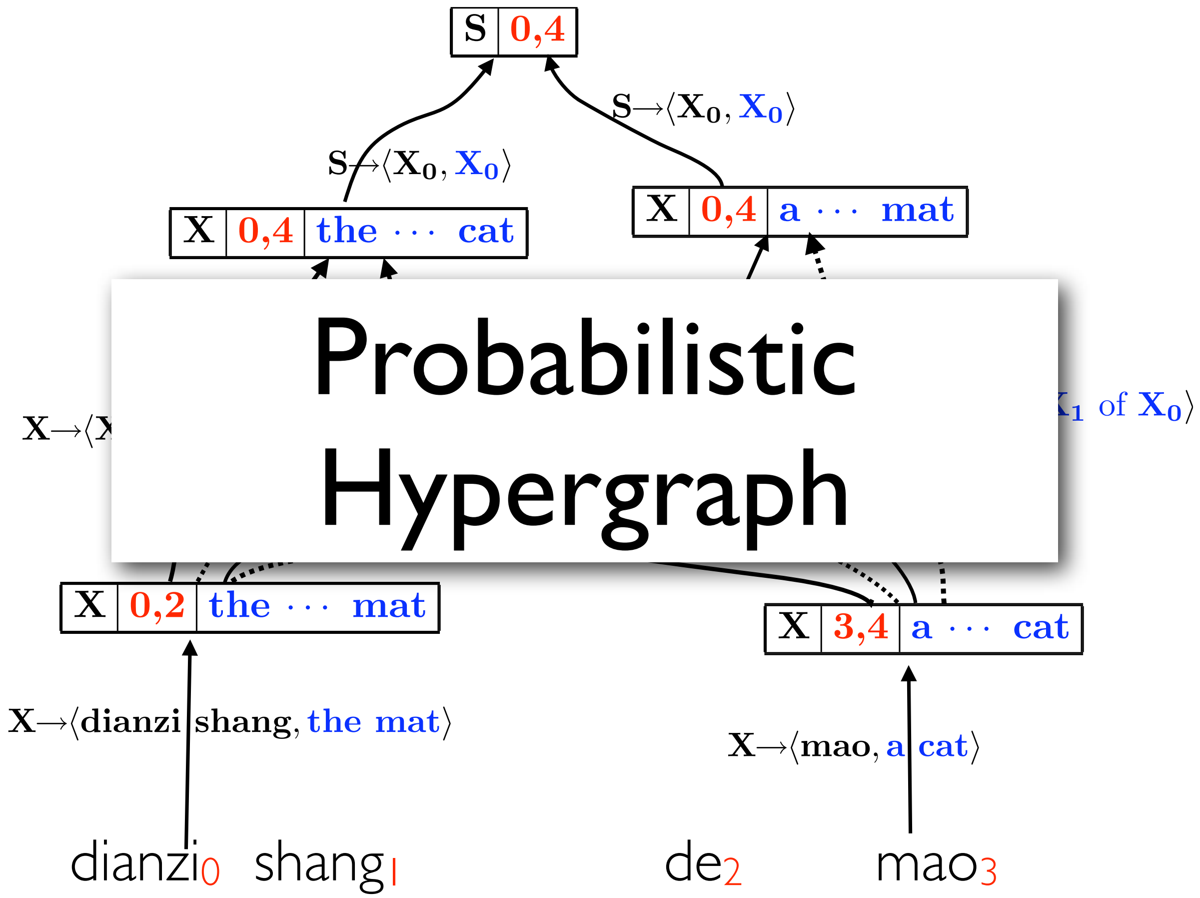
Summary



Probabilistic Hypergraph



- First-order quantities:
  - expectation
    - entropy
    - cross-entropy
    - KL divergence
    - Bayes risk
    - feature expectations
  - first-order gradient of Z



