Lambertian model of reflectance I: shape from shading and photometric stereo

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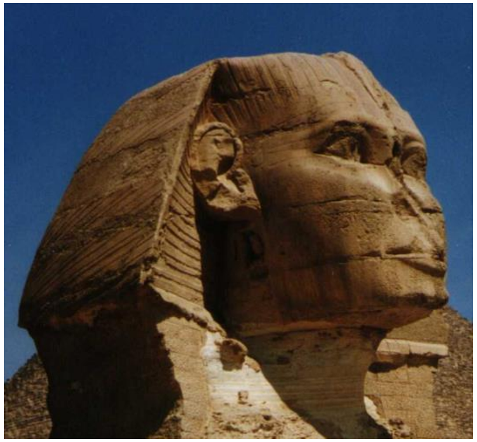
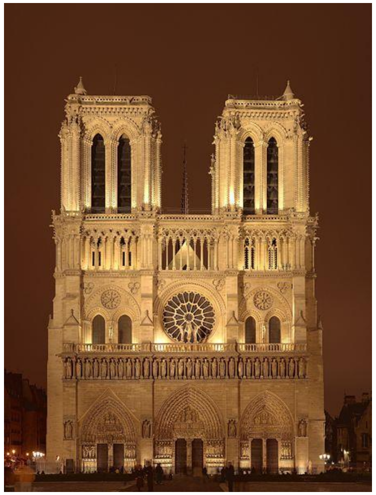
Variations due to lighting (and pose)

