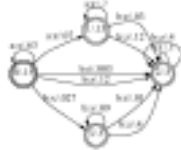


# Parameterized Finite-State Machines and their Training

Jason Eisner

Johns Hopkins University

October 16, 2002 — AT&T Speech Days

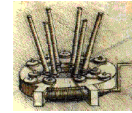


# Outline – The Vision Slide!

1. Finite-state machines as a shared modeling language.



2. The training gizmo (an algorithm).



Should use out-of-the-box finite-state gizmos to build and train most of our current models. Easier, faster, better, & enables fancier models.


# Training Probabilistic FSMs

- State of the world – surprising:
  - Training for HMMs, alignment, many variants
  - But no basic training algorithm for **all** FSAs
  - Fancy toolkits for building them, but no learning
- New algorithm:
  - Training for FSAs, FSTs, ... (collectively FSMs)
  - Supervised, unsupervised, incompletely supervised ...
  - Train components separately or all at once
  - Epsilon-cycles OK
  - Complicated parameterizations OK



"If you build it, it will train"

# Currently Two Finite-State Camps

	Vanilla FSTs	Probabilistic FSTs
What they represent	Functions on strings. Or nondeterministic functions (relations).	Prob. distributions $p(x,y)$ or $p(y x)$ .
How they're currently used	Encode expert knowledge about Arabic morphology, etc. 	Noisy channel models $p(x)p(y x)p(z y)...$ (much more limited)
How they're currently built	Fancy regular expressions (or sometimes TBL)	Build parts by hand For each part, get arc weights somehow Then combine parts (much more limited)

# Current Limitation

Knight & Graehl  
1997 - transliteration

- Big FSM must be made of separately trainable parts.

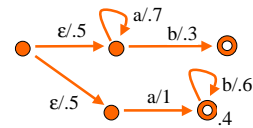
- $p(\text{English text})$
- $p(\text{English text} \rightarrow \text{English phonemes})$
- $p(\text{English phonemes} \rightarrow \text{Japanese phonemes})$
- $p(\text{Japanese phonemes} \rightarrow \text{Japanese text})$

Need explicit training data for this part (smaller loanword corpus).  
A pity – would like to use guesses.

Topology must be simple enough to train by current methods.  
A pity – would like to get some of that expert knowledge in here!

Topology: sensitive to syllable struct?  
Parameterization: /t/ and /d/ are similar phonemes ... parameter tying?

# Probabilistic FSA

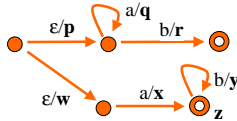


Example:  $ab$  is accepted along 2 paths  
 $p(ab) = (.5 \cdot .7 \cdot .3) + (.5 \cdot .6 \cdot .4) = .225$

Regex:  $(a^{*.7} b) +_{.5} (ab^{*.6})$

Theorem: Any probabilistic FSM has a regexp like this.

## Weights Need Not be Reals

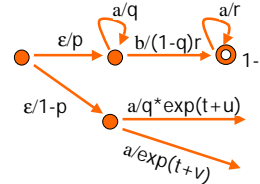


Example:  $ab$  is accepted along 2 paths  
 $\text{weight}(ab) = (p \otimes q \otimes r) \oplus (w \otimes x \otimes y \otimes z)$

If  $\otimes$   $\oplus$   $*$  satisfy "semiring" axioms, the finite-state constructions continue to work correctly.

## Goal: Parameterized FSMs

- Parameterized FSM:
  - An FSM whose arc probabilities depend on parameters: they are formulas.

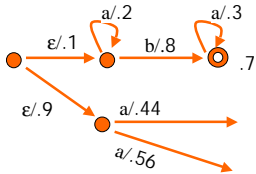


**Expert first:** Construct the FSM (topology & parameterization).

**Automatic takes over:** Given training data, find parameter values that optimize arc probs.

## Goal: Parameterized FSMs

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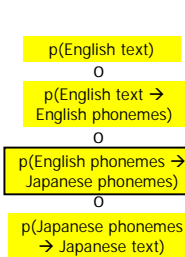
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## Goal: Parameterized FSMs

Knight & Graehl  
1997 - transliteration

- FSM whose arc probabilities are formulas.



"Would like to get some of that expert knowledge in here"

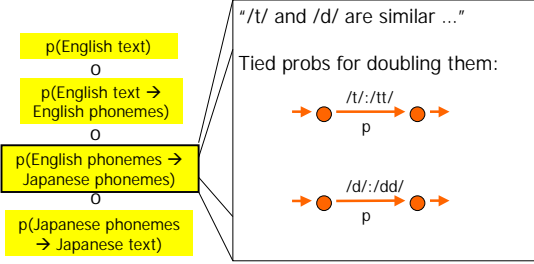
Use probabilistic regexps like  $(a^{*.7} b) +_5 (ab^{*.6}) \dots$

If the probabilities are variables  $(a^{*x} b) +_y (ab^{*z}) \dots$  then arc weights of the compiled machine are nasty formulas. (Especially after minimization!)

