Ray Casting

Ray Casting Algorithm

For each pixel

1. **Compute ray from eye through pixel**
2. **For each primitive**
   - **—Test for ray-object intersection**
3. **Shade pixel using nearest primitive (or set to background color)**

Computing the Rays

Choose eye point, view direction, up direction, fields of view (x and y)

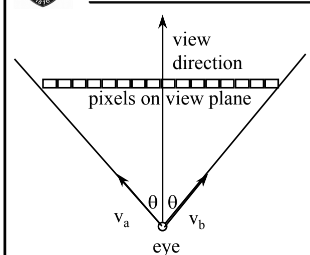
p_t = eye + **t***v (v **typically normalized)**

Compute rays to two opposite corners

Compute step sizes, Δx and Δy to go from pixel to pixel

To compute new ray: take step, then normalize

2D ray calculation



Note: take equal-sized steps in viewing plane, not equal angles!

view is normalized view direction

right = (**view**$_y$, -**view**$_x$)
v$_a$ = **view** - tanθ * **right**
v$_b$ = **view** + tanθ * **right**
step = (**v**$_b$ - **v**$_a$) / num_pixels
v$_0$ = **v**$_a$ + **step** / 2
v$_i$ = **v**$_{i-1}$ + **step**

In 3D, we have an additional step size and field-of-view angle as well as an up vector.

Computing Intersections

Ray is in parametric form (t is parameter)

- **x = p + t*v**

Represent primitive in implicit form:

- **f(x,y,z) = 0**
- **(any (x,y,z) on surface evaluates to zero)**

Substitute (x,y,z) of ray into f(x,y,z) and solve for t

- **degree n implicit function will be degree n in t**
- **quadric surfaces may be solved with quadratic equation -- pick real solution closest to eye**

Example Quadric Functions

Sphere: $(x-a)^2 + (y-b)^2 + (z-c)^2 - r^2 = 0$

Circular cylinder (parallel to z-axis):

$(x-a)^2 + (y-b)^2 - r^2 = 0$

Hyperbolic paraboloid:

$y^2/b^2 - x^2/a^2 - z = 0$

General Quadrics

General quadric has form:

$$Ax^2 + 2Bxy + 2Cxz + 2Dx + Ey^2 + 2Fyz + 2Gy + Hz^2 + 2Iz + J = 0$$

or...

$x^tQx = 0$, **where** $x^t = [x\ y\ z\ 1]$ **and**

$$Q = \begin{bmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \\ D & G & I & J \end{bmatrix}$$

Quadric Intersections

Quadric: $x^tQx = 0$

Ray: $x = p + tv$

Substituting ray for x**:**

$$(p + tv)^tQ(p + tv) = 0$$

$$p^tQp + p^tQtv + tv^tQp + tv^tQtv = 0$$

$$(v^tQv)t^2 + (p^tQv + v^tQp)t + p^tQp = 0$$

$$(v^tQv)t^2 + (2v^tQp)t + p^tQp = 0$$

(Q is symmetric)

Common Ray-tracing Primitives

Sphere, ellipsoid

Cylinders

Plane, triangle

- $Ax + By + Cz + D = 0$

Torus

Bezier/Nurbs patches

- **parametric, so use implicit form of ray**
  - **—intersection of two planes**

Local Illumination Shading

Compute normal at closest intersection

- $\nabla f = (\partial x, \partial y, \partial z)$ **is normal vector field for implicit function, f**

For each light

- **Use position and normal to compute light contribution**
- **Accumulate light contributions**

Color pixel

- **Clamp to avoid overflow**

Shadows

Only add contribution from a light if it is visible from the point (and vice versa)

- **test for intersections along ray in L direction**
- **accumulate contribution if no occlusion**

(illumination is no longer totally local)

Truncating Primitives

Use another implicit function

- **Test which side of the implicit function the intersection is on**
- **Keep intersection only if it is on the correct side**

For example, truncate a cylinder using two plane equations (or perhaps a sphere)

- **then cap using the two planes truncated by the cylinder**

Constructive Solid Geometry

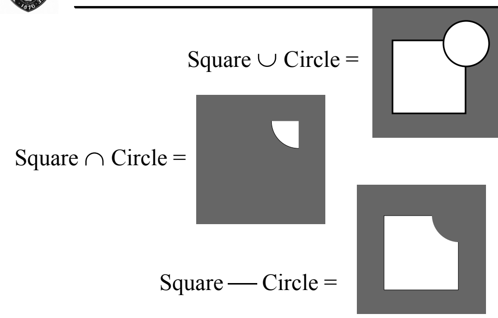
Perform hierarchical set operations on primitives

Union: ∪

Intersection: ∩

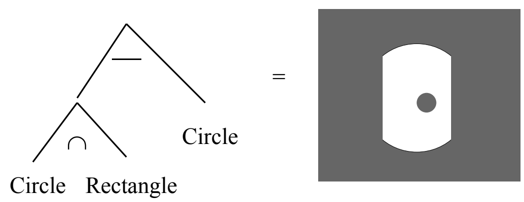
Difference: —

CSG Operators



Johns Hopkins Department of Computer Science
Course 600.456: Rendering Techniques, Professor: Jonathan Cohen

CSG Hierarchy



Johns Hopkins Department of Computer Science
Course 600.456: Rendering Techniques, Professor: Jonathan Cohen

Ray Tracing CSG

Each "object" may be a primitive or a CSG hierarchy

Find all **ray-primitive intersections for hierarchy**

Use CSG operators to determine which intervals **are solid or vacant**

Use start of nearest solid interval as ray-object intersection

Johns Hopkins Department of Computer Science
Course 600.456: Rendering Techniques, Professor: Jonathan Cohen

CSG Tracing Algorithm

Start at root of CSG Hierarchy

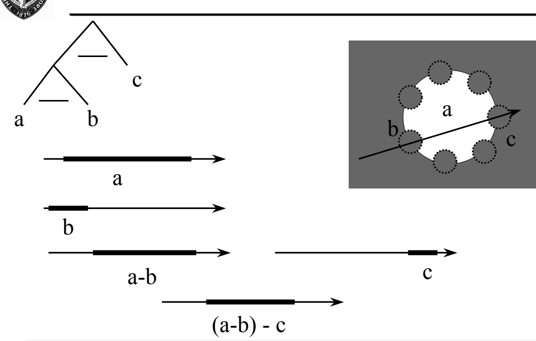
Trace ray through left child - result is ordered list of intersections, forming solid and vacant intervals

Trace ray through right child

Merge lists of intersections/intervals by applying CSG operator of current node

Johns Hopkins Department of Computer Science
Course 600.456: Rendering Techniques, Professor: Jonathan Cohen

CSG Example - golf ball



Johns Hopkins Department of Computer Science
Course 600.456: Rendering Techniques, Professor: Jonathan Cohen

Some CSG Details

Each interval endpoint associated with intersection of ray with some surface

Normal computed from surface of intersection

Material parameters may come from either primitive