Dyna
Evaluation of Logic Programs with Built-Ins and Aggregation: A Calculus for Bag Relations

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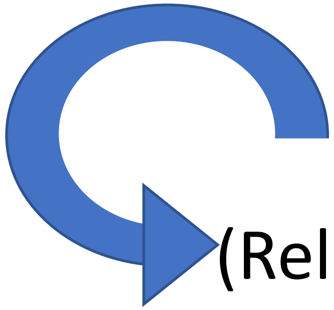
R-exprs

(Relational expressions)

Term Rewriting



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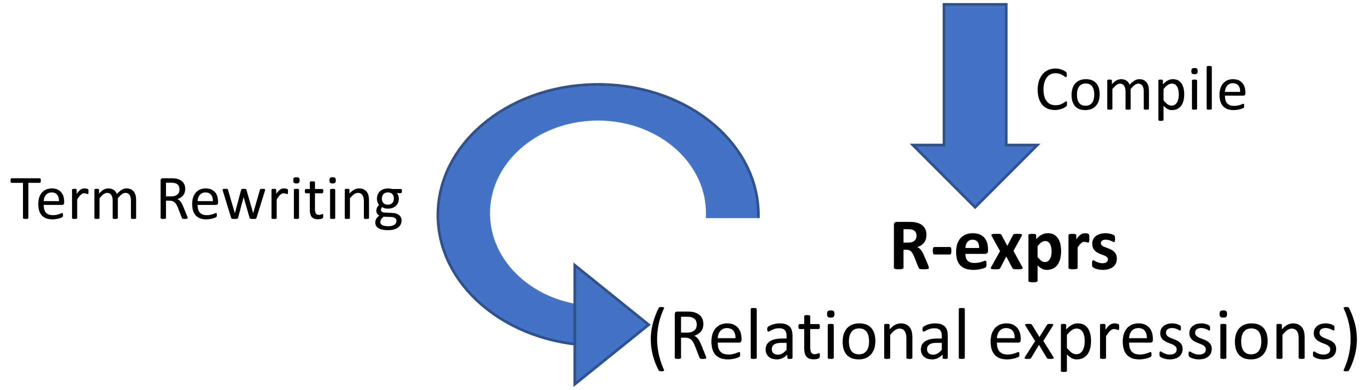
:-dyna.



Machine Learning

Search

Dynamic Programming

Database

AI

Deductive Databases

:-dyna.