## Goal: Parameterized FSMs

Knight & Graehl  
1997 - transliteration

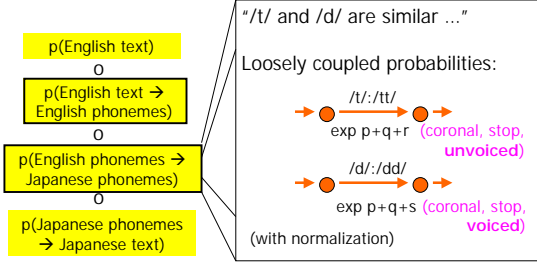
- An FSM whose arc probabilities are formulas.



## Goal: Parameterized FSMs

Knight & Graehl  
1997 - transliteration

- An FSM whose arc probabilities are formulas.



## Outline of this talk

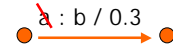
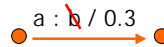
1. What can you build with parameterized FSMs?
2. How do you train them?

## Finite-State Operations

- Projection GIVES YOU marginal distribution

$$\text{domain}(p(x,y)) = p(x)$$

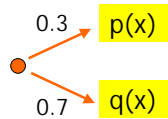
$$\text{range}(p(x,y)) = p(y)$$



## Finite-State Operations

- Probabilistic union GIVES YOU mixture model

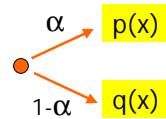
$$p(x) +_{0.3} q(x) = 0.3 p(x) + 0.7 q(x)$$



## Finite-State Operations

- Probabilistic union GIVES YOU mixture model

$$p(x) +_{\alpha} q(x) = \alpha p(x) + (1 - \alpha)q(x)$$



Learn the mixture parameter  $\alpha$ !

## Finite-State Operations

- Composition GIVES YOU chain rule

$$p(x|y) \circ p(y|z) = p(x|z)$$

$$p(x|y) \circ p(y|z) \circ z = p(x,z)$$

- The most popular statistical FSM operation
- Cross-product construction

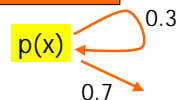
## Finite-State Operations

- Concatenation, probabilistic closure  
HANDLE unsegmented text

$$p(x) q(x)$$

$$p(x) \rightarrow q(x)$$

$$p(x)^{*0.3}$$



- Just glue together machines for the different segments, and let them figure out how to align with the text

## Finite-State Operations

- Directed replacement MODELS noise or postprocessing

$$p(x,y) \circ D = p(x, \text{noisy } y)$$

noise model defined by dir. replacement

- Resulting machine compensates for noise or postprocessing

## Finite-State Operations

- Intersection GIVES YOU product models
  - e.g., exponential / maxent, perceptron, Naive Bayes, ...
  - Need a normalization op too – computes  $\sum_x f(x)$  “pathsum” or “partition function”

$$p(x) \& q(x) = p(x) * q(x)$$

$$p(A(x)|y) \& p(B(x)|y) \& p(y) \propto p_{NB}(y|x)$$

- Cross-product construction (like composition)

## Finite-State Operations

- Conditionalization (new operation)

$$\text{condit}(p(x,y)) = p(y|x)$$

- Resulting machine can be composed with other distributions:  $p(y|x) * q(x)$
- Construction:  $\text{reciprocal}(\text{determinize}(\text{domain}(p(x,y)))) \circ p(x,y)$   
not possible for all weighted FSAs

## Other Useful Finite-State Constructions

- Complete graphs YIELD n-gram models
- Other graphs YIELD fancy language models (skips, caching, etc.)
- Compilation from other formalism  $\rightarrow$  FSM:
  - Wordlist (cf. trie), pronunciation dictionary ...
  - Speech hypothesis lattice
  - Decision tree (Sproat & Riley)
  - Weighted rewrite rules (Mohri & Sproat)
  - TBL or probabilistic TBL (Roche & Schabes)
  - PCFG (approximation!) (e.g., Mohri & Nederhof)
  - Optimality theory grammars (e.g., Eisner)
  - Logical description of set (Vaillette; Klarlund)

## Regular Expression Calculus as a Modelling Language

Programming Languages	The Finite-State Case
Function	Function on strings, or probability distrib.
Source code	Regular expression (can be probabilistic)
Object code	Finite state machine
Compiler	Regex compiler
Optimization of object code	Determinization, minimization, pruning

## Regular Expression Calculus as a Modelling Language

Many features you wish other languages had!

