Territory Partitioning for Minimalist Gossiping Robots

Francesco Bullo

Center for Control,
Dynamical Systems & Computation
University of California at Santa Barbara
`http://motion.mee.ucsb.edu`

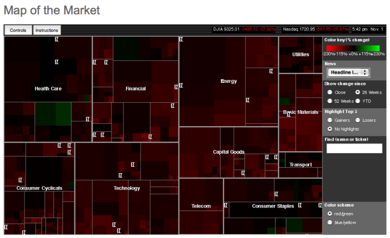
Johns Hopkins University
Baltimore, Nov 4, 2008

Collaborators: Paolo Frasca, Ruggero Carli

Territory partitioning is ... art

Ocean Park Paintings, by Richard Diebenkorn (1922-1993)

Territory partitioning is ... visualization



Map of the Market

DJIA 9325.01 -3495.12 -27.26% Nasdaq 1720.95 -691.85 -28.67% 5:42 pm Nov. 1

Color key (% change)
-230% -115% +0% +115% +230%

News
Headline I...

Show change since
○ Close ● 26 Weeks
○ 52 Weeks ○ YTD

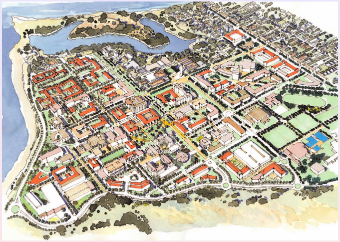
Highlight Top 5
○ Gainers ○ Losers
● No highlights

Find (name or ticker)

Color scheme
● red/green
○ blue/yellow

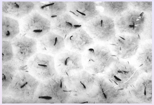
MarketMap applet by SmartMoney.com, Nov 1, 2008

Territory partitioning is ... centralized space allocation

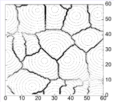


UCSB Campus Development Plan, 2008

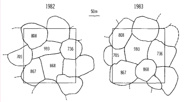
Territory partitioning is ... animal territory dynamics



Tilapia mossambica, "Hexagonal Territories," Barlow et al, '74



Red harvester ants, "Optimization, Conflict, and Nonoverlapping Foraging Ranges," Adler et al, '03

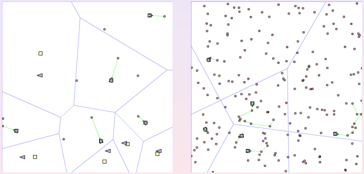


Sage sparrows, "Territory dynamics in a sage sparrows population," Petersen et al '87

Territory partitioning is ... robotic load balancing

Dynamic Vehicle Routing

- customers appear
- network provides service

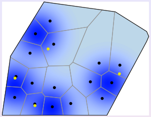


E. Frazzoli and F. Bullo. Decentralized algorithms for vehicle routing in a stochastic time-varying environment. In *Proc CDC*, pages 3357–3363, Paradise Island, Bahamas, December 2004

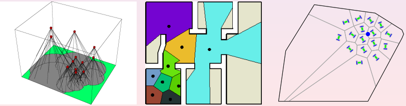
Distributed partitioning+centering algorithm

Partitioning+centering law

```
At each comm round:
 1: acquire neighbors' positions
 2: compute own dominance region
 3: move towards centroid of own
    dominance region
```



J. Cortés, S. Martínez, T. Karatas, and F. Bullo. Coverage control for mobile sensing networks. *IEEE Trans Robotics & Automation*, 20(2):243–255, 2004



Multi-center optimization

- take environment with density function $\phi : Q \to \mathbb{R}_{\geq 0}$
- place N robots at $p = \{p_1, \ldots, p_N\}$
- partition environment into $v = \{v_1, \ldots, v_N\}$
- define expected quadratic deviation

$$H(v,p) = \sum_{i=1}^{N} \int_{v_i} \|q - p_i\|^2 \phi(q) dq$$

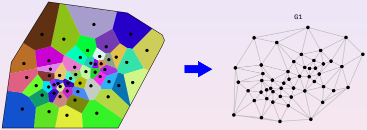
Theorem (Lloyd '57 "least-square quantization")

1. *at fixed partition, optimal positions are centroids*
2. *at fixed positions, optimal partition is Voronoi*
3. *Lloyd algorithm: alternate p-v optimization*

Today: partitioning with asynchronous peer-to-peer

Partitioning+centering law requires:

1. synchronous communication
2. communication along edges of dual graph



Minimalist robotics

- is synchrony necessary?
- is it sufficient to communicate peer-to-peer (gossip)?
- what are minimal requirements?

From standard to gossip algorithm

Standard partitioning+centering algorithm

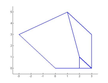
1. robot talks to all its neighbors in dual graph
2. robot computes its Voronoi region
3. robot moves to centroid of its Voronoi region

Gossip partitioning policy

1. robot/region talks to only one neighboring robot/region
2. two regions are updated according to

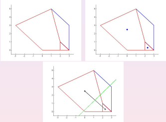
$$v_i^+ := \{q \in v_i \cup v_j \mid \quad \|q - \text{centroid}(v_i)\| \le \|q - \text{centroid}(v_j)\|\}$$

Gossip partitioning policy

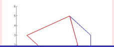


before meeting

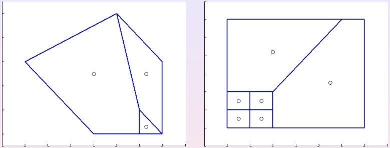
1. Randomly chose two neighboring regions
2. Compute two centroids
3. Compute bisector of centroids
4. Partition two regions by bisector



after meeting

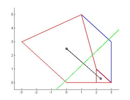


Simulations



Implementation: centralized, General Polygon Clipper (GPC) library

Technical challenges



1. state space is not finite-dimensional
   - non-convex disconnected polygons
   - arbitrary number of vertices
2. gossip map is not deterministic, ill-defined and discontinuous
   - two regions could have same centroid
   - disconnected/connected discontinuity
3. Lyapunov function missing