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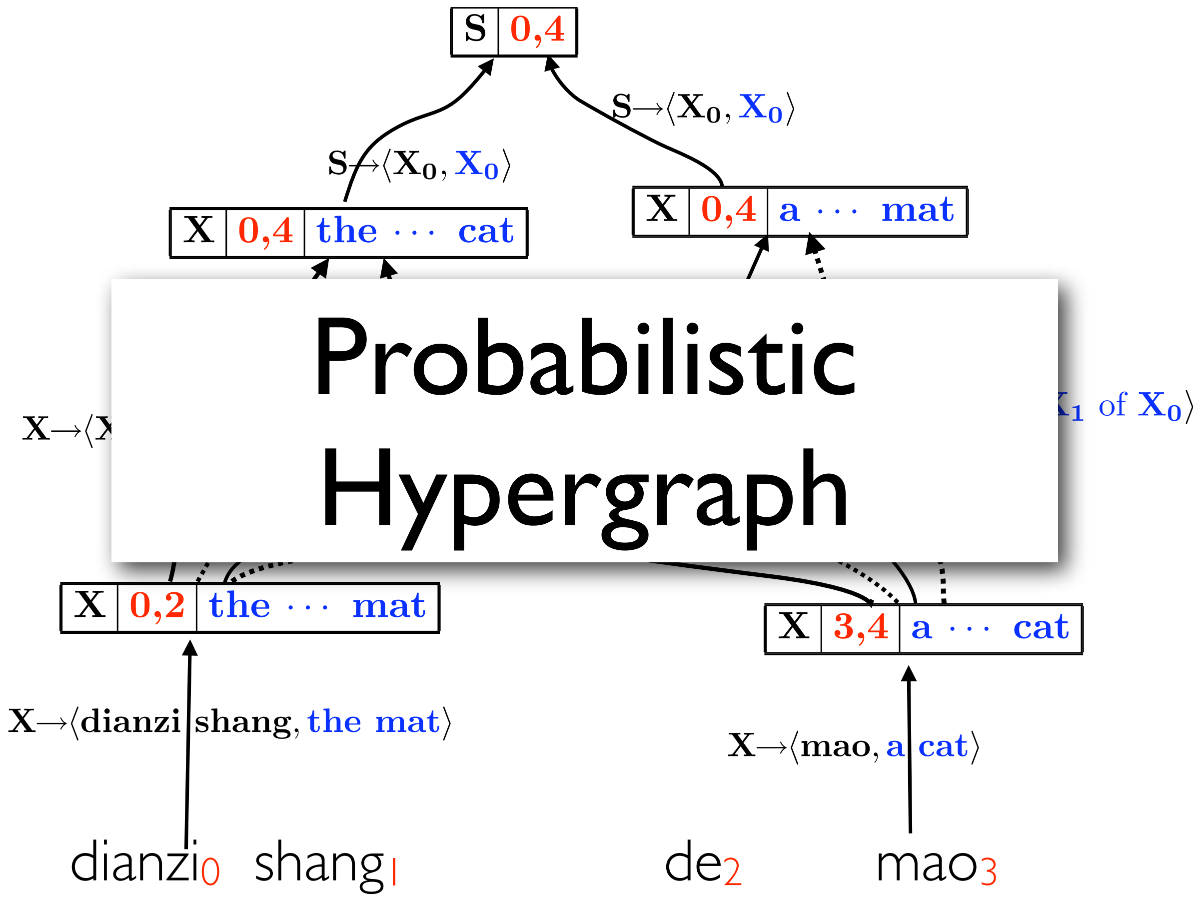
  - gradient of expectation
    - gradient of entropy or Bayes risk



Probabilistic Hypergraph

This work provides a unified, elegant, and efficient framework in computing all of these!