Programming Languages	The Finite-State Case
Function composition	Machine composition
Nondeterminism	Nondeterminism
Parallelism	Compose FSA with FST
Function inversion (cf. Prolog)	Function inversion
Higher-order functions	Transform object code (apply operators to it)

## Regular Expression Calculus as a Modelling Language

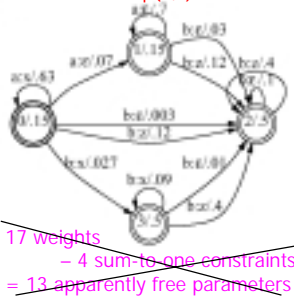
- Statistical FSMs still done in assembly language
  - Build machines by manipulating arcs and states
  - For training,
    - get the weights by some exogenous procedure and patch them onto arcs
    - you may need extra training data for this
    - you may need to devise and implement a new variant of EM
- Would rather build models **declaratively**
  - $((a^{*.7} b) +_{.5} (ab^{*.6})) \circ \text{repl}_g((a:(b +_{.3} \epsilon))^*, L, R)$

## Outline

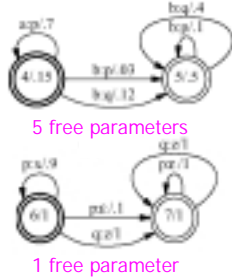
- What can you build with parameterized FSMs?
- How do you train them?
  - Hint: Make the finite-state machinery do the work.

## How Many Parameters?

Final machine  $p(x,z)$

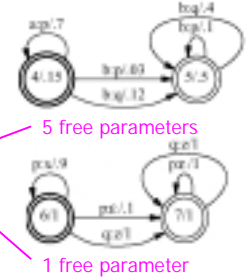


But really I built it as  $p(x,y) \circ p(z|y)$



## How Many Parameters?

But really I built it as  $p(x,y) \circ p(z|y)$



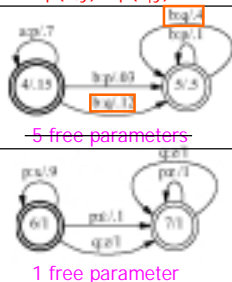
Even these 6 numbers could be tied ... or derived by formula from a smaller parameter set.

## How Many Parameters?

But really I built it as  $p(x,y) \circ p(z|y)$

Really I built this as

$(a:p)^{*.7} (b: (p +_{.2} q))^{*.5}$   
3 free parameters

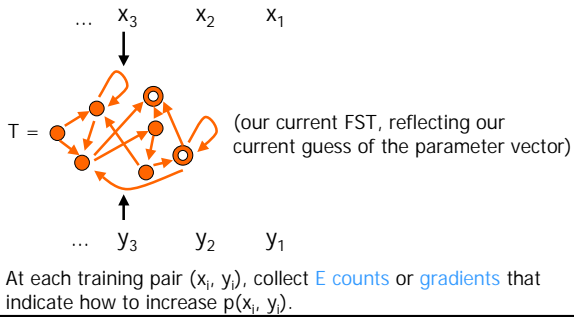


## Training a Parameterized FST

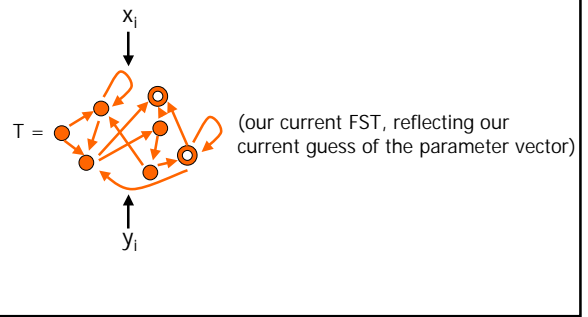
Given: an expression (or code) to build the FST from a parameter vector  $\theta$

- Pick an initial value of  $\theta$
- Build the FST – implements fast prob. model
- Run FST on some training examples to compute an objective function  $F(\theta)$
- Collect E-counts or gradient  $\nabla F(\theta)$
- Update  $\theta$  to increase  $F(\theta)$
- Unless we converged, return to step 2

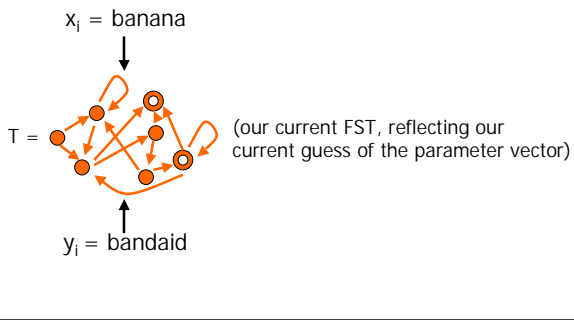
## Training a Parameterized FST



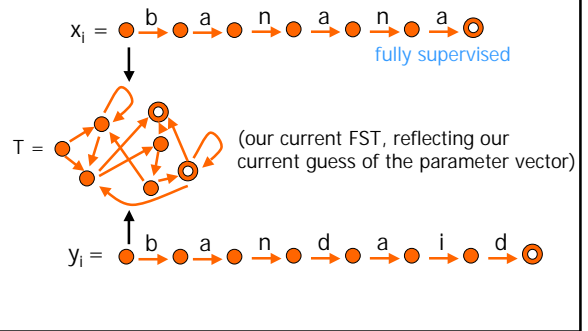
## What are $x_i$ and $y_i$ ?



