

Authentication

CS 601.642/442 Modern Cryptography

Fall 2019

The Setting

- Alice wants to send a message m to Bob in such a manner that upon receipt, Bob can determine whether the message arrived untampered or not
- Want: Digital analogue of physical signatures
- Alice (“signer”) signs a message m to produce a signature σ
- Bob (“verifier”) can verify that σ is indeed generated for m
- Adversary cannot *forge* a signature

Two Types

- 1 Private Key: Message Authentication Codes
- 2 Public Key: Digital Signatures

Message Authentication Code (MAC)

- Signer and Verifier “share a secret”
- **Key Generation:** $\text{Gen}(1^n)$ outputs secret key k
- **Sign:** $\text{Tag}_k(m)$ outputs a tag σ
- **Verify:** $\text{Ver}_k(m, \sigma)$ is 1 if and only if σ is a valid tag of m under the secret key k

Security: An adversary can observe multiple (message,tag) pairs of its choice, but still cannot forge a tag on a new message

MAC: Algorithms

- $k \leftarrow \text{Gen}(1^n)$
- $\sigma \leftarrow \text{Tag}_k(m)$
- $\text{Ver}_k: \mathcal{M} \times \mathcal{T} \rightarrow \{0, 1\}$
- Correctness: $\Pr[k \leftarrow \text{Gen}(1^n), \sigma \leftarrow \text{Tag}_k(m): \text{Ver}_k(m, \sigma) = 1] = 1$
- Security (UF-CMA): For all n.u. PPT adversary \mathcal{A} there exists a negligible $\nu(\cdot)$ such that:

$$\Pr \left[\begin{array}{c} k \leftarrow \text{Gen}(1^n) \\ (m, \sigma) \leftarrow \mathcal{A}^{\text{Tag}_k(\cdot)}(1^n) \end{array} : \begin{array}{c} \mathcal{A} \text{ did not query } m \wedge \\ \text{Ver}_k(m, \sigma) = 1 \end{array} \right] \leq \nu(n)$$

Theorem

$PRF \implies MAC$

- $\text{Gen}(1^n)$: Output $k \xleftarrow{\$} \{0, 1\}^n$
- $\text{Tag}_k(m)$: Output $f_k(m)$
- $\text{Ver}_k(m, \sigma)$: Output $f_k(m) \stackrel{?}{=} \sigma$
- Think: Proof?

One-time MAC

- Weaker Security: Adversary is allowed only one query
- Advantage: Unconditional security!
- Analogue of OTP for authentication
- Think & Read

Digital Signature

- Only Signer can sign but everyone can verify
- **Key Generation:** $(sk, pk) \leftarrow \text{Gen}(1^n)$
- **Sign:** $\sigma \leftarrow \text{Sign}_{sk}(m)$
- **Verify:** $\text{Ver}_{pk}(m, \sigma): \mathcal{M} \times \mathcal{S} \rightarrow \{0, 1\}$
- Correctness:

$$\Pr[(sk, pk) \leftarrow \text{Gen}(1^n), \sigma \leftarrow \text{Sign}_{sk}(m) : \text{Ver}_{pk}(m, \sigma) = 1] = 1$$

- Security (UF-CMA):

$$\Pr \left[\begin{array}{l} (sk, pk) \leftarrow \text{Gen}(1^n) \\ (m, \sigma) \leftarrow \mathcal{A}^{\text{Sign}_{sk}(\cdot)}(1^n, pk) \end{array} : \begin{array}{l} \mathcal{A} \text{ did not query } m \wedge \\ \text{Ver}_{pk}(m, \sigma) = 1 \end{array} \right] \leq \nu(n)$$

- One-time Signatures: Adversary is allowed only one query

One-time Signature: Construction [Lamport]