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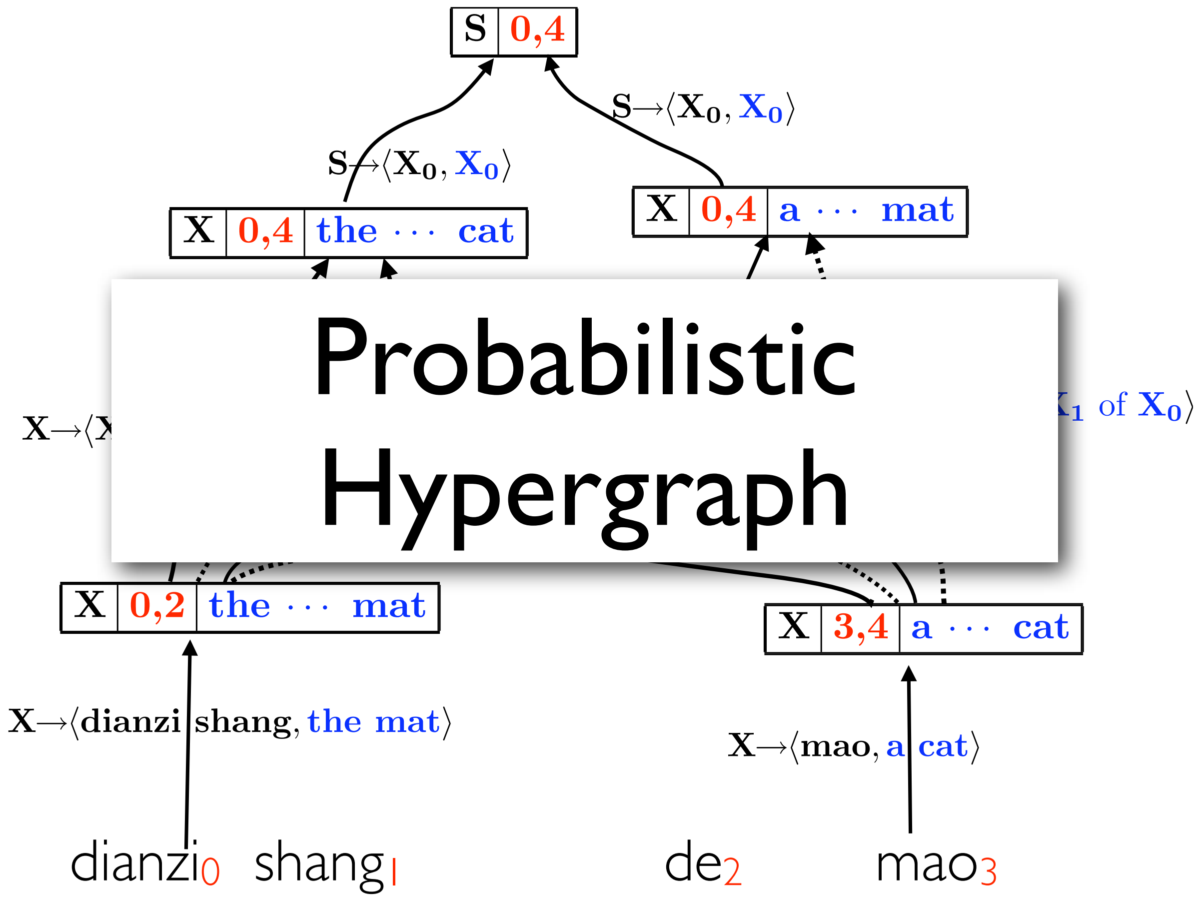
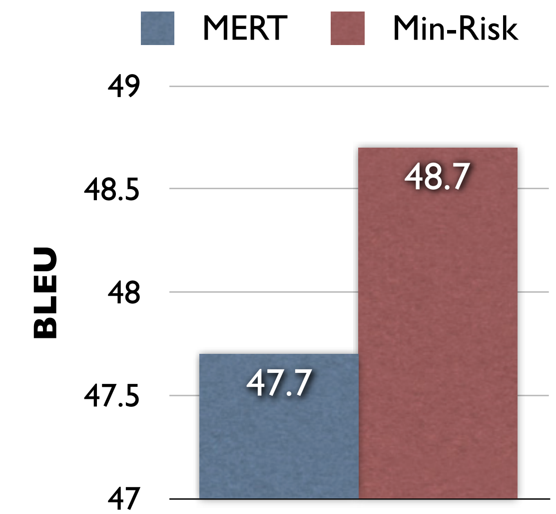


Probabilistic Hypergraph

Improved BLEU score!



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Future: machine learning for MT

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semirings for parameter estimation

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minimum risk

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minimum risk

deterministic annealing

semirings for parameter estimation

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minimum risk

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active learning

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Future: machine learning for MT

