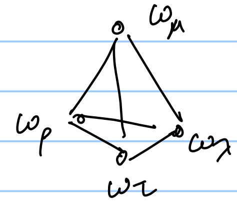
Lecture 12 — Hierarchical Models

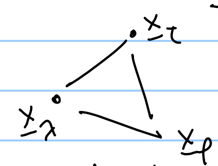
Note Title 2/26/2010



$(\rho, \tau, \lambda) \in ch(\mu)$

$\underline{\lambda} \cdot \underline{\phi}(\omega_\mu : \underline{\omega}_{ch(\mu)})$

$\underline{\omega}_\mu = f(\underline{\omega}_{ch(\mu)})$ deterministic function.



e.g. Gaussian $\underline{\omega}_\mu = (\underline{x}_\mu, \theta_\mu, s_\mu)$

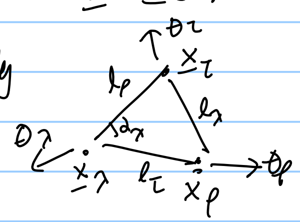
$$\underline{x}_\mu = \tfrac{1}{3}(\underline{x}_\rho + \underline{x}_\tau + \underline{x}_\lambda)$$
$$s_\mu = s_\rho + s_\tau + s_\lambda$$
$$\theta_\mu = \tfrac{1}{3}(\theta_\rho + \theta_\tau + \theta_\lambda) \quad \text{mod } 2\pi$$

Gaussian distribution.
on relative position $\underline{x}_\tau, \underline{x}_\lambda, \underline{x}_\rho$.
relative to center $\tfrac{1}{3}(\underline{x}_\rho + \underline{x}_\tau + \underline{x}_\lambda)$

Suff. stats $\underline{x}, \underline{x}\,\underline{x}^T$

$$\underline{\phi}(\underline{x}_{ch(\mu)}) = \left((\underline{x}_\tau, \underline{x}_\rho, \underline{x}_\mu), (\underline{x}_\tau, \underline{x}_\rho, \underline{x}_\mu)(\underline{x}_\tau, \underline{x}_\rho, \underline{x}_\mu)^T \right)$$

Alternatively

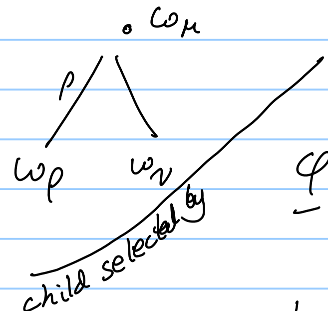


$\dfrac{l_\rho}{l_\lambda + l_\rho + l_\tau}, \ \alpha_\tau, \theta_\tau$ are independent of position, scale, orientation.

define $\underline{l} = \left(\alpha_\tau, \alpha_\rho, \alpha_\lambda, \dfrac{l_\rho}{l_\lambda + l_\rho + l_\tau}, \dfrac{l_\lambda}{l_\lambda + l_\rho + l_\tau}, \dfrac{l_\tau}{l_\lambda + l_\rho + l_\tau}, \theta_\tau, \theta_\rho, \theta_\lambda\right)$

Statistics $\underline{l}, \underline{l}\,\underline{l}^T$

Or nodes



switch variables
$t_\mu \in \{\rho, \nu\}$

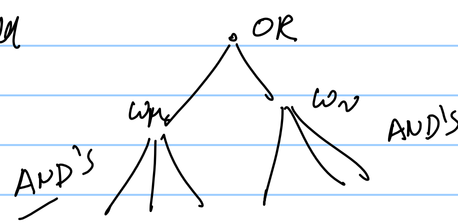
state of ω_μ is state of one of its children
$\omega_\mu = \omega_\rho$ or $\omega_\mu = \omega_\nu$

$$\underline{\phi}^{or}(\omega_\mu : \underline{\omega}_{ch(\mu)}) = \lambda_\rho \delta(\omega_\mu - \omega_\rho) + \lambda_\nu \delta(\omega_\mu - \omega_\nu)$$

Way to build a complex mixture distribution.

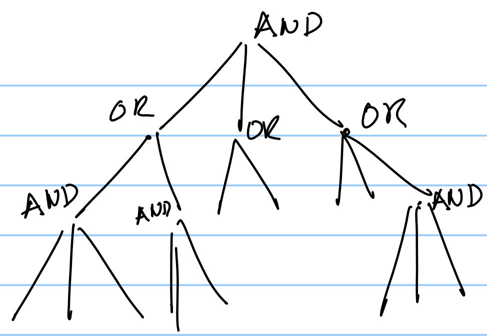
Or nodes allow topology of graph to change — to deal with different viewpoints and poses. (e.g. baseball players)

mixture model



standard mixture model.

(2)



Combinatorics:

AND / OR / AND in alternative levels.

reusable Parts

If n — no of child of OR's
m — no. of child of AND's
h — no. of OR level.

Then No. of Topologies $\sim O(n^{mh})$
No. of Nodes $\sim O(nm)^h$?

Efficient representation $\to$ Ratio $\frac{\text{No. of Nodes}}{\text{No. of Topologies}} \sim 0$ large h.

By comparison, standard mixture model.