Logic Programming

Compile

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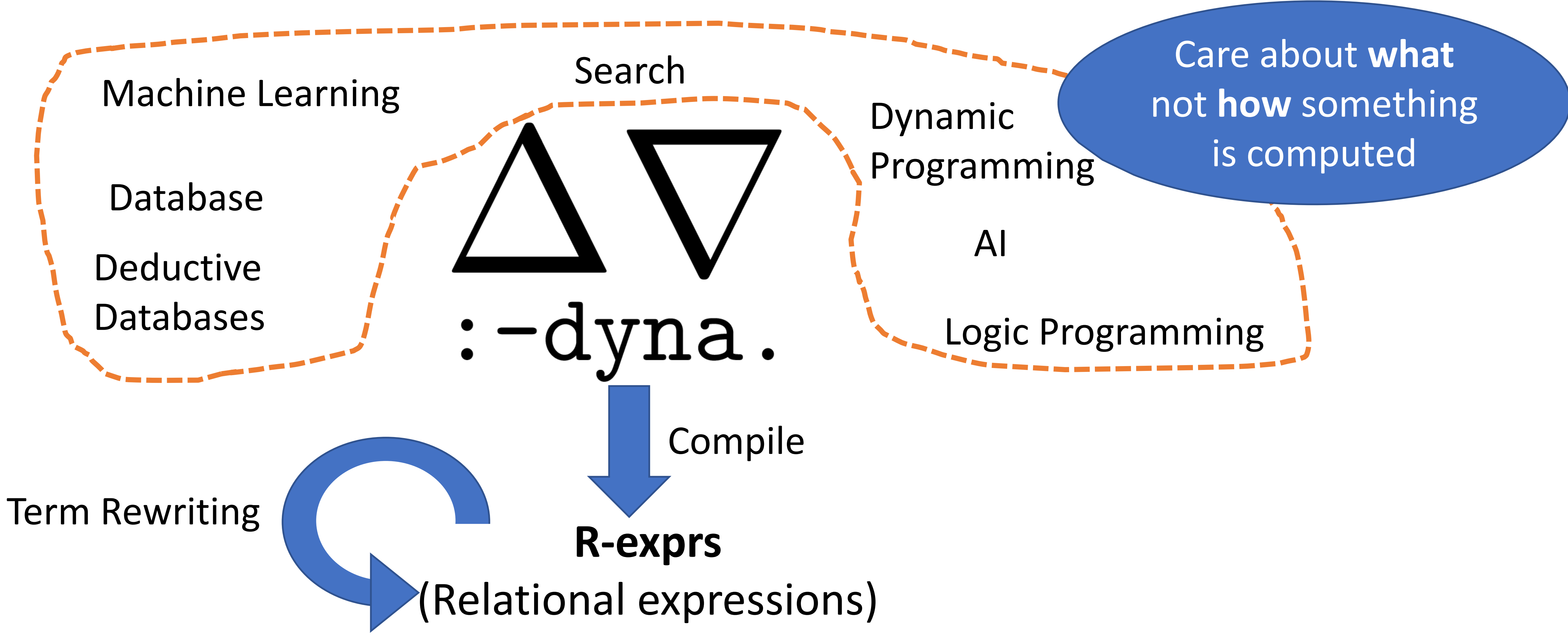
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Search
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Care about **what** not **how** something is computed
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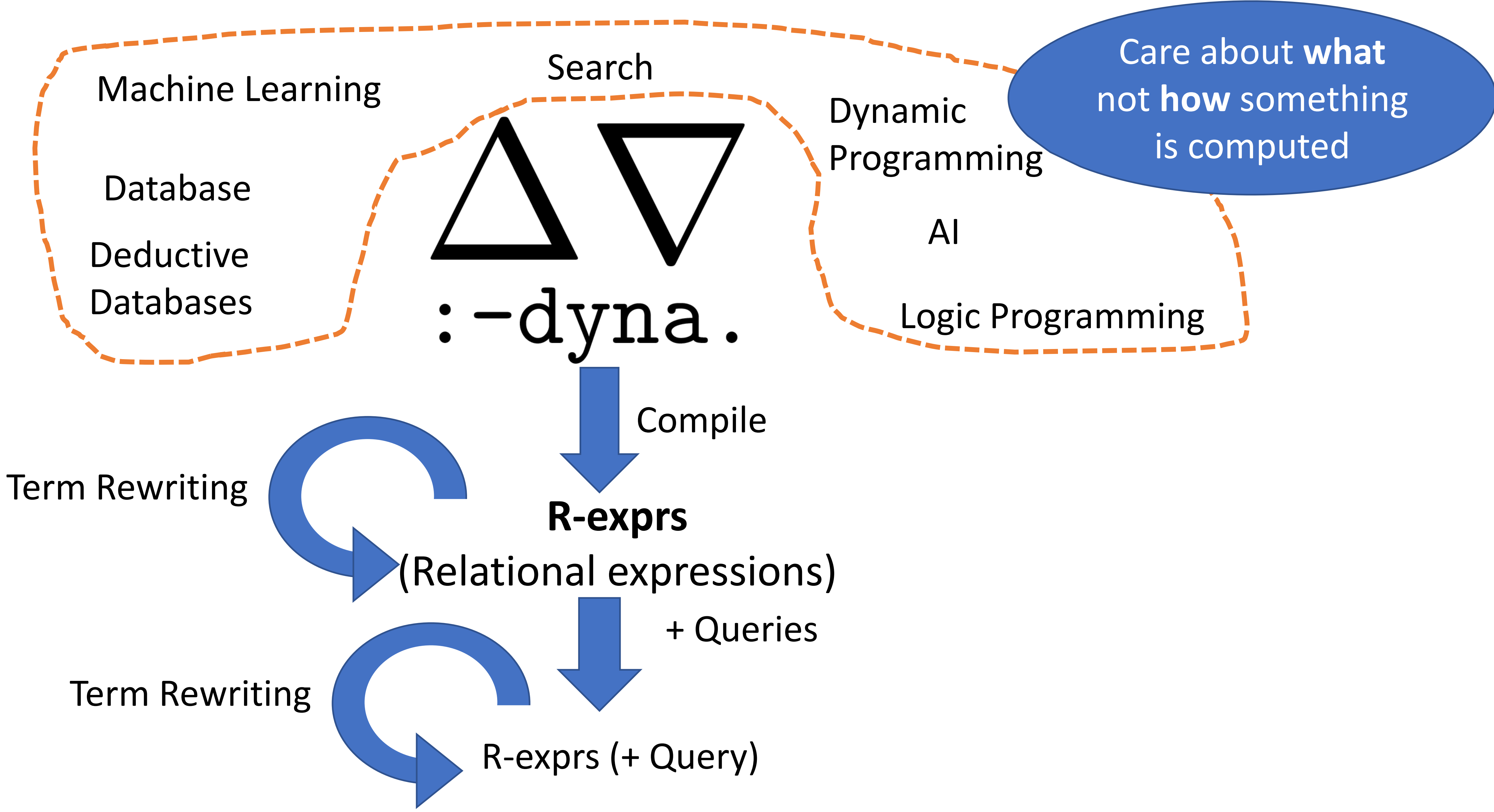