minimum risk	deterministic annealing	active learning	semi-supervised learning

semirings for parameter estimation

Future: machine learning for MT

feature interaction

minimum risk

deterministic annealing

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semi-supervised learning

semirings for parameter estimation

Future: machine learning for MT

feature interaction

second-order gradient descent

minimum risk

deterministic annealing

active learning

semi-supervised learning

semirings for parameter estimation

Future: machine learning for MT

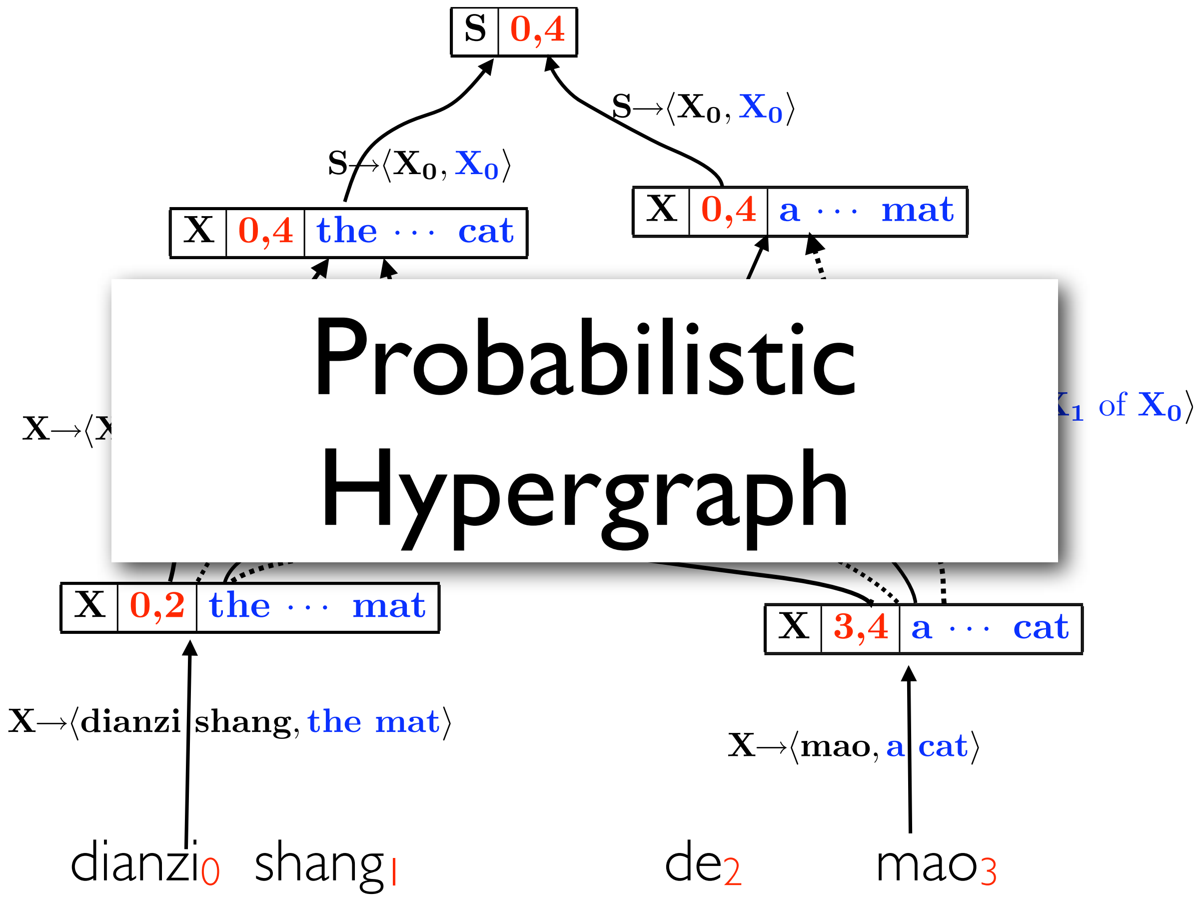
feature interaction	second-order gradient descent	....

minimum risk	deterministic annealing	active learning	semi-supervised learning

semirings for parameter estimation


Joshua
semiring

Thank you!
谢谢！



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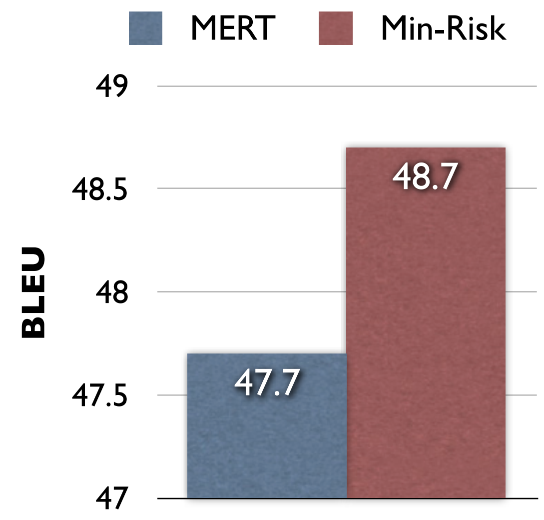

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