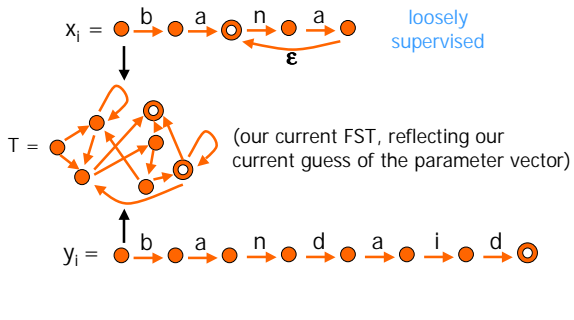
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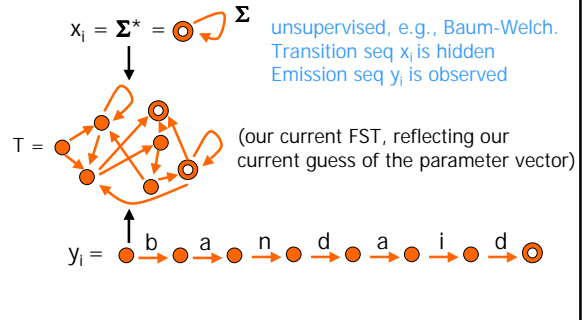
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## What are $x_i$ and $y_i$ ?



### Building the Trellis

$x_i = \dots$

$y_i = \dots$

$T =$

COMPOSE to get **trellis**:

$x_i \circ T \circ y_i =$

Extracts paths from T that are compatible with  $(x_i, y_i)$ .

Tends to unroll loops of T, as in HMMs, but not always.

### Summing the Trellis

$x_i \circ T \circ y_i =$

Extracts paths from T that are compatible with  $(x_i, y_i)$ . Tends to unroll loops of T, as in HMMs, but not always.

Let  $t_i$  = total probability of all paths in trellis  
 =  $p(x_i, y_i)$   $x_i, y_i$  are regexps (denoting strings or sets of strings)  
 This is what we want to increase!

How to compute  $t_i$ ?  
 If acyclic (exponentially many paths): dynamic programming.  
 If cyclic (infinitely many paths): solve sparse linear system.

### Summing the Trellis

$x_i \circ T \circ y_i =$

Let  $t_i$  = total probability of all paths in trellis  
 =  $p(x_i, y_i)$ .  
 This is what we want to increase!

**Remark:** In principle, FSM minimization algorithm already knows how to compute  $t_i$ , although not the best method.

minimize (epsilonify  $(x_i \circ T \circ y_i)$ ) =  $t_i$   
 ↑  
 replace all arc labels with  $\epsilon$

### Example: Baby Think & Baby Talk

observe talk

recover think, by composition

Mama/.005  
 Mama Iwant/.0005  
 Mama Iwant Iwant/.00005  
 XX/.032  
 Total = .037555555

### Joint Prob. by Double Composition

think

talk

compose

$p(\Sigma^* : mm) = .0375555 = \text{sum of paths}$

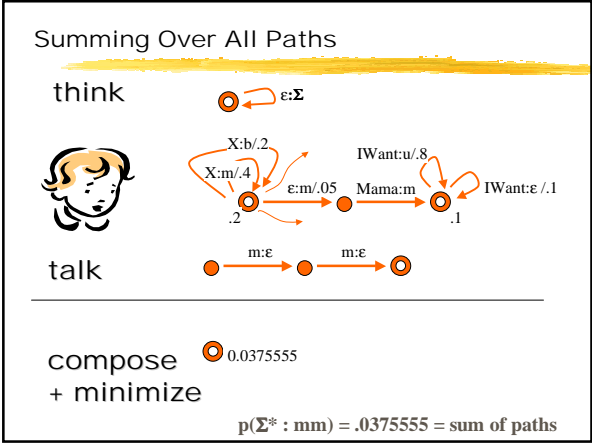
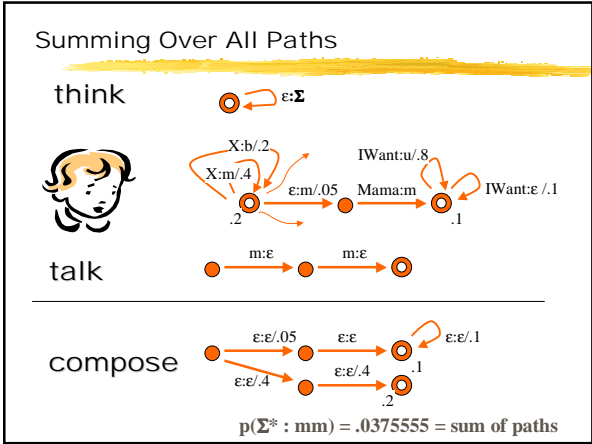
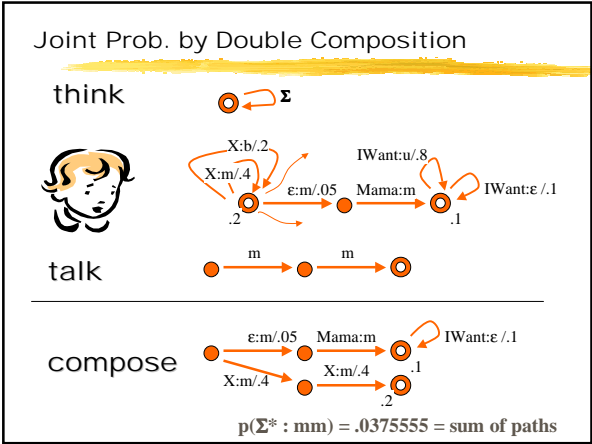
### Joint Prob. by Double Composition