4. **motion protocol for deterministic/random meetings**

From standard to Lyapunov functions for partitions

Standard coverage control

robot i moves towards centroid of its Voronoi region

$$H(p_1, \dots, p_N) = \sum_{i=1}^{N} \int_{V_i(p_1,\dots,p_N)} \|p_i - x\|^2 \phi(q) dq$$

Gossip coverage control

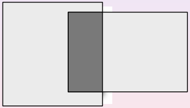
region v_i is modified to appear like a Voronoi region

$$H(v_1, \dots, v_N) = \sum_{i=1}^{N} \int_{v_i} \| \text{centroid}(v_i) - x\|^2 \phi(q) dq$$

Symmetric difference

Given sets A, B, symmetric difference and distance are:

$$A \Delta B = (A \cup B) \setminus (A \cap B), \qquad d_\Delta(A, B) = \text{measure}(A \Delta B)$$



The space of partitions

Definition (space of N-partitions)

$\mathcal{V}_N$ is collections of N subsets of Q, $v = \{v_i\}_{i=1}^N$, such that

1. $v_i \neq \emptyset$ and $v_i = \overline{\text{interior}(v_i)}$
2. $\text{interior}(v_i) \cap \text{interior}(v_j) = \emptyset$ if $i \neq j$, and
3. $\bigcup_{i=1}^N v_i = Q$

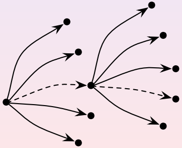
Theorem (space of partitions is metric and compact)

$\mathcal{V}_N$ with metric $d_\Delta(u, v) = \sum_{i=1}^N d_\Delta(u_i, v_i)$ is compact metric

LaSalle invariance principle: persistent switches

- X is metric space
- set-valued $T : X \rightrightarrows X$ with $T(x) = \{T_i(x)\}_{i \in I}$ for finite I
- consider sequences $\{x_n\}_{n \geq 0} \subset X$ with

$$x_{n+1} \in T(x_n)$$



LaSalle invariance principle: persistent switches

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- consider sequences $\{x_n\}_{n \geq 0} \subset X$ with

$$x_{n+1} \in T(x_n)$$

Assume:

1. $W \subset X$ compact and positively invariant for T
2. $U : W \to \mathbb{R}$ is non-decreasing along T
3. U and T_i are continuous on W
4. for all $i \in I$, there are infinite $m \in \mathbb{N}$ such that $x_{m+1} = T_i(x_m)$

Then

$x_n \to$ largest T-invariant subset of

$$\{x \in W \mid \forall y \in T(x), \;\; U(y) = U(x)\}$$

LaSalle invariance principle: random switches

- X is metric space
- set-valued $T : X \rightrightarrows X$ with $T(x) = \{T_i(x)\}_{i \in I}$ for finite I
- consider sequences $\{x_n\}_{n \geq 0} \subset X$ with

$$x_{n+1} \in T(x_n)$$

Assume:

1. $W \subset X$ compact and positively invariant for T
2. $U : W \to \mathbb{R}$ is non-decreasing along T
3. U and T_i are continuous on W
4. random sequences with $x_{n+1} = T_i(x_n)$ with probability p_i

Then almost surely

$x_n \to$ largest T-invariant subset of

$$\{x \in W \mid \forall y \in T(x), \;\; U(y) = U(x)\}$$

Conclusions

Summary

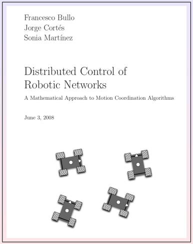
1. novel gossip partitioning algorithm
2. space of partitions
3. LaSalle invariance principles
4. convergence to centroidal Voronoi partition

P. Frasca, R. Carli, and F. Bullo. Gossip coverage control: dynamical systems on the space of partitions. In *Proc ACC*, St. Louis, MO, June 2009. Submitted

Ongoing work

1. motion laws to maximize peer-to-peer meeting frequencies
2. convergence rates: known in 1D; unknown in 2D
3. robots arriving/departing
4. more general version of partitioning:
   nonsmooth, equitable, nonconvex, 3D

“Distributed Control of Robotic Networks”


Francesco Bullo
Jorge Cortés
Sonia Martínez

Distributed Control of
Robotic Networks
A Mathematical Approach to Motion Coordination Algorithms

June 3, 2008

1. intro to distributed algorithms (graph theory, synchronous networks, and averaging algos)
2. geometric models and geometric optimization problems
3. model for robotic, relative sensing networks, and complexity
4. algorithms for rendezvous, deployment, boundary estimation

Status: Freely downloadable at `http://coordinationbook.info` with tutorial slides and (ongoing) software libraries. To appear, Princeton University Press.

Emerging discipline: robotic networks

- **network modeling**
  network, ctrl+comm algorithm, task, complexity
- **coordination algorithm**
  deployment, task allocation, boundary estimation

Open problems

1. algorithmic design for minimalist robotic networks
   scalable, adaptive, asynchronous, agent arrival/departure
   tasks: search, exploration, identify and track
2. integrated coordination, communication, and estimation
3. Very few results available on:
   1. scalability analysis: time/energy/communication/control
   2. robotic networks over random geometric graphs
   3. complex sensing/actuation scenarios