Lecture 12

Bayes-Kalman and Particle Filters

Note Title 4/23/2006

Bayes-Kalman update equations. (or $\int dx_t \int dx_{t+1}$)

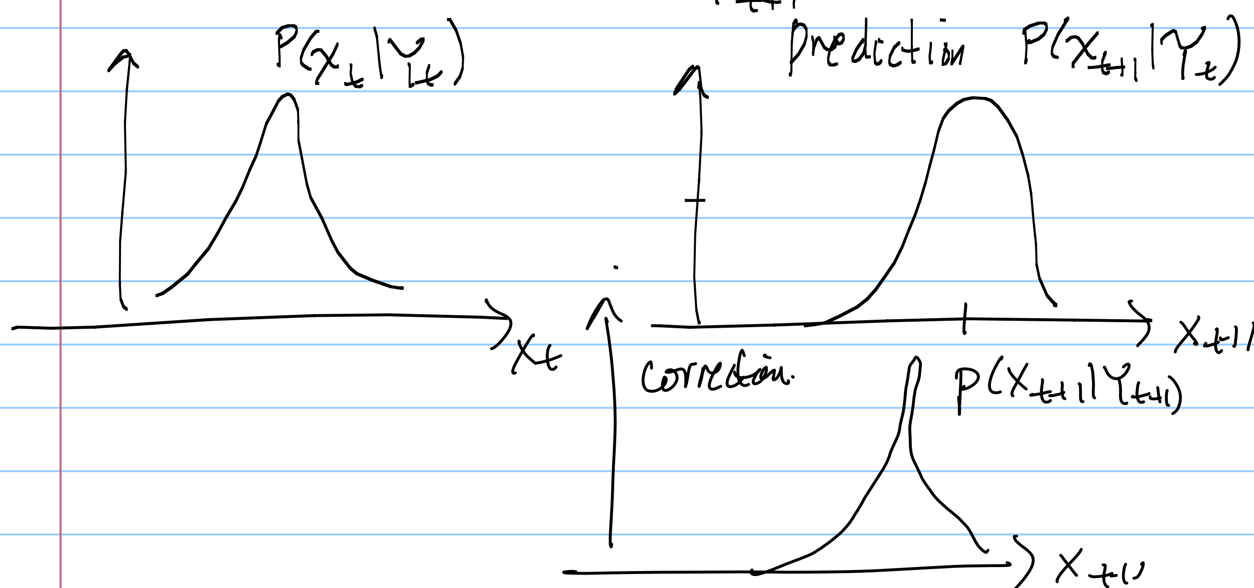
Prediction

(1) $$p(x_{t+1} \mid Y_t) = \sum_{x_t} p(x_{t+1} \mid x_t)\, p(x_t \mid Y_t)$$

Correction.

(2) $$p(x_{t+1} \mid Y_{t+1}) = \frac{p(y_{t+1} \mid x_{t+1})\, p(x_{t+1} \mid Y_t)}{p(y_{t+1} \mid Y_t)}$$

with $$p(y_{t+1} \mid Y_t) = \sum_{x_{t+1}} p(y_{t+1} \mid x_{t+1})\, p(x_{t+1} \mid Y_t)$$



Problem: Hard to compute Bayes-Kalman update.

If the distributions are Gaussians – prior and likelihood – then both stages can be reduced to algebra, previous lecture

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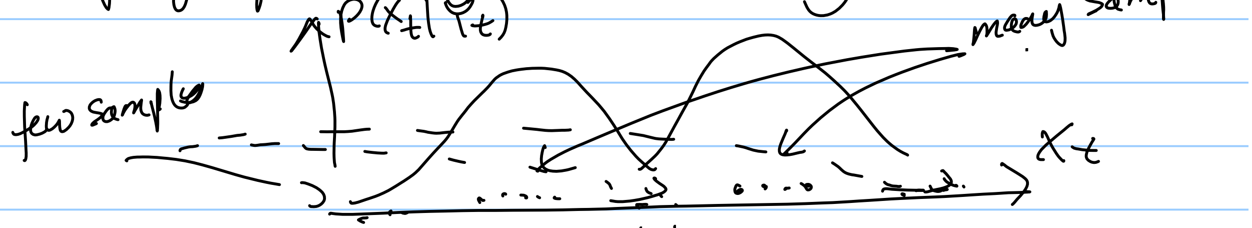
(Section 3.3 Jun Liu)

This motivates a sampling approach based on particle filters (sometimes called Bootstrap Filter)

Represent the distribution — e.g. $p(x_t | Y_t)$ — by a set of particles $\{x_t^1, \ldots, x_t^m\}$. These are random samples from $p(x_t|Y_t)$, so big density of particles at places of high probability and small density elsewhere.



(a) Draw samples $\{x_{t+1}^{*j} : j=1,..m\}$ (predict) from $p(x_{t+1}|x_t^{(j)})$ for $j = 1 \ldots m$.

(b) Weight each sample by $\omega^{(j)} \propto p(y_{t+1}|x_{t+1}^{*j})$ (uses the new observation y_{t+1})

(c) Resample from $\{x_{t+1}^{*1}, \ldots, x_{t+1}^{*m}\}$ with probability proportional to $\omega^{(j)}$ to produce random sample $\{x_{t+1}^1, \ldots, x_{t+1}^{(m)}\}$ for time $t+1$.

Claim: It can be shown that if the $\{x_t^1 \ldots x_t^m\}$ follow $P(x_t|Y_t)$ and if m is suff. big, then the $\{x_{t+1}^1, \ldots x_{t+1}^m\}$ follow $P(x_{t+1}|Y_{t+1})$

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<u>Note:</u> the weights are big for particles which are consistent with the new observation y_{t+1} – i.e. for x^{*j}_{t+1} s.t. $p(y_{t+1}|x^{*j}_{t+1})$ is large – and weights are small for particles which are inconsistent.

<u>Note:</u> this is like importance sampling. You sample from $p(X_{t+1}|Y_t)$ (like $g(.)$ in importance sampling.) when you really want to sample from $p(X_{t+1}|Y_{t+1})$

<u>Note:</u> you can extend particle filtering to cases where the distribution change with time → e.g. $p_t(X_{t+1}|X_t)$, $p_t(y_t|x_t)$.

Particle Filter / Bootstrap were developed in the past 10 years. They have had considerable success.

Limitation:

(a) They do not use the current available information y_{t+1} in the sampling step. (b) The use of resampling may cause inefficiency.

Some concern about how well they work in high dims.

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Bootstrap / Particle Filters are a special case of Sequential Monte Carlo.

(Chp. 3 of Jun Liu).

Justification for Claim that Particle Filters give samples from $p(X_{t+1} | Y_{t+1})$.

1. <u>Prediction</u>. $P(b,a) = P(b|a)P(a)$

samples $\{(b^i, a^i)\}$ from $p(b,a)$

sample a^i from $P(a)$

b^i from $P(b^i | a^i)$

Then $\{b^i\}$ are samples from $p(b)$

2. <u>Correction</u>. $p(a|b) = \frac{p(b|a)P(a)}{P(b)}$

sample $\{a^i\}$ from $p(a)$

accept each sample with prob $\propto p(b|a^i)$

gives new samples $\{a^{i*}\}$ from $P(a|b)$