$\frac{\text{No of Nodes}}{\text{No. of Topologies}} \simeq C$ — average no of nodes for each mixture.

Models: OR, AND.

How to Learn:

Case(1): Assume structure is known. / AND nodes only.

If potential obeys summarization — $\omega_\mu = f(\underline{\omega}_{ch(\mu)})$
then ground truth for leaf nodes gives ground truth for all nodes.

Then we can apply standard learning techniques
$\to$ maximum likelihood estimation, structure learning, etc

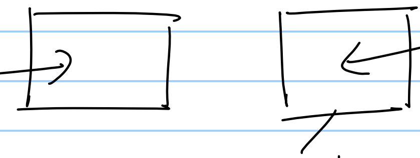
If OR nodes, then they must be specified by hand.

(3)

(2) Suppose we know the rough location of the object – i.e. it is within a box

Latent SVM

object present →

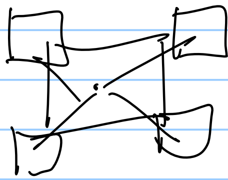


← object not present.

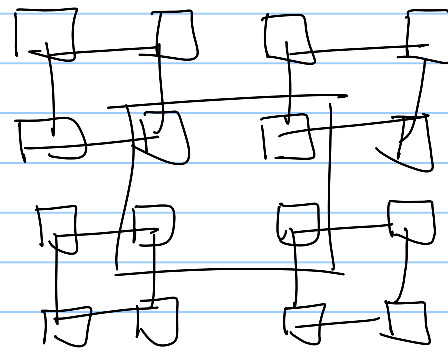
known variable is $y \in \{\pm 1\}$ $y = 1$ $y = -1$

Has only been applied to simple hierarchy

2 levels



3 levels

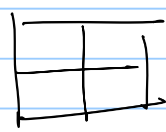


Recall — latent SVM requires good initial conditions

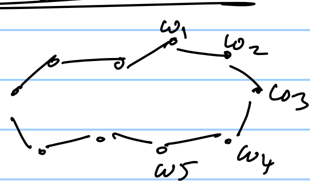
(3) Hierarchical Image Models

segmentation template (S_μ, S_μ)

groundtruth → at lattice level enables us to estimate the groundtruth for higher levels.



(4) Learning a Hierarchy in one stage.



affinity measure

$$A(\omega_i, \omega_{i+1}) = e^{-|x_i - x_{i+1}|} e^{-|\theta_i - \theta_{i+1}|}$$

Cluster:

$C_1 = \{\omega_1\}$

if $A(\omega_1, \omega_2) > T$, $C_1 = \{\omega_1\}$, $C_2 = \{\omega_2\}$

otherwise $C_1 = \{\omega_1, \omega_2\}$.

repeat to form clusters $C_1, \dots C_m$

represent each cluster by the summary of its elements

e.g. $C_1 = \{\omega_1, \omega_2, \omega_3\}$, $\tilde{\omega}_1 = f(\omega_1, \omega_2, \omega_3)$

repeat process on these $\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_m$.

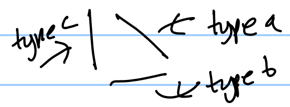
yield hierarchy

(4) (5) Unsupervised Learning of Hierarchy:

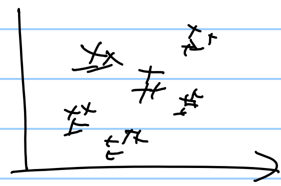
Dictionaries: Level 1. $\underline{\lambda}^a \cdot \underline{\Phi}^a(\omega_\mu; I)$ tuned to different orientations | — \ |

$a \in D_1$ hand specified.

Detect all instances and all composites.



cluster instances $\{\omega^a, \omega^b, \omega^c\}$ based on geometric properties



Each cluster gives a possible level 2 dictionary element.

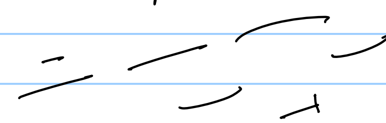
prune: by "suspicious coincidences"

— require the cluster to have a suff. large no of elements

next prune by "competitive exclusion"

— remove models which overlap too much.

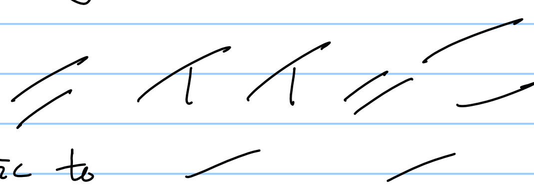
This gives Level-2 dictionary e.g.



Continue to higher levels.

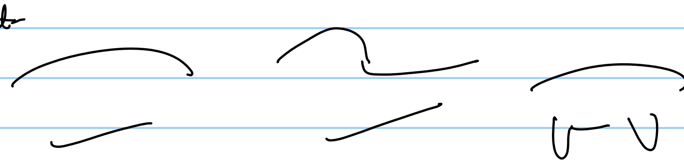
Stop automatically when there are no suspicious structures.

Level-3



Generic → not specific to object being learnt

Level-4



→ more like the object (e.g. horse)

Level-5.

models at level-5 are not complete models of the horse.

But can expand ~~these~~ models by adding elements from the lower-level dictionaries.

For details, see handout.