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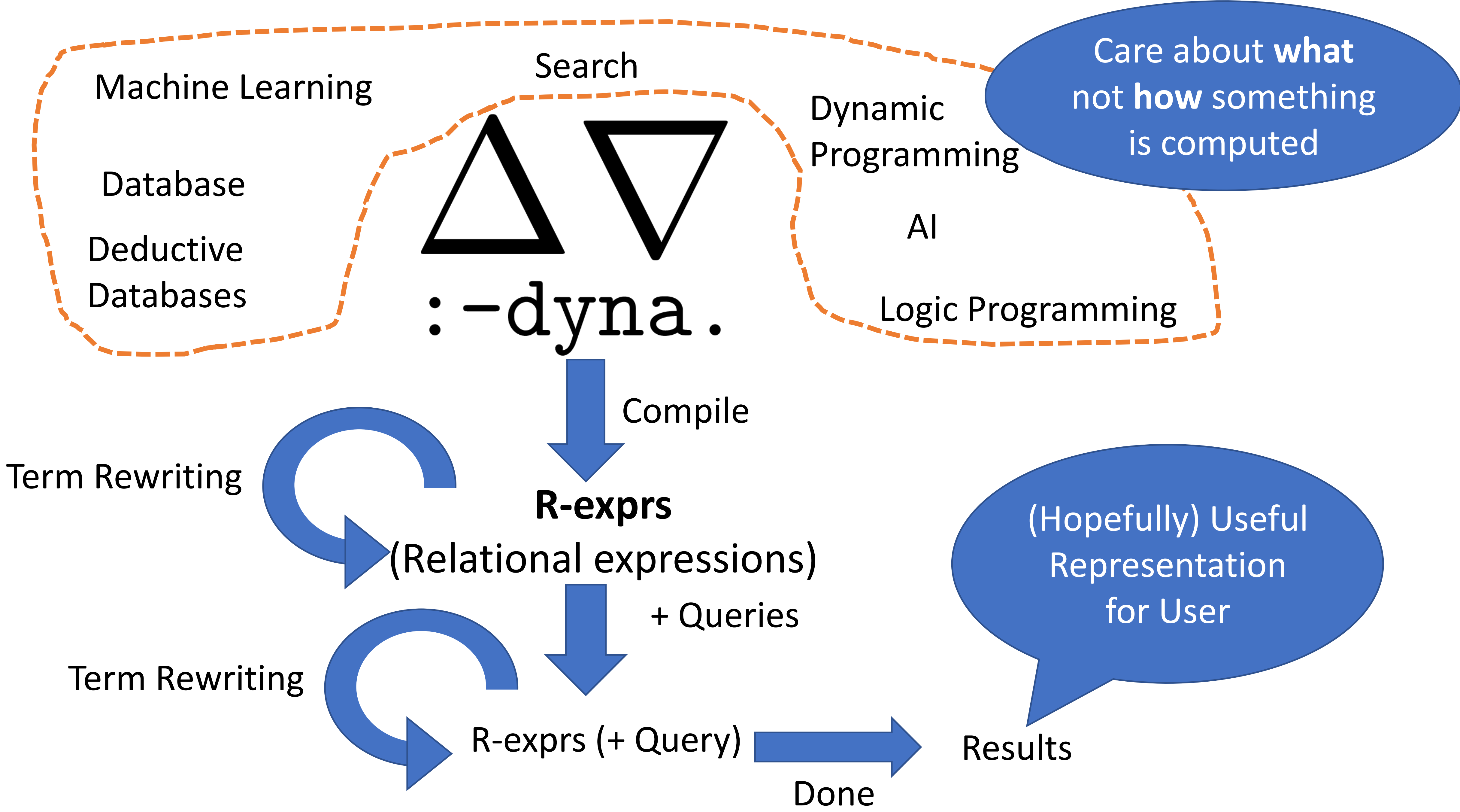
(Relational expressions)
+ Queries
Term Rewriting
R-exprs (+ Query)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(Relational expressions)
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(Hopefully) Useful Representation for User
Term Rewriting
R-exprs (+ Query)
Results
Done