Let f be a one-way function

- $sk := \begin{pmatrix} x_1^0 & x_2^0 & \dots & x_n^0 \\ x_1^1 & x_2^1 & \dots & x_n^1 \end{pmatrix}$, where $x_i^b \xleftarrow{\$} \{0, 1\}^n$ for all $i \in [n]$ and $b \in \{0, 1\}$
- $pk := \begin{pmatrix} y_1^0 & y_2^0 & \dots & y_n^0 \\ y_1^1 & y_2^1 & \dots & y_n^1 \end{pmatrix}$, where $y_i^b = f(x_i^b)$ for all $i \in [n]$ and $b \in \{0, 1\}$
- $\text{Sign}_{sk}(m) : \sigma := (x_1^{m_1}, x_2^{m_2}, \dots, x_n^{m_n})$
- $\text{Ver}_{pk}(m, \sigma) : \bigwedge_{i \in [n]} f(\sigma_i) \stackrel{?}{=} y_i^{m_i}$
- Think: Proof?

Think: How to sign long messages?

Collision-resistant Hash Functions

- Intuition: A compressing function h for which it is hard to find x, x' s.t. $x \neq x'$ but $h(x) = h(x')$
- Impossible for non-uniform adversary notion
 - Think: Why?
- Need to consider a family of hash functions

Collision-resistant Hash Function Family

Definition (Collision-resistant Hash Function Family)

A family of functions $H = \{h_i : D_i \rightarrow R_i\}_{i \in I}$ is a collision-resistant hash function family (CRHF) if:

- **Easy to Sample:** There exists a PPT Gen s.t.: $i \leftarrow \text{Gen}(1^n)$, $i \in I$
- **Compression:** $|R_i| < |D_i|$
- **Easy to Evaluate:** There exists a poly-time algorithm Eval s.t. given $x \in D_i$, $i \in I$, $\text{Eval}(x, i) = h_i(x)$
- **Collision Resistance:** For all n.u. PPT \mathcal{A} , \exists negligible function $\mu(\cdot)$ s.t.

$$\Pr \left[\begin{array}{l} i \stackrel{\$}{\leftarrow} \text{Gen}(1^n), \\ (x, x') \leftarrow \mathcal{A}(1^n, i) \end{array} : x \neq x' \wedge h_i(x) = h_i(x') \right] \leq \mu(n)$$

- One-bit compression implies arbitrary bit compression
 - Think: Proof?
 - Read: Merkle Trees
- Range cannot be too small
 - Enumeration Attacks
 - Birthday Attack
- **Existence**:
 - Unlikely to be constructed from OWF or OWP [Simon98]
 - Can be constructed from number-theoretic assumptions such as factoring, discrete log

Remarks (contd.)

- **Weaker notion:** Universal One-way Hash Functions (UOWHF)

$$\Pr \left[\begin{array}{l} (x, \text{state}) \leftarrow \mathcal{A}(1^n), \\ i \xleftarrow{\$} \text{Gen}(1^n), \\ x' \leftarrow \mathcal{A}(i, \text{state}) \end{array} : \begin{array}{l} x \neq x' \wedge \\ h_i(x) = h_i(x') \end{array} \right] \leq \mu(n)$$

- Can be constructed from OWF [Rompel90]
- Suffices for Digital Signatures [Naor-Yung89]
- More efficient construction
[Haitner-Holenstein-Reingold-Vadhan-Wee10]

One-time Signatures for Long Messages

- Let $H = \{h_i : \{0, 1\}^* \rightarrow \{0, 1\}^n\}_{i \in I}$ be a CRHF
- Idea: Sign $h_i(m)$ instead of m using Lamport signature
- Think: Proof?

Multi-message Signatures (via chain)

- $(sk_0, pk_0) \xleftarrow{\$} \text{Gen}(1^n)$
- Initialize: $\tilde{\sigma}_i = \emptyset, i = 1$
- To sign m_i :
 - $(sk_i, pk_i) \xleftarrow{\$} \text{Gen}(1^n)$
 - $\tilde{\sigma}_i \leftarrow \text{Sign}_{sk_{i-1}}(m_i \| pk_i)$
 - Output: $\sigma_i = (i, \tilde{\sigma}_i, m_i, pk_i, \sigma_{i-1})$
 - Increment i
- Think: Proof?
- Think: How to reduce signature size?
- Read: Tree-based signatures
- Read: Efficient Signatures from Trapdoor Permutations in the Random Oracle Model
- Read: Full Domain Hash (RSA-based signatures)