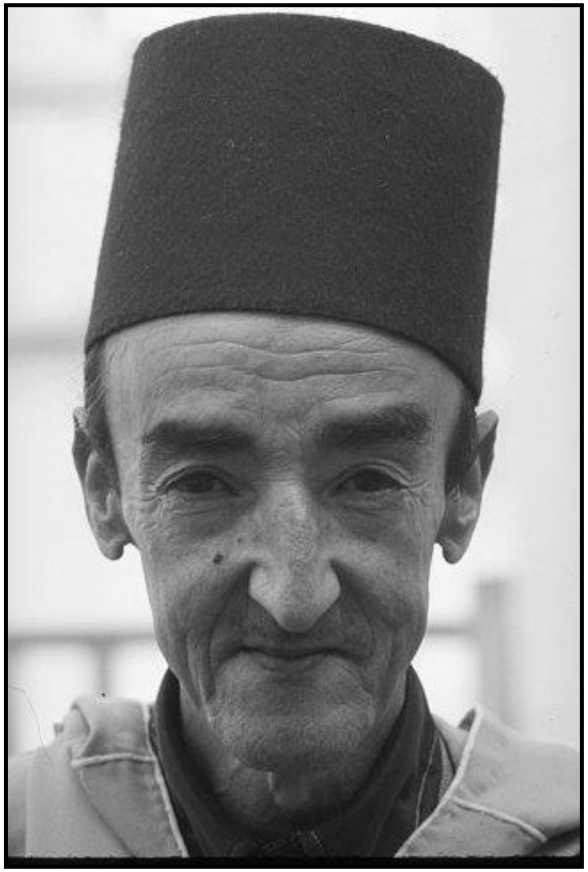



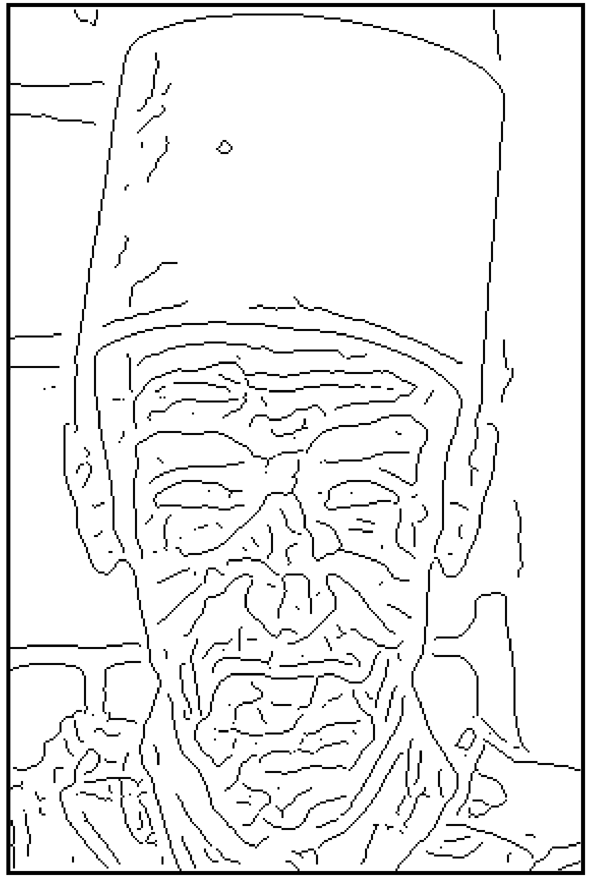


Dumitru Verdianu
Flying Pregnant Woman

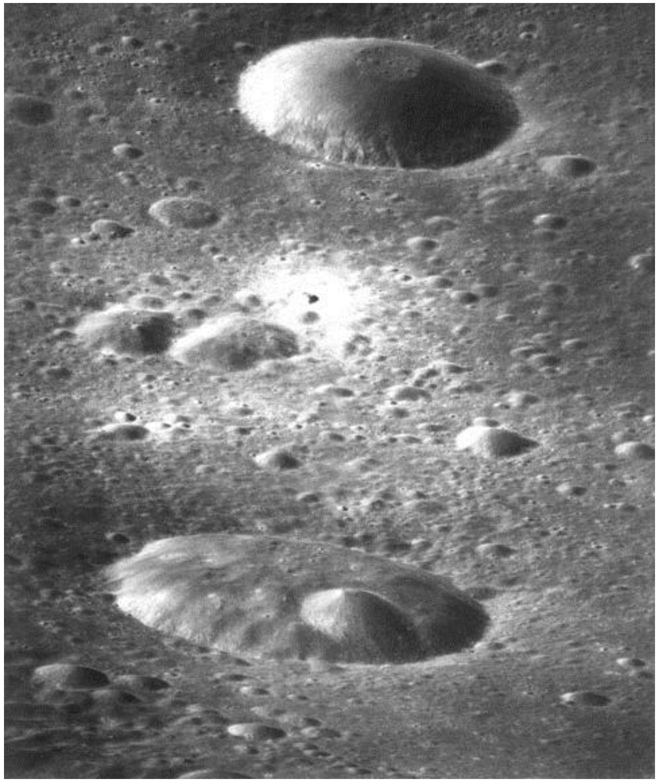
Edges

Edges on smooth surfaces

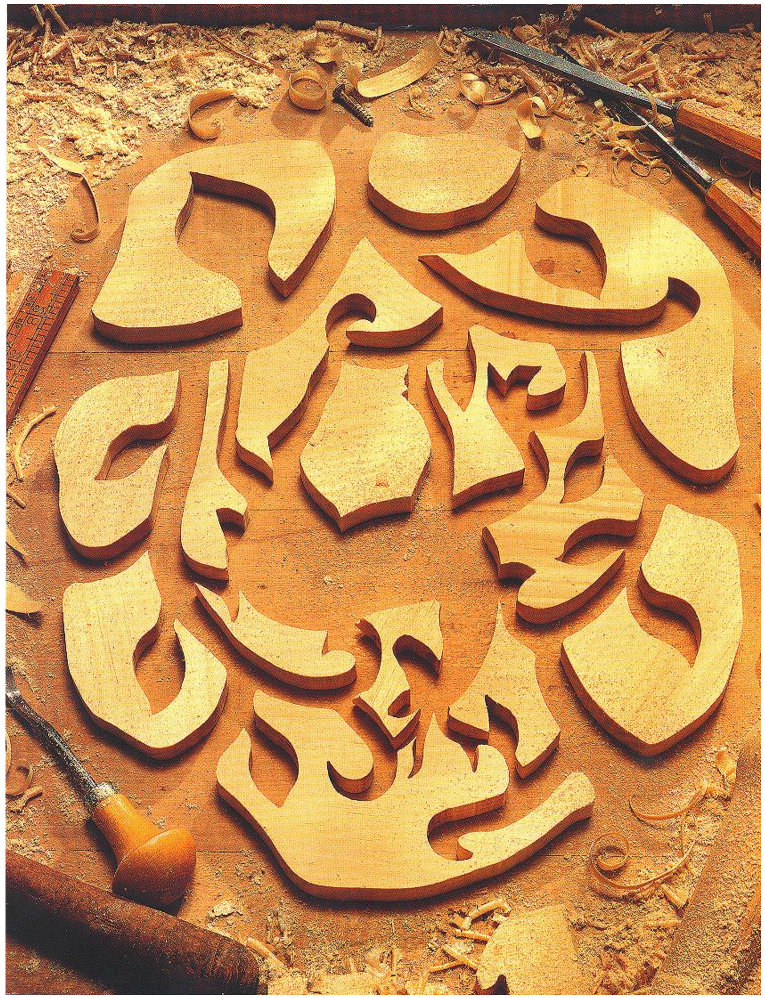




Bump or dip?



What sticks out?

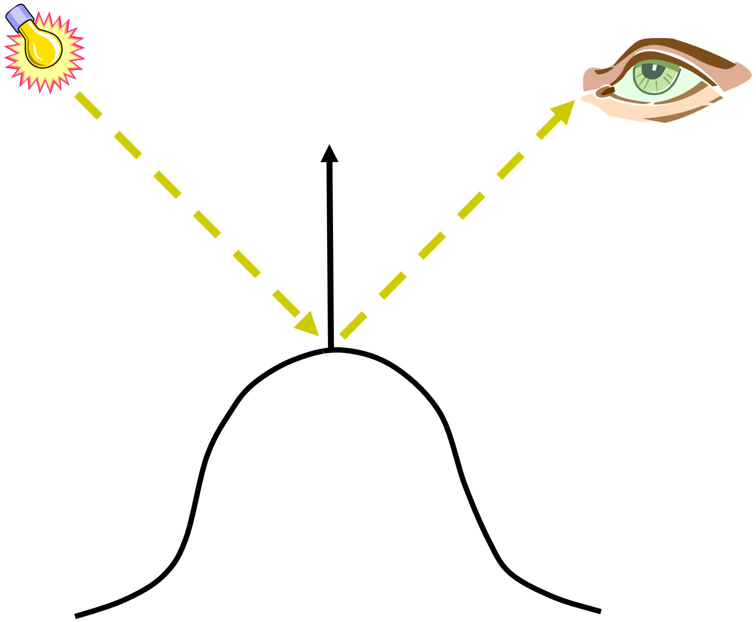


Light field

- Rays travel from sources to objects
- There they are either absorbed or reflected
- Energy of light decreases with distance and number of bounces
- Camera is designed to capture the set of rays that travel through the focal center
- We will assume the camera response is linear

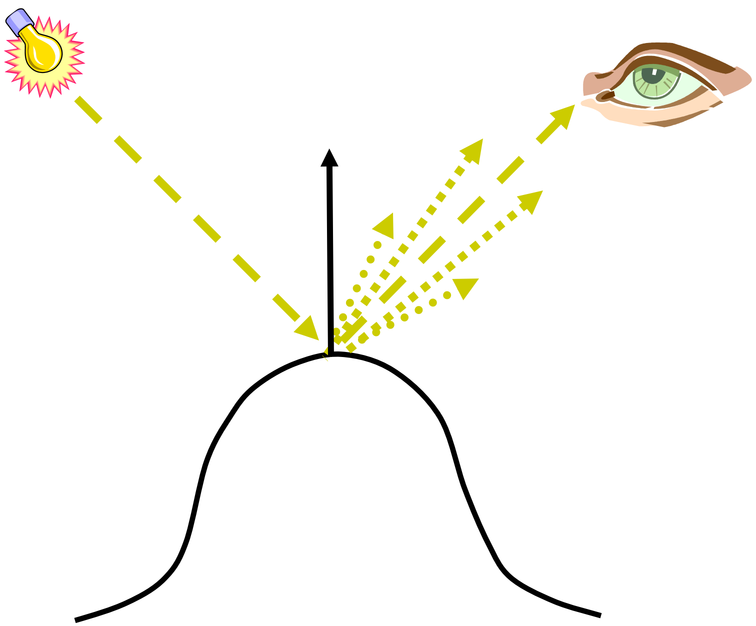
Specular reflectance (mirror)