Dyna vs. Prior Work

Dyna vs. Prior Work

	SQL	Datalog	Prolog	CLP	Dyna

Dyna vs. Prior Work

	SQL	Datalog	Prolog	CLP	Dyna
Finite	✓	✓	✓	✓	✓

Supported by all.
Naïve strategies terminate due to finite.

Dyna vs. Prior Work

	SQL	Datalog	Prolog	CLP	Dyna
Finite	✓	✓	✓	✓	✓
Deductive	✗	✓	✓	✓	✓

Combining rules and "facts" to infer new "facts"

Dyna vs. Prior Work

	SQL	Datalog	Prolog	CLP	Dyna
Finite	✓	✓	✓	✓	✓
Deductive	✗	✓	✓	✓	✓
Infinite relations	✗	✗	✓	✓	✓

E.g. can we represent the set of all positive integers, or all prime numbers

Dyna vs. Prior Work

	SQL	Datalog	Prolog	CLP	Dyna
Finite	✓	✓	✓	✓	✓
Deductive	✗	✓	✓	✓	✓
Infinite relations	✗	✗	✓	✓	✓
Aggregation	✓	✓	✗	✗	✓

`SELECT sum(column) FROM x`

Important for weighted programs

Dyna vs. Prior Work

	SQL	Datalog	Prolog	CLP	Dyna
Finite	✓	✓	✓	✓	✓
Deductive	✗				✓
Infinite relations	✗				✓
Aggregation	✓	✓	✗	✗	✓
Turing complete	✗	✗	✓	✓	✓

Is this a full programming language

Dyna vs. Prior Work

	SQL	Datalog	Prolog	CLP	Dyna
Finite	✓	✓	✓	✓	✓
Deductive	✗			✓	✓
Infinite relations					✓
Aggregation					✓
Turing complete				✓	✓
Constraints	✗	✗	✗	✓	✓

Can expressions like:
$X < Y \ \&\& \ Y < X$
be identified as impossible

Dyna vs. Prior Work

	SQL	Datalog	Prolog	CLP	Dyna
Finite	✓	✓	✓	✓	✓
Deductive	✗	✓	✓	✓	✓
Infinite relations	✗	✗	✓	✓	✓
Aggregation	✓	✓	✗	✗	✓
Turing complete	✗	✗	✓	✓	✓
Constraints	✗	✗	✗	✓	✓

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Turing complete	✗	✗	✓	✓	✓
Constraints	✗	✗	✗	✓	✓

WANT ALL THE THINGS

Hard to mix

Aggregation + Infinite

-
  - $\mathrm{m}(\{X : X \geq 5\}) = \infty$
  - $\{X : X \geq 5\}) = 5$
- $\frac{1}{2^i}$ = 2

Aggregation + Infinite

Aggregators

- OR – Exists A True Branch
  - Used in Prolog ($:-$)
  - Can stop early if find true value

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Infinite Relations

- Infinite …..
  - Can't use a naïve enumerate strategy unless it stops early
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Infinite Relations

- ∑ i=0 ∞ ii=0 ∑ i=0 ∞ ∞ ∑ i=0 ∞ 1 2 i 1 1 2 i 2 i 2 2 i ii 2 i 1 2 i = 2
- X :X≥5 XX :XX≥*5 X :X≥5)=5*
- *m*(X :X≥5 XX *:XX≥5 X :X≥5)=∞*
- *: XX*≥*5 X : X≥5*
- *Infinite* …..
  - Can't use a naïve enumerate strategy unless it stops early
  - Require special rules to understand sequencesm$(\{X : X \geq 5\}) = \infty$
  - $\{X : X \geq 5\}) = 5$
- $\frac{1}{2^i}$ = 2