think

talk

compose

$p(\Sigma^* : mm) = .0005 = \text{sum of paths}$



### Where We Are Now

"minimize (epsilonify (  $x_i$  o T o  $y_i$  ))" =  $\epsilon/t_i$

obtains  $t_i$  = sum of trellis paths =  $p(x_i, y_i)$ .

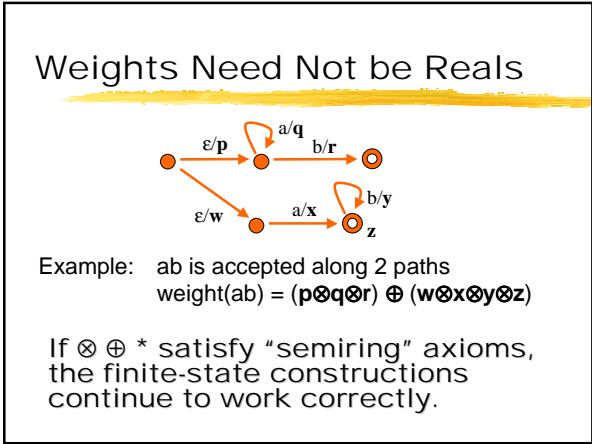
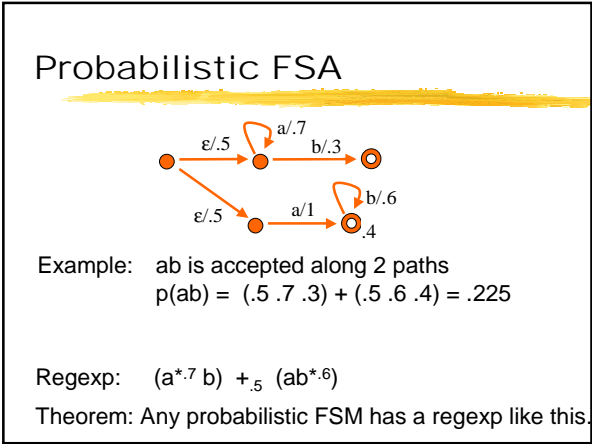
Want to change parameters to make  $t_i$  increase.

a vector

**Solution:** Annotate every probability with bookkeeping info. So probabilities know how they depend on parameters.

Then the probability  $t_i$  will know, too!  
It will emerge annotated with info about how to increase it.

The machine T is built with annotations from the ground up.





## Semiring Definitions

Weight of a string is total weight of its accepting paths.

Union: $\oplus$		$p \oplus q$
Concat: $\otimes$		$p \otimes q$
Closure: $*$		$p^*$
Intersect, Compose: $\boxtimes$		$p \boxtimes q$

## The Probability Semiring

Weight of a string is total weight of its accepting paths.

Union: $\oplus$		$p \oplus q = p+q$
Concat: $\otimes$		$p \otimes q = pq$
Closure: $*$		$p^* = 1+p+p^2 + \dots = (1-p)^{-1}$
Intersect, Compose: $\boxtimes$		$p \boxtimes q = pq$

## The (Probability, Gradient) Semiring

Base case		where $\nabla p$ is gradient
Union: $\oplus$		$(p,x) \oplus (q,y) = (p+q, x+y)$
Concat: $\otimes$		$(p,x) \otimes (q,y) = (pq, py + qx)$
Closure: $*$		$(p,x)^* = ((1-p)^{-1}, (1-p)^{-2}x)$
Intersect, Compose: $\boxtimes$		$(p,x) \boxtimes (q,y) = (pq, py + qx)$

## We Did It!

- We now have a clean algorithm for computing the gradient.

$$x_i \circ T \circ y_i =$$

Let  $t_i$  = total annotated probability of all paths in trellis  
 $= (p(x_i, y_i), \nabla p(x_i, y_i))$ . Aggregate over  $i$  (training examples).