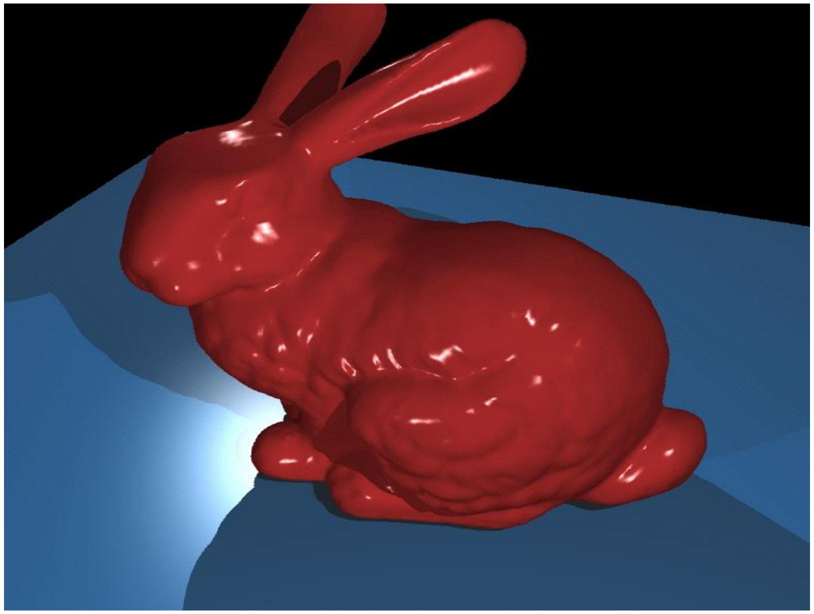
- When a surface is smooth light reflects in the opposite direction of the surface normal



Specular reflectance

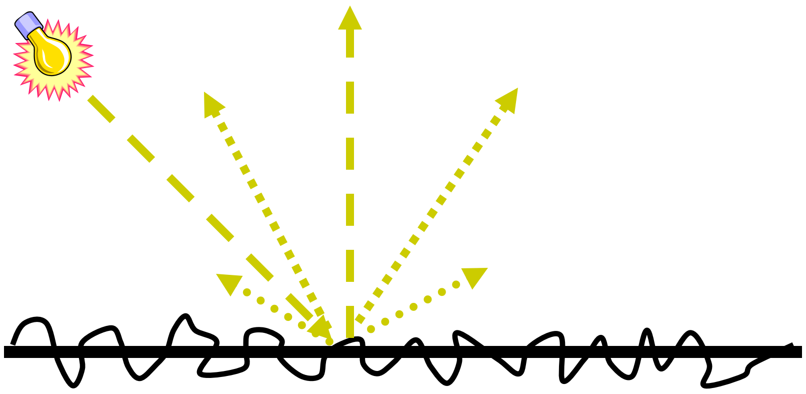
- When a surface is slightly rough the reflected light will fall off around the specular direction





Diffuse reflectance

- When the surface is very rough light may be reflected equally in all directions



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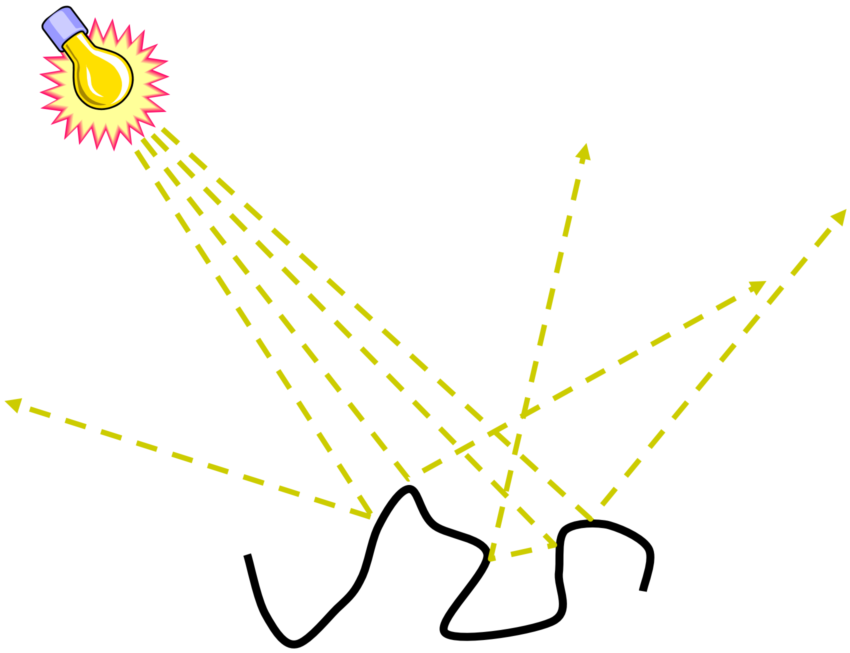
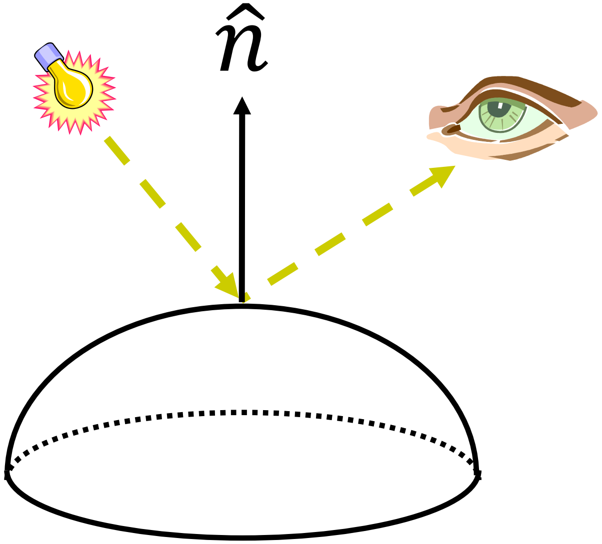


