Divisive Normalization

An important example is the use of probabilistic models [170] to account for divisive normalization. This is a mechanism whereby cells mutually inhibit one another, effectively normalizing their responses with respect to stimulus inputs. Originally developed to explain non-linear responses to contrast in V1 [59], divisive normalization has been proposed as a basic cortical computation that underlies various effects of context (see next section), as well as higher-level processes such as attention [20].

The probabilistic approach give a theoretical justification for divisive normalization in V1. The main idea is that filters with similar preferences for orientation representing nearby spatial locations in a scene have striking statistical dependencies, which can be removed by divisive normalization. Specifically, if we plot the statistics of two linear filters f_c, f_s (center and surround) then the magnitudes of f_c, f_s are coordinated in a straightforward way, which has a characteristic shape of a Bow-Tie.

Modeling Divisive Normalization using hidden variables

This can be modeled by assuming there are hidden variables ν which affect both responses and hence induces correlation between the responses. E.g., ν could represent the local average image intensity which could affect the response of both filters but, after the filter response could be made independent by conditioning on the average intensity. Suppose ν has a prior distribution $P(\nu) = \nu \exp\{-\nu^2/2\}$ for $\nu \geq 0$. We have a pair of filters $\{l_i : i = 1, 2\}$ which are related to gaussian models $\{g_1 : i = 1, 2\}$. Then we can model the activation of the set of filter responses:

$$P(l_1, l_2) = \int d\nu P(\nu) \prod_{i=1}^{2} P(l_i|\nu, g_i)) P(g_i), \quad (19)$$

where $P(l_i|\nu, g_i) = \delta(l_i - \nu g_i)$. In this model the filter responses are generated by independent processes, g_1, g_2, but then are multiplied by the common factor ν. This is illustrated in figure (25).

Figure for Divisive Normalization Model

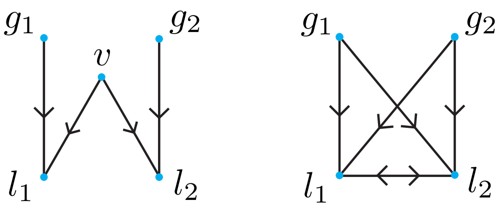

Figure 25: Left Panel: The graphical structure of the divisive normalization model. The filter responses l_1, l_2 are generated from stimuli g_1, g_2 respectively and by the common factor ν. The distributions of l_1, l_2 are factorized if we condition on ν. Right Panel: But if we integrate out ν then almost all the variables become dependent as reflected by the complexity of the graph structure.

Divisive Normalization Model

In particular, for each filter we can compute $P(g_i|l_1, l_2)$. After some algebra, this is computed to be:

$$P(g_1|l_1, l_2) = \frac{g_1^{-1} \exp\{-\frac{g_1^2 l^2}{2\sigma^2 l_1^2} - \frac{l_1^2}{2g_1^2}\}}{B(0, l/\sigma)}, \tag{20}$$

where $l = \sqrt{l_1^2 + l_2^2}$, and $B(.,.)$ is a Bessel's function. To get intuition, note that $g_1 = l_1/\nu$ and $g_1 = l_1/\nu$. So if ν is small then $|l_1|$ and $|l_2|$ are likely to be small together, while if ν is large, then $|l_1|$ and $|l_2|$ are both likely to be large. Assume that the goal of a model unit is to estimate the g_i from the observed filter responses $\{l_i : i = 1, 2\}$, which gives the non-linear response of the cell. It follows, from analysis above, that

$$E(g_1|l_1, l_2) \propto \mathrm{sign}\{l_1\}\sqrt{|l_1|}\sqrt{\frac{|l_1|}{\sqrt{l_1^2 + l_2^2 + k}}}. \tag{21}$$

The $\sqrt{l_1^2 + l_2^2 + k}$ term sets the gain and performs the divisive normalization.

Application to the Tilt Illusion

The model has also been applied to explain the classic tilt illusion in perception [152, 136]. In the “simultaneous” tilt illusion, a set of vertically oriented lines appears to tilt right when surrounded by an annulus of lines tilted left–an effect called “repulsion”. But for large differences between the center orientation and surround (tilted left), the center vertical lines can appear to tilt left–an effect called “attraction”. In their model, the population of neurons responding to the surround tilted lines contribute to divisive normalizing of the neurons responding to the center stimulus. This results in a change of their neural tuning curves which, together with the degree of coupling between center and surrounds, accounts for repulsion and attraction.

The suppressive effect of surround contrast on a central region is an example of local spatial context.