Dyna = Logic Programming + Aggregation

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```
a(I) :- b(I), c(I).
```

- pointwise logical AND

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```
a(I) = b(I) * c(I).
```

- pointwise multiplication

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a(I) = b(I) * c(I).
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- pointwise multiplication

```
a += b(I) * c(I).
```

- dot product

$$\left(a = \sum_i b_i * c_i\right)$$

Dyna = Logic Programm… gation

I can range over any value, not just integers

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- dot product

$$\left(a = \sum_i b_i * c_i\right)$$

```
a(I,K) += b(I,J) * c(J,K).
```

$$\left(a_{i,k} = \sum_j b_{i,j} * c_{j,k}\right)$$

- matrix multiplication; could be sparse
- `J` is free on the right-hand side, so we sum over it

Dyna = Logic Programming + Aggregation

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$$\left(a_{i,k} = \sum_j b_{i,j} * c_{j,k}\right)$$

```
b(I,I) += 1.    b(I,J) += 0.
```

- *Infinite* identity matrix

Example Program: Shortest path

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```
distance(Start, Y) min= distance(Start, X) + edge(X, Y).
distance(Start, Start) min= 0.
```

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```

Variables not present in the head of an expression are aggregated over like with the dot product example.

Example Program: Shortest path

```
distance(Start, Y) min= distance(Start, X) + edge(X, Y).
distance(Start, Start) min= 0.
```

Here the "`min=`" aggregator only keeps the minimal value that we have computed

Example Program: Shortest path

```
distance(Start, Y) min= distance(Start, X) + edge(X, Y).
distance(Start, Start) min= 0.
```

```
edge("a", "b") = 10.
edge("b", "c") = 2.
edge("c", "d") = 7.
```

Example Program: Shortest p

```
distance(Start, Y) min= distance(Start, X)
distance(Start, Start) min= 0.
```

Dyna programs are equivalent to the set of values they define

```
edge("a", "b") = 10.
edge("b", "c") = 2.
edge("c", "d") = 7.
```

Start	Y	distance(Start, Y)
"a"	"a"	0
"a"	"b"	10
"a"	"c"	12
"a"	"d"	19
"b"	"b"	0
"b"	"c"	2
"b"	"d"	9
"c"	"c"	0
"c"	"d"	7
"d"	"d"	0

Example Program: Shortest path

```
distance(Start, Y) min= distance(Start, X) + edge(X, Y).
distance(Start, Start) min= 0.
```

```
= 10.
) = 2.
d") = 7.
```

Defined for all cases where both arguments are equal

Start	Y	distance(Start, Y)
"foo"	"foo"	0
7	7	0
3.1415	3.1415	0

Start	Y	distance(Start, Y)
"a"	"a"	0
"a"	"b"	10
"a"	"c"	12
"a"	"d"	19
"b"	"b"	0
"b"	"c"	2
"b"	"d"	9
"c"	"c"	0
"c"	"d"	7
"d"	"d"	0

Shortest Path (cont.)

```
distance(S, S) = 0.
```

Shortest Path (cont.)

```
distance(S, S) = 0.
```

S	Y	distance(S, Y)
"foo"	"foo"	0
7	7	0
3.1415	3.1415	0

Shortest Path (cont.)

```
distance(S, S) = 0.
```

S	Y	distance(S, Y)
"foo"	"foo"	0
7	7	0
3.1415	3.1415	0

$$\{\langle Arg_1, Arg_2, Result\rangle : Arg_1 = Arg_2 \textbf{ AND } Result = 0\}$$

Shortest Path (cont.)

```
distance(S, S) = 0.
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S	Y	distance(S, Y)
"foo"	"foo"	0
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3.1415	3.1415	0

$$\{\langle Arg_1, Arg_2, Result \rangle : Arg_1 = Arg_2 \textbf{ AND } Result = 0\}$$

Tuple of Named Variables

Shortest Path (cont.)

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distance(S, S) = 0.
```

S	Y	distance(S, Y)
"foo"	"foo"	0
7	7	0
3.1415	3.1415	0

$$\{\langle Arg_1, Arg_2, Result \rangle : Arg_1 = Arg_2 \textbf{ AND } Result = 0\}$$

Tuple of Named Variables

Executable Code Defines the Rule

Shortest Path (cont.)