BRDF

- *Bidirectional Reflectance Distribution Function*

$$f(\theta_{in}, \emptyset_{in}; \theta_{out}, \emptyset_{out})$$

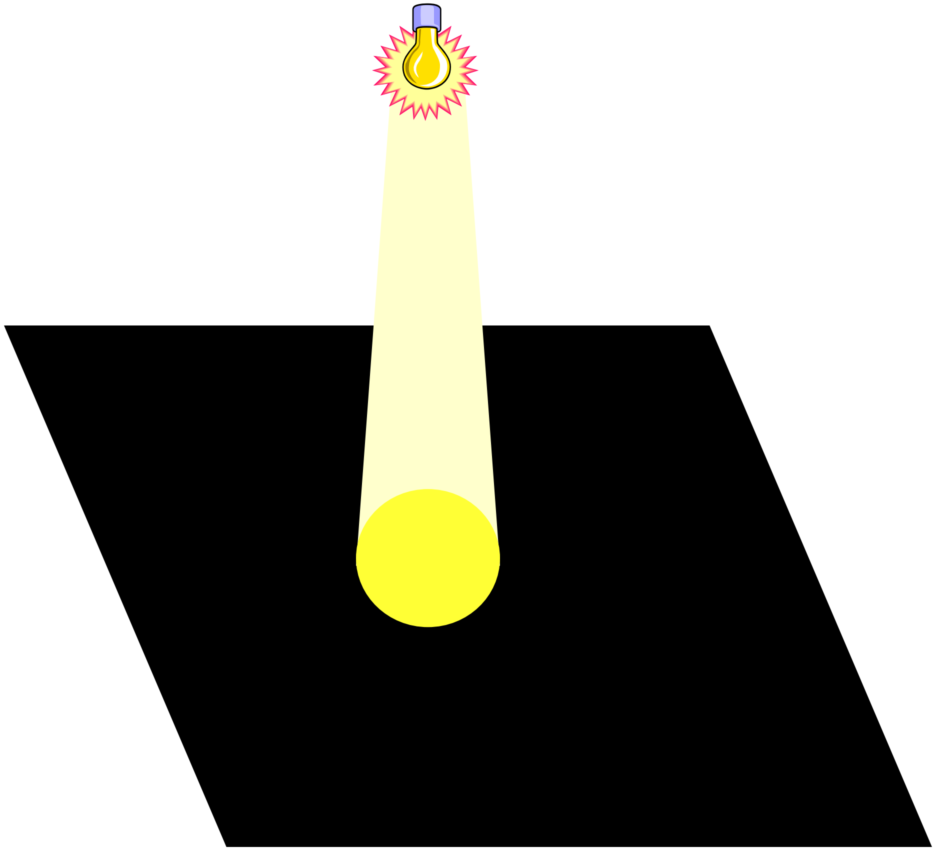
- Specifies for a unit of incoming light in a direction $(\theta_{in}, \emptyset_{in})$ how much light will be reflected in a direction $(\theta_{out}, \emptyset_{out})$



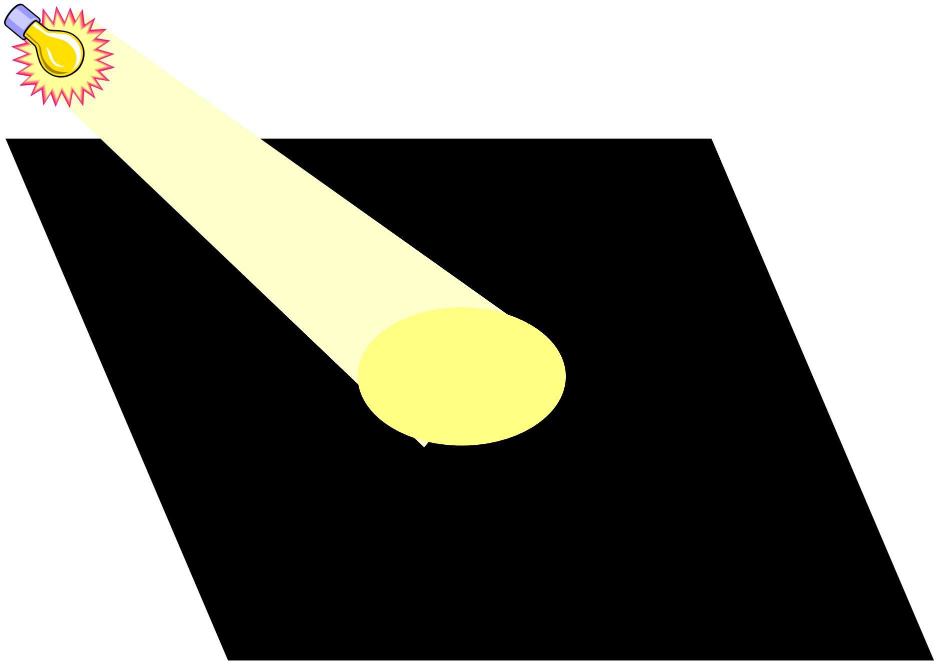
BRDF

Light from front   Light from back

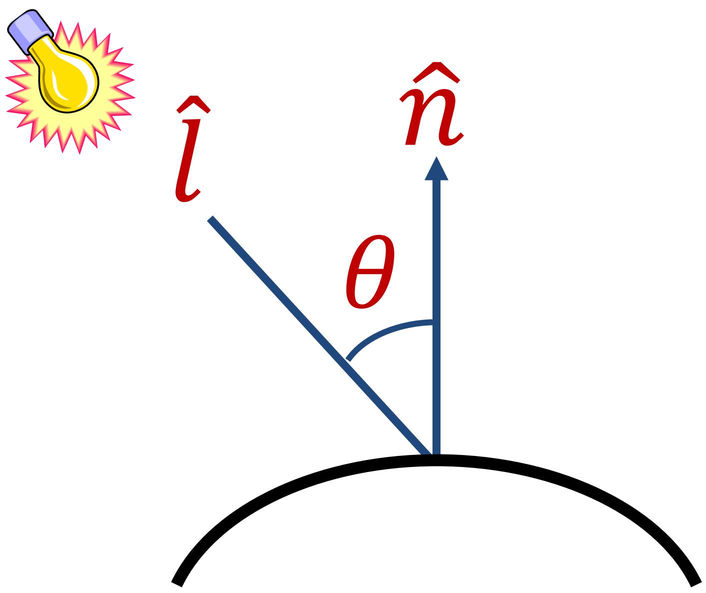
Lambertian reflectance



Lambertian reflectance



Lambert's law



$$I = E\rho\cos\theta \;\; (\theta \le 90^0)$$

$$I = \overrightarrow{l^T}\vec{n} \;\; (\vec{l} = E\hat{l}, \vec{n} = \rho\hat{n})$$