How to compute  $t_i$ ?

Just like before, when  $t_i = p(x_i, y_i)$ . But in **new semiring**.

If acyclic (exponentially many paths): dynamic programming.

If cyclic (infinitely many paths): solve sparse linear system.

Or can always just use minimize (epsilonify  $(x_i \circ T \circ y_i)$ ).

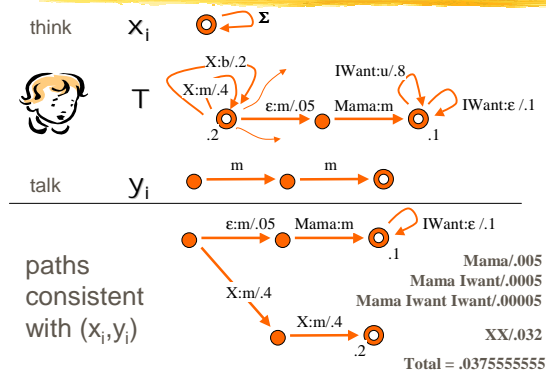
## An Alternative: EM

Would be easy to train probabilities  
*if we'd seen the paths the machine followed*

- E-step:** Which paths probably generated the observed data? (according to current probabilities)
- M-step:** Reestimate probabilities (or  $\theta$ ) as if those guesses were right
- Repeat**

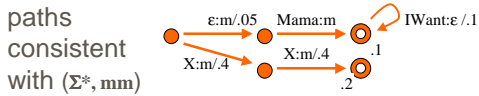
Guaranteed to converge to local optimum.

## Baby Says mm



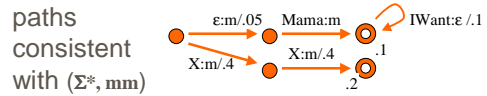
### Which Arcs Did We Follow?

$p(\text{Mama} : \text{mm}) = .005$   
 $p(\text{Mama Iwant} : \text{mm}) = .0005$   
 $p(\text{Mama Iwant Iwant} : \text{mm}) = .00005$  etc.  
 $p(\text{XX} : \text{mm}) = .032$   
 $p(\text{mm}) = p(\Sigma^* : \text{mm}) = .037555 = \text{sum of all paths}$   
 $p(\text{Mama} | \text{mm}) = .005/.037555 = 0.13314$   
 $p(\text{Mama Iwant} | \text{mm}) = .0005/.037555 = 0.01331$   
 $p(\text{Mama Iwant Iwant} | \text{mm}) = .00005/.037555 = 0.00133$   
 $p(\text{XX} | \text{mm}) = .032/.037555 = 0.85207$



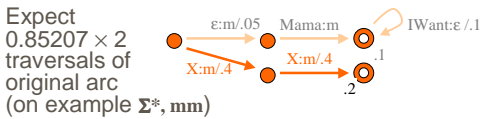
### Count Uses of Original Arcs

$p(\text{Mama} | \text{mm}) = .005/.037555 = 0.13314$   
 $p(\text{Mama Iwant} | \text{mm}) = .0005/.037555 = 0.01331$   
 $p(\text{Mama Iwant Iwant} | \text{mm}) = .00005/.037555 = 0.00133$   
 $p(\text{XX} | \text{mm}) = .032/.037555 = 0.85207$

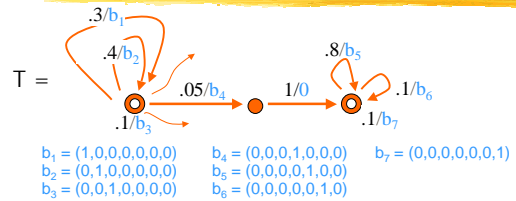


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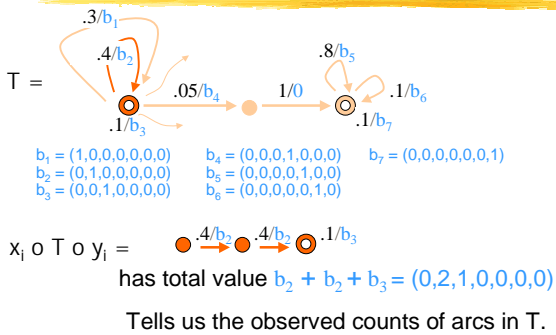
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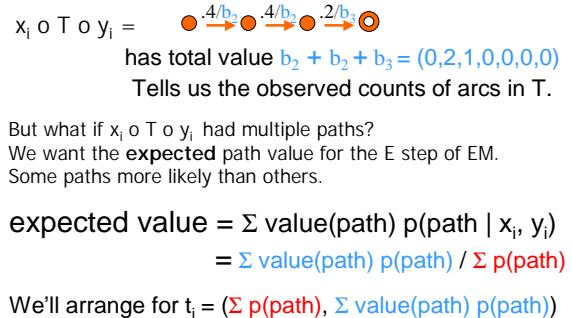
### Associate a *value* with each arc we wish to track Expected-Value Formulation