```
distance(S, S) = 0.
```

S	Y	distance(S, Y)
"foo"	"foo"	0
7	7	0
3.1415	3.1415	0

$$\{\langle Arg_1, Arg_2, Result\rangle : Arg_1 = Arg_2 \textbf{ AND } Result = 0\}$$

Tuple of Named Variables — Executable Code Defines the Rule

```
distance(S, Y) = distance(S, X) + edge(X, Y).
```

Shortest Path (cont.)

```
distance(S, S) = 0.
```

S	Y	distance(S, Y)
"foo"	"foo"	0
7	7	0
3.1415	3.1415	0

$$\{\langle Arg_1, Arg_2, Result\rangle : Arg_1 = Arg_2 \textbf{ AND } Result = 0\}$$

Tuple of Named Variables — Executable Code Defines the Rule

```
distance(S, Y) = distance(S, X) + edge(X, Y).
```

Because of recursion, it can not be expressed using the set builder notation

```
distance(Start, Y) = edge(X, Y) + distance(Start, X).
```

```
distance(Start, Y) = edge(X, Y) + distance(Start, X).
```

Normalize with standard names for all arguments

```
Result is distance(Arg1, Arg2) :-
    Result = edge(Arg2, X) + distance(Arg1, X).
```

```
distance(Start, Y) = edge(X, Y) + distance(Start, X).
```

```
Result is distance(Arg1, Arg2) :-
     Result = edge(Arg2, X) + distance(Arg1, X).
```

```
(E is edge(Arg2, X))
```

R-expr to Call function by name

```
distance(Start, Y) = edge(X, Y) + distance(Start, X).
```

Intermediate results are mapped to variables

```
istance(Arg1, Arg2) :-
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(E is edge(Arg2, X))
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R-expr to Call function by name

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distance(Start, Y) = edge(X, Y) + distance(Start, X).
```

```
Result is distance(Arg1, Arg2) :-
      Result = edge(Arg2, X) + distance(Arg1, X).
```

```
(E is edge(Arg2, X))
(D is distance(Arg1, X))
```

Recursive call to distance

```
distance(Start, Y) = edge(X, Y) + distance(Start, X).
```

```
Result is distance(Arg1, Arg2) :-
      Result = edge(Arg2, X) + distance(Arg1, X).
```

```
(E is edge(Arg2, X))
(D is distance(Arg1, X))
builtin_plus(Result, E, D)
```

Built-in
represented in the
R-expr

```
distance(Start, Y) = edge(X, Y) + distance(Start, X).
```

```
Result is distance(Arg1, Arg2) :-
    Result = edge(Arg2, X) + distance(Arg1, Y)
```

```
(E is edge(Arg2, X)) ∩*
(D is distance(Arg1, X)) ∩*
builtin_plus(Result, E, D)
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Intersect the bag by multiplying the multiplicities and joining these expressions using the same variable names

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```

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(E is edge(Arg2, X)) ⋒*
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Over the tuple $\langle Arg1, Arg2, Result, E, D, X \rangle$

```
distance(Start, Y) = edge(X, Y) + distance(Start, X).
```

```
Result is distance(Arg1, Arg2) :-
    Result = edge(Arg2, X) + distance(Arg1, X).
```

```
(E is edge(Arg2, X)) ∩*
(D is distance(Arg1, X)) ∩*
builtin_plus(Result, E, D)
```

```
proj(E, proj(D, proj(X,      )))
```

Now Over the tuple ⟨Arg1, Arg2, Result⟩

Project out all local variables

What about Aggregation?

```
distance(S, X) min= edge(X, Y) + distance(S, Y).
```

- Any semi-group: min, max, sum, product, logical OR, logical AND

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```
(Result=min(MinInputVariable, R))
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R-expr composed on previous slide

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New intermediate variable introduced (Like project)

R-expr composed on previous slide

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distance(S, X) min= edge(X, Y) + distance(S, Y).
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```
(Result=min(MinInputVariable, R))
```

Resulting value from aggregation

New intermediate variable introduced (Like project)

R-expr composed on previous slide

Shortest Path All Together Now

```
distance(S, S) min= 0.
distance(S, X) min= edge(X, Y) + distance(S, Y).
```

Shortest Path All Together Now

