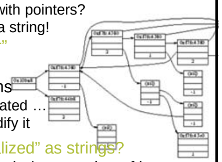
Encodings and reducibility

Designing your little language

- You have to encode your problem as a string
  - How would you encode a tree?
    - That is, what's a nice little language for trees?
      - More than one option?
    - What solvers could you write?
    - Does every string encode some tree?
  - How would you encode an arbitrary graph?
    - Is it okay if there are two encodings of the same graph?
    - What solvers could you write?

Remind me why we're using strings?

- Why not design a Java graph object & write solver to work on that?
  - Then your solver can only take input from another Java program
  - Can only send its output (if a graph) to another Java program
  - Where do those programs get *their* input and send *their* output?
  - All programs must be running at once, to see same object
    - How do you *save* or *transmit* a structure with pointers?
    - Solution is serialization – turn object into a string!
- Strings are a "least common denominator"
  - Simple storage
  - Simple communication between programs
    - can even peek at what's being communicated …
    - and even run programs to analyze or modify it



- Can all finite data structures really be "serialized" as strings?
  - Sure … computer's memory can be regarded as a *string of bytes*.
  - Theoretical CS even regards all problems as string problems!
    - A Turing machine's input and output tapes can be regarded as *strings*
    - **P** = "polynomial time" = class of decision problems that are O(n^k) for some constant k, where n = *length of the input string*

Encoding mismatches

- How do you run graph algorithms on a string that represents a tree?
- How do you run tree algorithms on a string that represents a graph?
  - Assuming you know the graph is a tree
  - Should reject graphs that aren't trees (syntax error)
- (What's the corresponding problem & solution for Java classes?)
- How do you run a graph algorithm to sort a sequence of numbers?

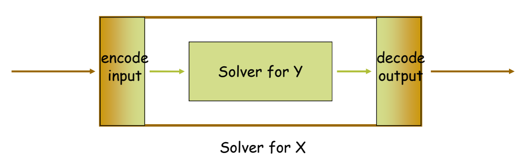
Encoding integers

- You want to write a solver that determines if an integer is prime
  - How hard is this? How hard is it to factor an integer?
- How do you encode the number 2010 as a string?
  - "2010" No harder: First convert from d digits to b bits in O(d) time
  - "MMX" Then test primality in time $O(b^6) = O((4d)^6) = O(4^6 d^6) = O(d^6)$
    - Slightly harder for solver So decimal encoding isn't harder; is it easier?
    - But easier for some users (ancient Romans)
  - "11111011010" If b = # bits, can factor in $O\left(\exp\left(\left(\frac{64}{9}b\right)^{\frac{1}{3}}(\log b)^{\frac{2}{3}}\right)\right)$.
    - Slightly easier for solver (if a PC) But test primality in about $O(b^6)$.
    - But harder for most users
  - **Does it matter which of the above we use?**

  - "2*3*5*67" (encode 2010 as its unique factorization)
    - Qualitatively different! Why? Can test primality in O(length of input)
  - "1111111111111111111111111111111111111…." (2010 times)
    - Qualitatively different! Why? Can test primality in O(length of input)

Reducibility

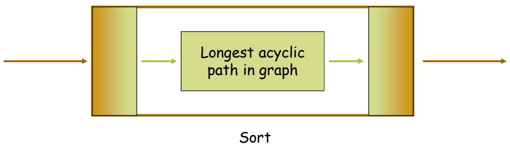
- One way to build a solver is to "wrap" another solver



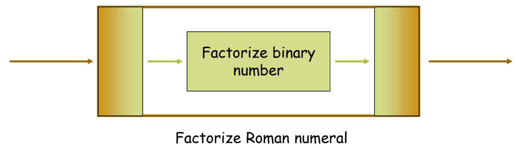
X(input) = decode(Y(encode(input)))

- Can set this up without knowing how to solve Y!
  - As we find (faster) solvers for Y, automatically get (faster) solvers for X

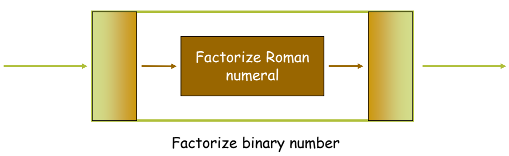
Reducibility



Reducibility



Reducibility



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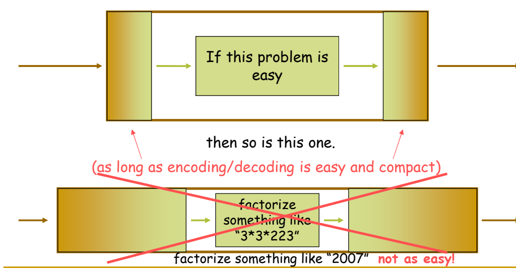