Preliminaries

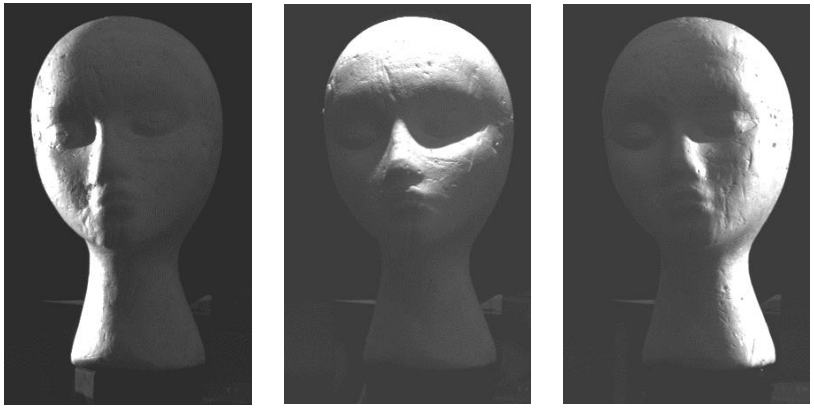
- A surface is denoted $z(x, y)$
- A point on z is $(x, y, z(x, y))$
- The tangent plane is spanned by

$$(1,0, z_x) \qquad (0,1, z_y)$$

- The surface normal is given by

$$\hat{n} = \frac{1}{\sqrt{{z_x}^2 + {z_y}^2 + 1}}(-z_x, -z_y, 1)$$

Photometric stereo



- Given several images of the a lambertian object under varying lighting
- Assuming single directional source

$$M = LS$$

Photometric stereo

$$M = LS$$

$$\begin{bmatrix} I_{11} & \cdots & I_{1p} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ I_{f1} & \cdots & I_{fp} \end{bmatrix}_{f\times p} = \begin{bmatrix} l_{1x} & l_{1y} & l_{1z} \\ & \vdots & \\ & \vdots & \\ l_{fx} & l_{fy} & l_{fz} \end{bmatrix}_{f\times 3} \begin{bmatrix} n_{x1} & & & n_{xp} \\ n_{y1} & \cdots & \cdots & n_{xp} \\ n_{z1} & & & n_{xp} \end{bmatrix}_{3\times p}$$

- We can solve for *S* if *L* is known (Woodham)
- This algorithm can be extended to more complex reflection models (if known) through the use of a lookup table

Factorization
(Hayakawa)

- Use SVD to find a rank 3 approximation

$$M = U\Sigma V^T$$

- Define $\Sigma_3 = diag(\sigma_1, \sigma_2, \sigma_3)$, where $\sigma_1, \sigma_2, \sigma_3$ are the largest singular values of M

$$\hat{L} = U\sqrt{\Sigma_3}, \quad \hat{S} = \sqrt{\Sigma_3}V^T \text{ and } M \approx \hat{L}\hat{S}$$

- Factorization is not unique, since

  $\widehat{M} = (\hat{L}A^{-1})(A\hat{S}),\ A$ is 3×3 invertible

- We can reduce ambiguity by imposing integrability

Generlized bas-relief ambiguity

(Belhumeur, Kriegman and Yuille)

- Linearly related surfaces: given a surface $z(x, y)$ the surfaces related linearly to z are:

$$\tilde{z}(x, y) = ax + by + cz(x, y)$$

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & c \end{bmatrix} \qquad G^{-1} = \frac{1}{c} \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ -a & -b & 1 \end{bmatrix}$$

- Forms a sub-group of GL(3)

Bas-relief

Integrability

- Objective: given a normal field $n(x,y) \in S^2$ recover depth $z(x,y) \in \mathbb{R}$

- Recall that

$$n = (n_x, n_y, n_z) = \frac{\rho}{\sqrt{{z_x}^2 + {z_y}^2 + 1}}(-z_x, -z_y, 1)$$