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






### Associate a *value* with each arc we wish to track Expected-Value Formulation



$$t_i = (\sum p(\text{path}), \sum \text{value}(\text{path}) p(\text{path}))$$

## The Expectation Semiring

Base case		where v is arc value
Union: $\oplus$		$(p,x) \oplus (q,y) = (p+q, x+y)$
Concat: $\otimes$		$(p,x) \otimes (q,y) = (pq, py + qx)$
$\overset{\curvearrowright}{\cap}$		$(p,x)^* = ((1-p)^{-1}, (1-p)^{-2}x)$
Intersect, Compose: $\otimes$		$(p,x) \otimes (q,y) = (pq, py + qx)$

same as before!

## That's the algorithm!

- Existing mechanisms do all the work
- Keeps count of original arcs despite composition, loop unrolling, etc.
- Cyclic sums handled internally by the minimization step, which heavily uses semiring closure operation
- Flexible: can define arc values as we like
  - Example: Log-linear (maxent) parameterization
  - M-step: Must reestimate  $\theta$  from feature counts (e.g., Iterative Scaling)
  - If arc's weight is  $\exp(\theta_x + \theta_y)$ , let its value be  $(0, 1, 0, 0, 1, \dots)$
  - Then total value of correct path for  $(x_i, y_i)$  – counts observed features
  - E-step: Needs to find expected value of path for  $(x_i, y_i)$

## Log-Linear Parameterization

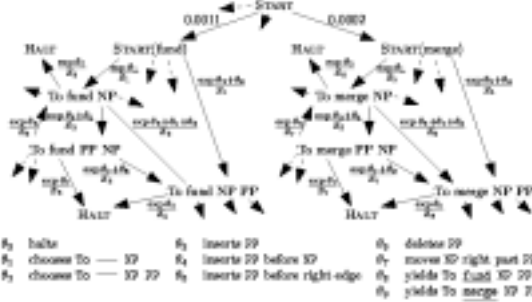


Figure 1.3: How the arc probabilities in Fig. 1.2 were determined from feature weights  $\theta$ . The  $Z$  values are chosen so that the arcs leaving each vertex have total probability 1.

## Some Optimizations



Let  $t_i$  = total annotated probability of all paths in trellis =  $(p(x_i, y_i), \text{bookkeeping information})$ .

- Exploit (partial) acyclicity
- Avoid expensive vector operations
- Exploit sparsity
- Rebuild quickly after parameter update

## Need Faster Minimization

- Hard step is the minimization:
  - Want total semiring weight of all paths
  - Weighted  $\epsilon$ -closure must invert a semiring matrix



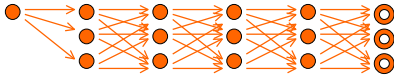
- Want to beat this! (takes  $O(n^3)$  time)
- Optimizations exploit features of problem

## All-Pairs vs. Single-Source

- For each  $q, r$ ,  $\epsilon$ -closure finds total weight of all  $q \rightsquigarrow r$  paths
- But we only need total weight of init  $\rightsquigarrow$  final paths
- Solve linear system instead of inverting matrix:
  - Let  $\alpha(r)$  = total weight of init  $\rightsquigarrow r$  paths
  - $\alpha(r) = \sum_q \alpha(q) * \text{weight}(q \rightarrow r)$
  - $\alpha(\text{init}) = 1 + \sum_q \alpha(q) * \text{weight}(q \rightarrow \text{init})$
- But still  $O(n^3)$  in worst case

## Cycles Are Usually Local

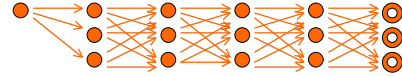
- In HMM case,  $T_i = (\epsilon \times x_i) \circ T \circ (y_i \times \epsilon)$  is an acyclic lattice:



- Acyclicity allows linear-time dynamic programming to find our sum over paths
- If not acyclic, first decompose into minimal cyclic components (Tarjan 1972, 1981; Mohri 1998)
  - Now full  $O(n^3)$  algorithm must be run for several small  $n$  instead of one big  $n$  – and reassemble results
  - More powerful decompositions available (Tarjan 1981); *block-structured matrices*

## Avoid Semiring Operations

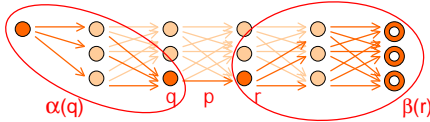
- Our semiring operations aren't  $O(1)$ 
  - They manipulate vector values
- To see how this slows us down, consider HMMs:



- Our algorithm computes sum over paths in lattice.
  - If acyclic, requires a forward pass only.
- Where's backward pass?**
- What we're pushing forward is  $(p, v)$ 
  - Arcs  $v$  go forward to be downweighted by later probs, instead of probs going backward to downweight arcs.
  - The vector  $v$  rapidly loses sparsity, so this is slow!