```
distance(S, S) min= 0.
distance(S, X) min= edge(X, Y) + distance(S, Y).
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```
Result is distance(Arg1, Arg2) min= Arg1=Arg2, Result=0.
Result is distance(Arg1, Arg2) min= Result=edge(Arg2, Y) + distance(Arg1, Y).
```

Shortest Path All Together Now

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distance(S, S) min= 0.
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```

(Arg1=Arg2) $\cap_{\star}$ (MinInput=0)

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```
(Arg1=Arg2) ⍝ (MinInput=0)

proj(E, proj(D, proj(Y,
(E is edge(Arg2, Y)) ⍝ (D is distance(Arg1, Y)) ⍝ builtin_plus(MinInput, E, D)
)))
```

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```
((Arg1=Arg2) ∩* (MinInput=0)) ⊎

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Result is distance(Arg1, Arg2) min= Result=edge(Arg2, Y) + distance(Arg1, Y).
```

The complete distance rule as a R-expr

```
(Result=min(MinInput,
  ((Arg1=Arg2) ⋒* (MinInput=0)) ⊎+
(proj(E, proj(D, proj(Y,
 (E is edge(Arg2, Y)) ⋒* (D is distance(Arg1, Y)) ⋒* builtin_plus(MinInput, E, D)
 )))
)
))
```

Manipulating R-exprs via Rewrites

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- A series of *semantic preserving* rewrites which attempt to *simplify* the expression
  - Look for a sub-R-expr which can be rewritten to be simpler, do so!

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- A series of *semantic preserving* rewrites which attempt to *simplify* the expression
  - Look for a sub-R-expr which can be rewritten to be simpler, do so!
- Non-deterministic: Any order of rewrites is acceptable
  - Requires searching through the entire R-expr to identify what can be rewritten/run

Manipulating R-exprs via Rewrites

- A series of *semantic preserving* rewrites which attempt to *simplify* the expression
  - Look for a sub-R-expr which can be rewritten to be simpler, do so!
- Non-deterministic: Any order of rewrites is acceptable
  - Requires searching through the entire R-expr to identify what can be rewritten/run
- Fair rewrites: non-normal form sub-expression are eventually rewritten
  - Important in the case of recursive programs

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  - Requires searching through the entire R-expr to identify what can be rewritten/run
- Fair rewrites: non-normal form sub-expression are eventually rewritten
  - Important in the case of recursive programs
- Core rewrites are presented in the paper

R-expr Rewrites—Built-ins

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R-expr Rewrites—Built-ins

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`builtin_plus(1,2,Z) → (Z=3)`

`builtin_plus` runs and its result is assigned **Z**

R-expr Rewrites—Built-ins

`builtin_plus(X,Y,Z)` $\equiv \{\langle X,Y,Z\rangle : X+Y=Z\}$

`builtin_plus(1,2,Z) → (Z=3)`

`builtin_plus(1,Y,Z)`

No rewrites available for:
1+Y=Z

Y=1, Z=2
Y=2, Z=3
Y=3, Z=4
….

R-expr Rewrites—Built-ins

Propagate the assignment to Z

builtin_plus(… $Y = Z\}$

builtin_plus(1, …

builtin_plus(1,Y,Z)

(Z=3)*builtin_plus(1,Y,Z)→(Z=3)*builtin_plus(1,Y,3)

R-expr Rewrites—Built-ins

builtin_plus(… $+ Y = Z\}$

builtin_plus(1, …

builtin_plus(1,Y,Z)

Propagate the assignment to Z

```
(Z=3)*builtin_plus(1,Y,Z)→(Z=3)*builtin_plus(1,Y,3)
(Z=3)*builtin_plus(1,Y,3)→(Z=3)*(Y=2)
```

Built-ins support multiple *modes* for computation

R-expr Rewrites—Built-ins

* and + are over the bag's multiplicity

(X,Y,Z) $\equiv \{\langle X,Y,Z\rangle : X + Y = Z\}$

Maps to the multiplicity of being contained in the bag

s(1,2,Z) → (Z=3)

buil n_plus(1,Y,Z)

(Z=3)*builtin_plus(1,Y,Z)→(Z=3)*bui

(Z=3)*builtin_plus(1,Y,3)→(Z=3)* 2)

builtin_plus(1,2,3) → 1
builtin_plus(1,2,4) → 0

Check assignment is consistent

Rewriting Example: Shortest Path

```
Distance is distance("a", "c")
```

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```

Program

```
(Result=min(MinInput,
  (Arg1=Arg2)*(MinInput=0) +
  proj(E, proj(D, proj(X,
     (E is edge(Arg2, X))*(D is
distance(Argr1,X)*bultin_plus(E,D,MinInput))))
```

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Rewrites Rules