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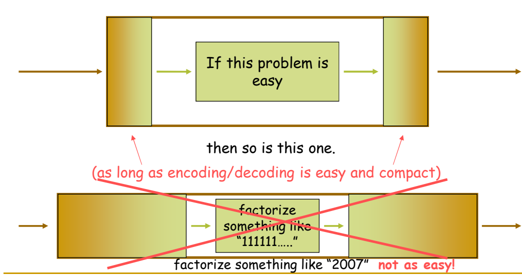

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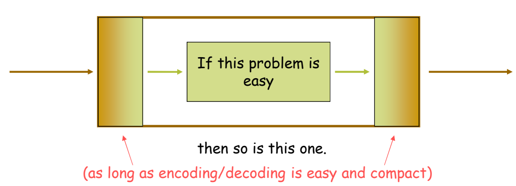
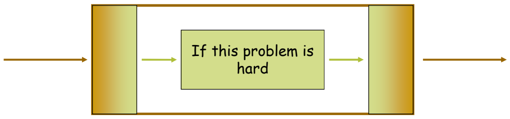


- What do we mean by a hard vs. an easy problem?
  - A hard problem has no fast solvers
  - An easy problem has at least one fast solver

Reducibility



then is this one necessarily hard?

(Nope. There might be a different, faster way to sort.)

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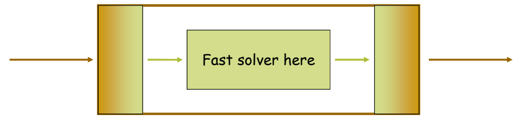
Reducibility



gives us a fast solver here.

(Even faster ones might exist.)

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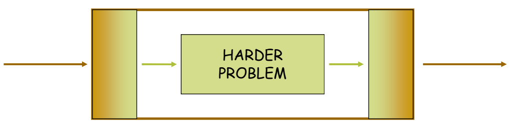

EASIER PROBLEM (or a tie)

(Even faster solvers might or might not exist.)

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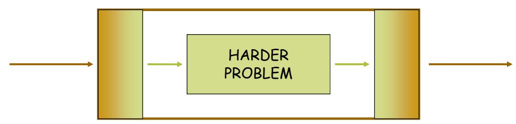
Interreducibility



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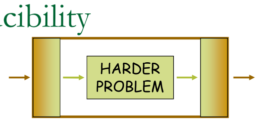
Interreducibility



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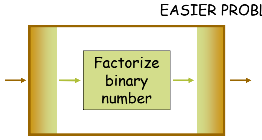
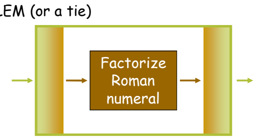


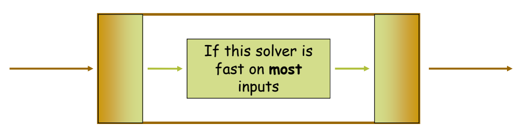
Factorize Roman numeral



Factorize binary number

Interreducible problems
They must tie!
Equally easy (or equally hard)
if we're willing to ignore encoding/decoding cost.

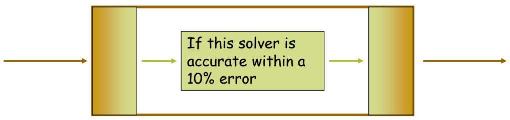
Reducibility & "typical" behavior



then is this solver fast on **most** inputs?

Not necessarily – the encoding might tend to produce hard inputs for the inner solver

Reducibility & approximation

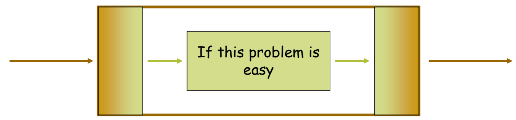


then is this solver equally accurate?

(Does almost-longest path lead to an almost-sort?)
(Not the way I measure "almost"!)

- **<u>Some</u>** reductions (encode/decode funcs) may preserve your idea of "almost."
- In such cases, any accurate solver for the inner problem will yield one for the outer problem. Just be careful to check: don't assume reduction will work!

Proving hardness



then so is this one.

So what can we conclude if outer problem is believed to be hard?

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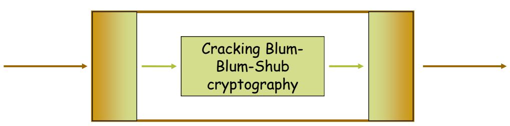
Proving hardness



Factoring integers

So what can we conclude if outer problem is believed to be hard?

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