- Given n, we set

$$\rho = \|n\|\ ;\ p = -\frac{n_x}{n_z}, q = -\frac{n_y}{n_z}$$

Integrability

- Solve

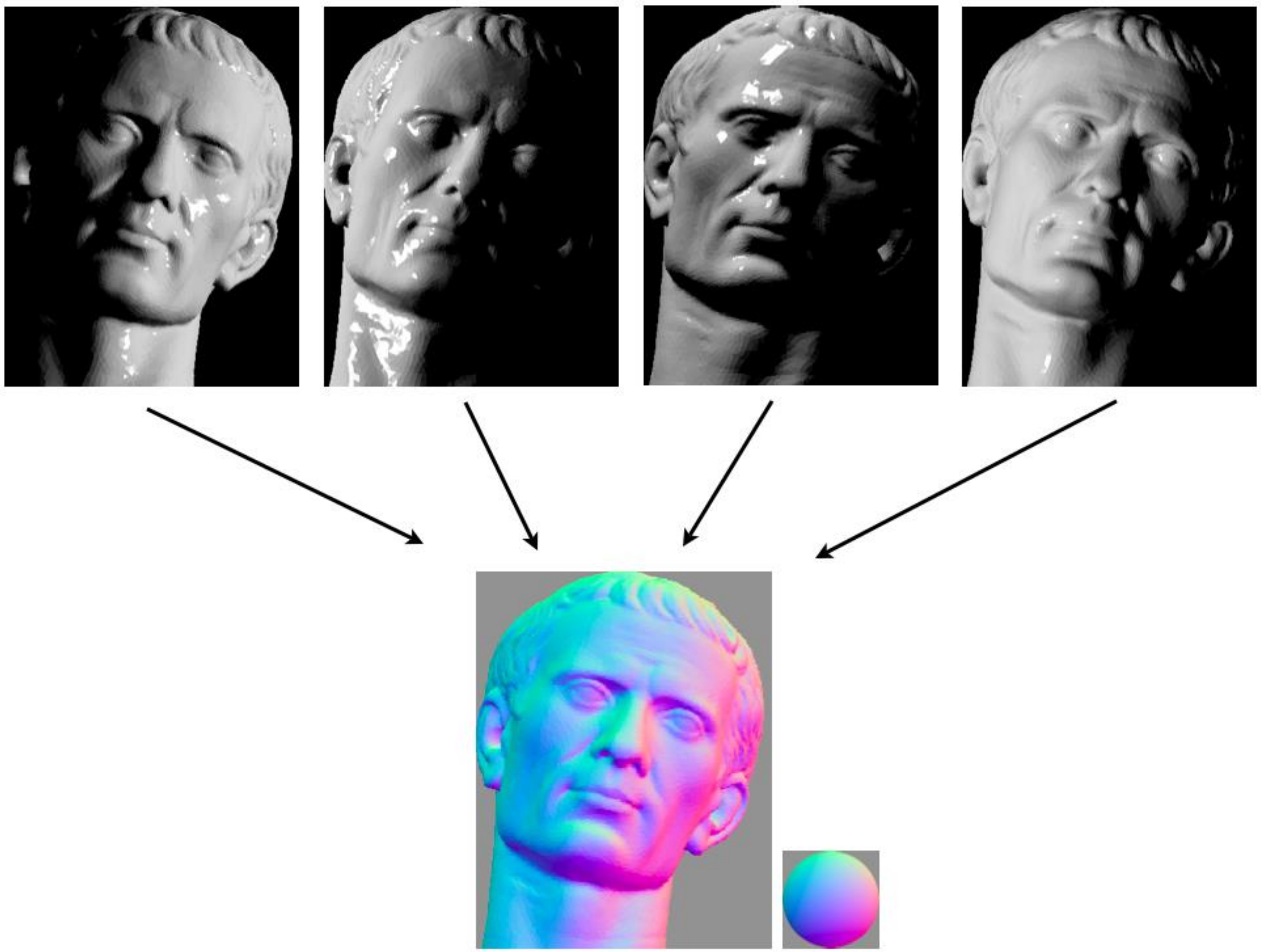
$$\min_{z(x,y)} (z(x+1,y) - z(x,y) - p)^2 + (z(x,y+1) - z(x,y) - q)^2$$

where

$$p = -\frac{n_x}{n_z}, q = -\frac{n_y}{n_z}$$

Photometric stereo with matrix completion

(Wu et al.)

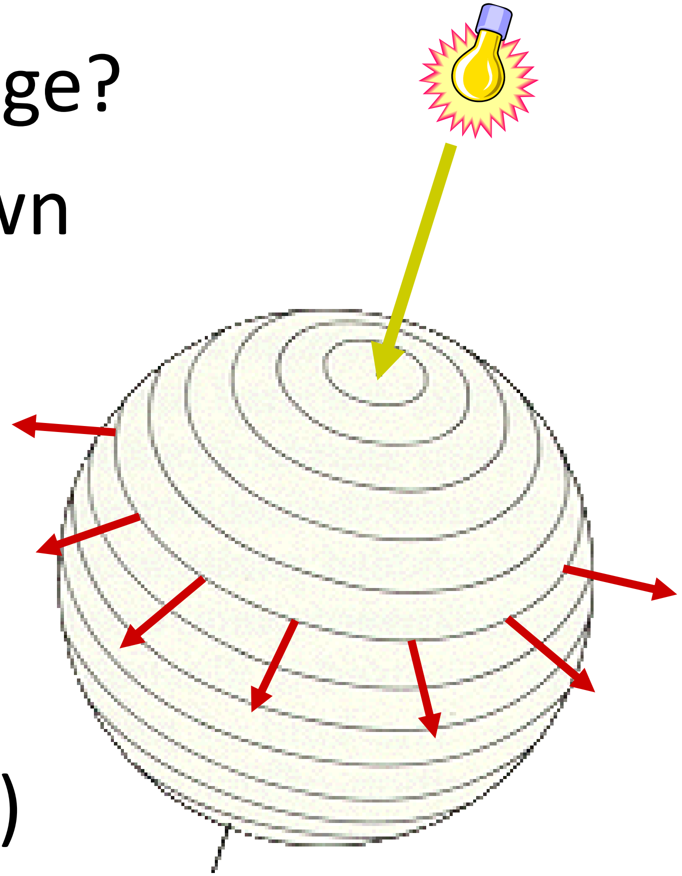


Shape from shading (SFS)

- What if we only have one image?
- Assuming that lighting is known and albedo is uniform

$$I = \overrightarrow{l^T}\vec{n} \propto \cos\theta$$

- Every intensity determines a circle of possible normals
- There is only one unknown (z) for each pixel (since uniform albedo is assumed)



Shape from shading

- Denote $n = (-z_x, -z_y, 1)$
- Then

$$E = \frac{l^T n}{\|n\|}$$

- Therefore

$$E^2 n^T n = n^T l l^T n$$

- We obtain

$$n^T (l l^T - E^2 I) n = 0$$

- This is a first order, non-linear PDE (Horn)