```
 0      Variables not equal
("a"="c") → 0
```

Variables not equal

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Rewrites Rules

```
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0      Variables not equal
0 * R → 0          Multiplicative annihilation
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```

Rewrites Rules

```
R          Additive identity
0          Multiplicative annihilation
0      Variables not equal
```

$0 + R \rightarrow R$ Additive identity

Rewriting Example: Shortest Path

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Distance is distance("a", "c")
```

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0 + R → R   Additive identity

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```

0 + R → R Additive identity

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```
(Distance=min(MinInput, proj(E, proj(D, proj(X,
     (E is edge("c", X))*(D is distance("a",X)*bultin_plus(E,D,MinInput)))
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```

Program

```
(Result is edge(Arg1, Arg2)) :-
   (Arg1="a")*(Arg2="b")*(Result=10) +
   (Arg1="b")*(Arg2="c")*(Result=2) +
   (Arg1="c")*(Arg2="d")*(Result=7)
```

```
(Distance=min(MinInput, proj(E, proj(D, proj(X,
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```
(Distance=min(MinInput, proj(E, proj(D, proj(X,
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    *(D is distance("a",X)*bultin_plus(E,D,MinInput)))))
```

```
 1
 0
 0      Equality checks
("c"="c") → 1
```

```
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     (E is edge("c", X))*(D is distance("a",X)*bultin_plus(E,D,MinInput))))
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    *(D is distance("a",X)*bultin_plus(E,D,MinInput))))
```

```
R          Multiplicative identity
1
0
0      Equality checks
```

$1 * R \rightarrow R$ Multiplicative identity

```
(Distance=min(MinInput, proj(E, proj(D, proj(X,
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1 * R → R Multiplicative identity

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```

```
R         Multiplicative identity
R         Multiplicative identity
1
0
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```
1 * R → R   Multiplicative identity
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```

```
(Distance=min(MinInput, proj(E, proj(D, proj(X,
    ((X="d")*(E=7))
   *(D is distance("a",X)*bultin_plus(E,D,MinInput))))
```

Propagate values

```
(Distance=min(MinInput, proj(D,
    (D is distance("a","d")*bultin_plus(7,D,MinInput))))
```

Rewrites for Aggregators

Rewrites for Aggregators

```
(Result=min(MinInput, (MinInput=789))) → (Result=789)
```

A final value has been determined. Assign it to the Result Variable

Rewrites for Aggregators

```
(Result=min(MinInput, (MinInput=789))) → (Result=789)

(Result=min(MinInput, R+S)) → builtin_min(MR, MS, Result)*
                              (MR=min(MinInput, R))*(MS=min(MinInput, S))
```

Two disjunctive R-exprs can be split and processed individually

Rewrites for Aggregators

```
(Result=min(MinInput, (MinInput=789))) → (Result=789)

(Result=min(MinInput, R+S)) → builtin_min(MR, MS, Result)*
                              (MR=min(MinInput, R))*(MS=min(MinInput, S))

(Result=min(MinInput, 0)) → (Result=identity) ≡ (Result=∞)
```

Rewrites for Aggregators

```
(Result=min(MinInput, (MinInput=789))) → (Result=789)


(Result=min(MinInput, R+S)) → builtin_min(MR, MS, Result)*
                              (MR=min(MinInput, R))*(MS=min(MinInput, S))


(Result=min(MinInput, 0)) → (Result=identity) ≡ (Result=∞)


not_identity(identity) → 0
not_identity(V) → 1   if ground(V) && V != identity

(Result=min(MinInput, 0))*not_identity(Result) → 0
```

More "traditional" for aggregation to map empty to empty

Ongoing and Future Work

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- Memoization and Mixed-chaining of computation
  - R-exprs serve as a basis for representing incomplete computations and can be run in a myriad of different execution orders
  - Extended version of this paper to (hopefully) appear soon

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  - R-exprs capture *what* needs to be computed while leaving the order and *how* open to the runtime to decide
  - Much like a database optimizer, but for full, long running programs

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- Compilation and optimization of R-exprs
- github.com/matthewfl/dyna-R arxiv.org/abs/2010.10503

Thank you

Questions?

github.com/matthewfl/dyna-R

arxiv.org/abs/2010.10503

mfl@cs.jhu.edu