

Lecture 26, Overview of Course

Note Title

6/8/2006

Overview

Part (1) Sampling basic (unstructured)
distributions $\pi(x)$
(Homework)

Main Methods:

Rejection Sampling.

Importance Sampling.

sample from $q(x)$
weight by $\frac{\pi(x)}{q(x)}$

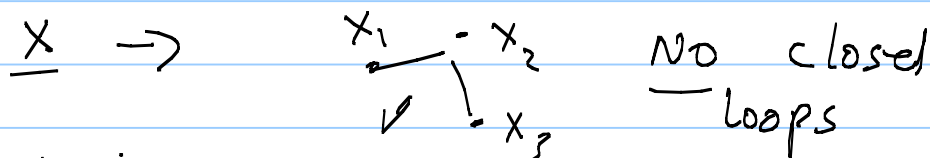
Both special cases of Weighted Importance Sampling
— eg. Rejection Control.

Variance of Sampler determines
sampling efficiency (want small variance)
If normalization factor of distribution is
unknown, then normalize by the weights.

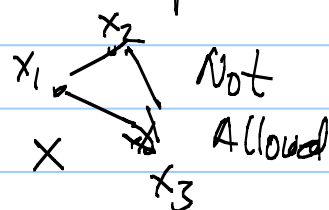
Rao-Blackwellization. $\rightarrow$ do as much as
possible analytically (to decrease variance)

Page 2.

(Part 2) Sampling with Structured Distributions
(Homework 2) where structure is a tree or polytree.



Exploit the linear structure of the graph — ordering of nodes.



Two types of algorithm:

- (1) Dynamic Programming
- (2) Bayes-Kalman Filtering (eg. particle filter)
- (3) Self Avoiding Walks.

particle filtering is a big success story of Monte Carlo sampling in the last 5-10 years

Before, people used Kalman filters (propagate the means and covariances) and tricks when the distributions were not Gaussian (Kalman only works for Kalman).

Page 3

Part 324
Homework 3
Homework 4

Structured Distributions of Any Distribution

MCMC

Repeated sampling from a transition kernel $K(x|y)$.

obey detailed balance

eg. $K(x|y)p(y) = K(y|x)p(x)$

$p(x)$ - target distribution

(Some people advocate transition kernels which do not obey detailed balance, but these are non-standard)

Best MCMC $\rightarrow$ Metropolis-Hastings

$K(x|y)$ divided into two steps

proposal $T(x|y)$ & acceptance

(Automatically satisfies detailed balance.) $\min \left\{ 1, \frac{p(x)T(y|x)}{p(y)T(x|y)} \right\}$

Variants of Metropolis-Hastings

Multiple-Try Metropolis-Hastings.

Hybrid Monte Carlo.

Simulated Annealing.

Page 4

Alternative MCMC

Gibbs Sampling. $P(x, y)$

Sample x from $P(x|y)$

y from $P(y|x)$

Obeys Detailed Balance.

(Gibbs can be thought of as a proposal distribution for Metropolis-Hastings where the proposals are always accepted - due to detailed balance)

Gibbs Sampling can be extended to

Data Augmentation — MCMC's alternative to the EM algorithm.

Swendsen-Wang.

Note: extensions of Swendsen-Wang involve Metropolis-Hastings.

Page 5.

More Alternatives

Genetic Algorithms — not an MCMC, but it sets the stage for

Evolutionary MCMC — a population of Markov-Chains that interact with each other (e.g. by competing for selection, or by cross-over).

Deterministic approximations

Belief Propagation and Mean Field Theory — Deterministic, and can get stuck in bad solutions.

But can work very fast and reliably. Need to be evaluated by computer simulations (very hard to prove useful theoretical results).