## Avoid Semiring Operations

- We're already computing forward probabilities  $\alpha(q)$
- Also compute backward probabilities  $\beta(r)$



- Total probability of paths through this arc =  $\alpha(q) * p * \beta(r)$
- $E[\text{path value}] = \sum_{q,r} (\alpha(q) * p(q \rightarrow r) * \beta(r)) * \text{value}(q \rightarrow r)$
- Exploits structure of semiring
- Now  $\alpha, \beta$  are probabilities, not vector values

## Avoid Semiring Operations

- Now our linear systems are over the reals:**
  - Let  $\alpha(r)$  = total weight of  $\text{init} \rightsquigarrow r$  paths
  - $\alpha(r) = \sum_q \alpha(q) * \text{weight}(q \rightarrow r)$
  - $\alpha(\text{init}) = 1 + \sum_q \alpha(q) * \text{weight}(q \rightarrow \text{init})$
- Well studied!** Still  $O(n^3)$  in worst case, but:
  - Proportionately faster for sparser graph
    - $O(|\text{states}| |\text{arcs}|)$  by iterative methods like conj. gradient
    - Usually  $|\text{arcs}| \ll |\text{states}|^2$
  - Approximate solutions possible
    - Relaxation (Mohri 1998) and back-relaxation (Eisner 2001); or stop iterative method earlier
  - Lower space requirement:  $O(|\text{states}|)$  vs.  $O(|\text{states}|^2)$

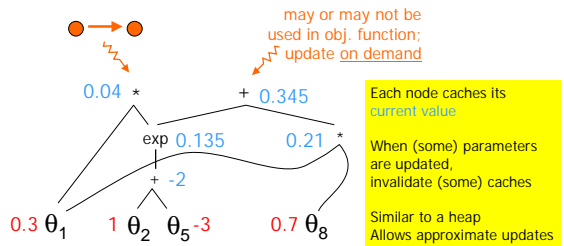
## Fast Updating

- Pick an initial value of  $\theta$
- Build the FST – implements fast prob. model
- ...
- Unless we converged, return to step 2

- But step 2 might be slow!
- Recompiles the FST from its parameterized regexp, using the new parameters  $\theta$ .
- This involves a lot of structure-building, not just arithmetic
  - Matching arc labels in intersection and composition
  - Memory allocation/deallocation
  - Heuristic decisions about time-space tradeoffs

## Fast Updating

- Solution: Weights remember underlying formulas
- A weight is a pointer into a formula DAG



## The Sunny Future



- Easy to experiment with interesting models.
- Change a model = edit declarative specification
- Combine models = give a simple regexp
- Train the model = push a button
- Share your model = upload to archive
- Speed up training = download latest version (conj gradient, pruning ...)
- Avoid local maxima = download latest version (deterministic annealing ...)
- p.s. Expectation semirings extend naturally to context-free case, e.g., Inside-Outside algorithm.

## Marrying Two Finite-State Traditions

Classic stat models & variants  $\Rightarrow$  *simple FSMs*  
HMMs, edit distance, sequence alignment, *n*-grams, segmentation

*Trainable from data*

Expert knowledge  $\Rightarrow$  *hand-crafted FSMs*

Extended regexps, phonology/morphology, info extraction, syntax ...

*Tailored to task*

*Tailor model, then train end-to-end*

*Design complex finite-state model for task*

**Any extended regexp**

**Any machine topology; epsilon-cycles ok**

*Parameterize as desired to make it probabilistic*

**Combine models freely, tying parameters at will**

*Then find best param values from data (by EM or CG)*

## Ways to Improve Toolkit

- Experiment with other learning algs ...
  - Conjugate gradient is a trivial variation; should be faster
  - Annealing etc. to avoid local optima
- Experiment with other objective functions ...
  - Trivial to incorporate a Bayesian prior
  - Discriminative training: maximize  $p(y | x)$ , not  $p(x,y)$
- Experiment with other parameterizations ...
  - Mixture models
  - Maximum entropy (log-linear): track expected feature counts, not arc counts
- Generalize more: Incorporate graphical modelling

## Some Applications

- Prediction, classification, generation; more generally, "filling in of blanks"
  - Speech recognition
  - Machine translation, OCR, other noisy-channel models
  - Sequence alignment / Edit distance / Computational biology
  - Text normalization, segmentation, categorization
  - Information extraction
  - Stochastic phonology/morphology, including lexicon
  - Tagging, chunking, finite-state parsing
  - Syntactic transformations (smoothing PCFG rulesets)
- Quickly specify & combine models
- Tie parameters & train end-to-end
- Unsupervised, partly supervised, erroneously supervised

# FIN

*that's all folks*  
*(for now)*

wish lists to [eisner@cs.jhu.edu](mailto:eisner@cs.jhu.edu)