Shape from shading

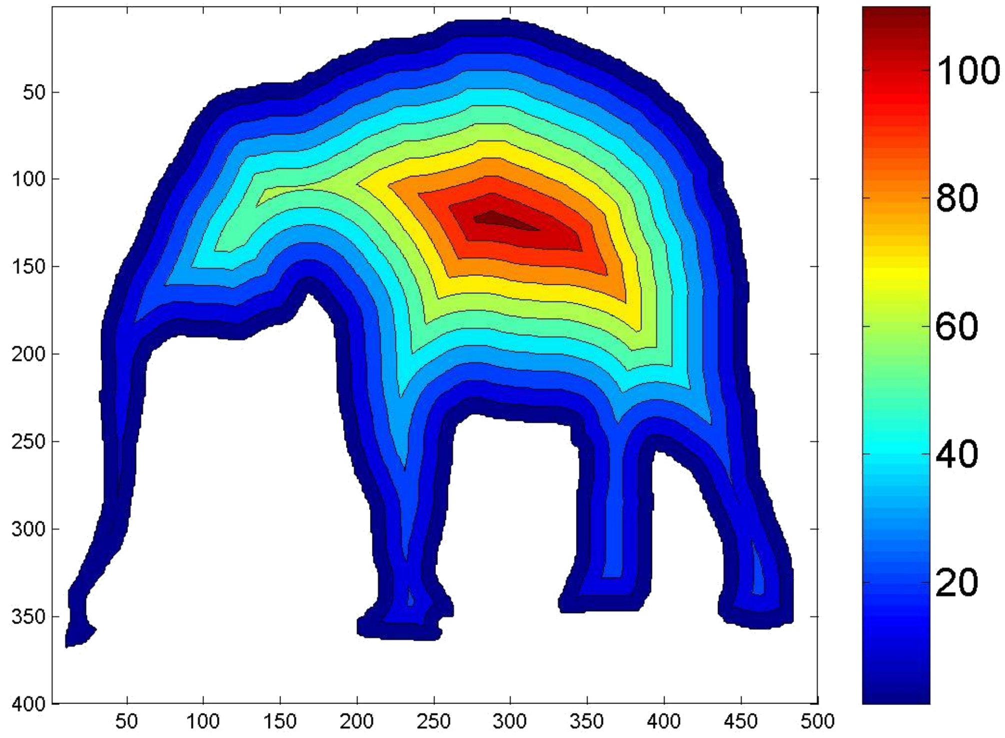
- Suppose $l = (0,0,1)$, then

$$z_x^{\ 2} + z_y^{\ 2} = \frac{1}{E^2} - 1$$

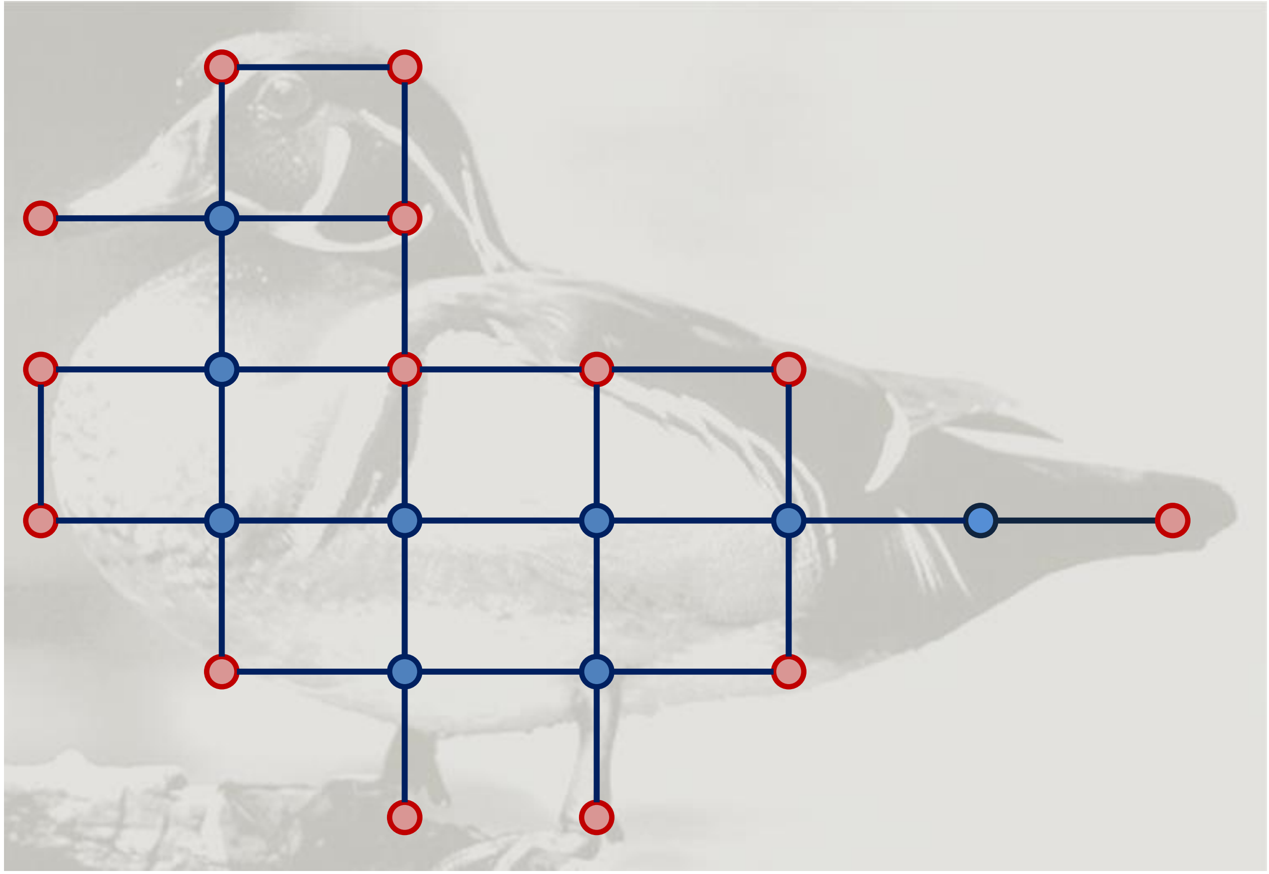
- This is called an *Eikonal* equation

Distance transform

- The distance of each point to the boundary



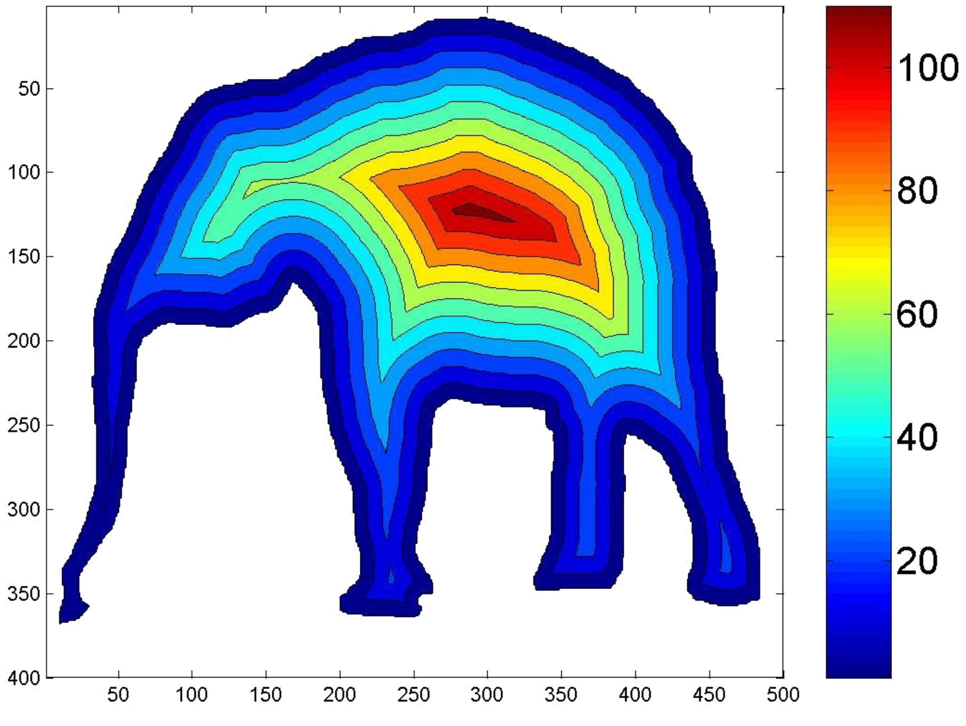
Shortest path



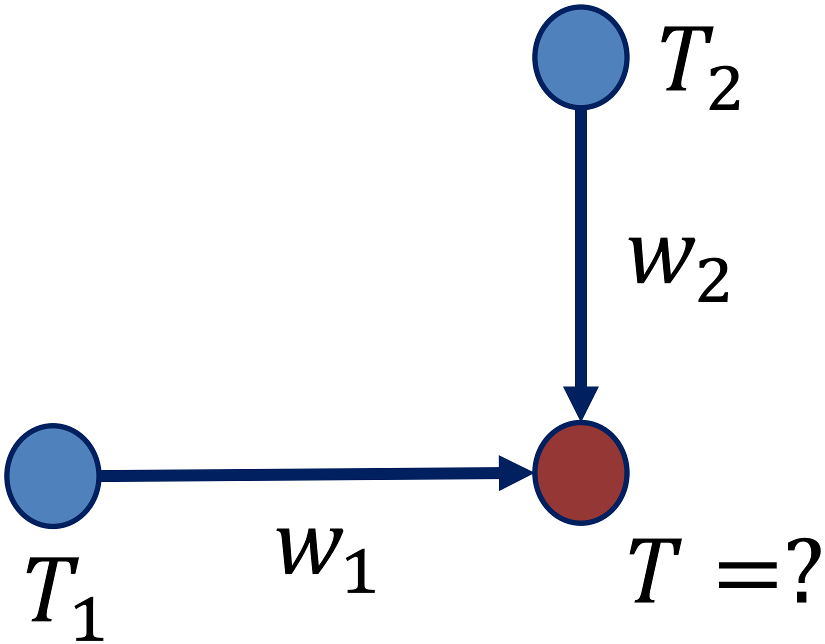
Distance transform

- Posed as an Eikonal equation

$$\|\nabla z\|^2 = {z_x}^2 + {z_y}^2 = 1$$



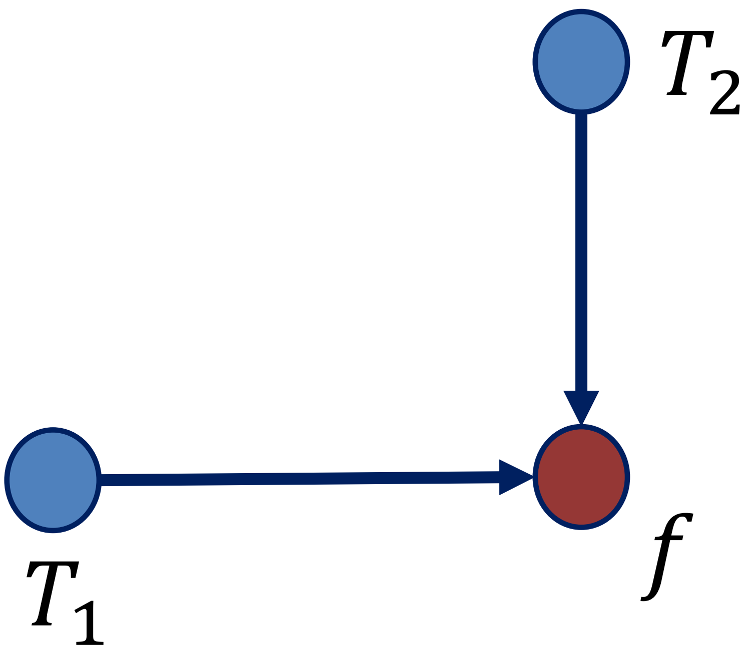
Update for shortest path



$$T = \min\{T_1 + w_1, T_2 + w_2\}$$

Update for fast marching

(Tsitsiklis, Sethian)



$$T = \begin{cases} T_1 + T_2 + \sqrt{2f^2 - (T_1 - T_2)^2} & \text{if real} \\ f + \min\{T_1, T_2\} & \text{otherwise} \end{cases}$$

SFS Solution

(Kimmel & Sethian)

- Lambertian SFS produces an eikonal equation

$$\|\nabla z\|^2 = {z_x}^2 + {z_y}^2 = \frac{1}{E^2} - 1$$

- Right hand side determines "speed"
- Boundary conditions are required: depth values at local maxima of intensity and possibly in shape boundaries
- The case $l \neq (0,0,1)$ is handled by change of variables

Example

Summary

- Understanding the effect of lighting on images is challenging, but can lead to better interpretation of images
- We considered Lambertian objects illuminated by single sources
- We surveyed two problems:
  - Photometric stereo
  - Shape from shading

Challenges

- General reflectance properties
  - Lambertian
  - specular
  - general BRDFs
- Generic lighting
  - multiple light sources ("attached shadows")
  - near light
- Cast shadows
- Inter-reflections
- Dynamic scenes
- Local approaches (eg. direction of gradients)

Can we hope to model this complexity?