# Homework Assignment 3 – 600.455/655 Fall 2020 (Circle One)

# Instructions and Score Sheet (hand in with answers)

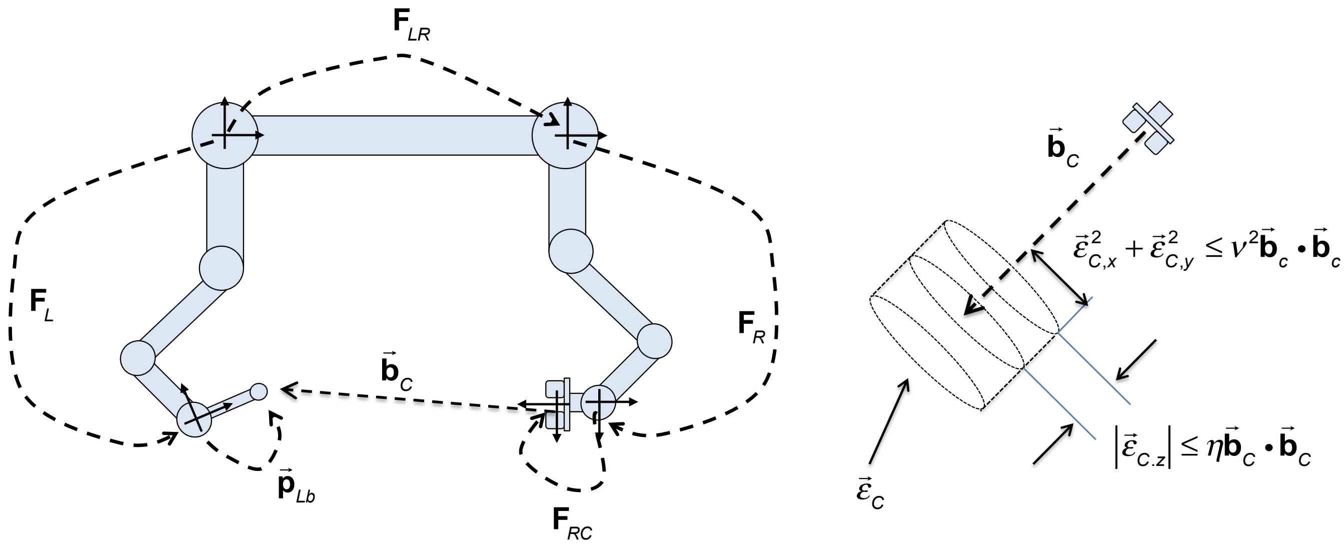
| | |
|---|---|
| Name | Name |
| Email | Email |
| Other contact information (optional) | Other contact information (optional) |
| Signature (required)<br>I/We have followed the rules in completing this assignment<br><br>____________________________ | Signature (required)<br>I/We have followed the rules in completing this assignment<br><br>____________________________ |

| Question | Points (455,655) | Totals |
|---|---|---|
| 1A | 2 | |
| 1B | 5 | |
| 1C | 10 | |
| 1D | 10 | |
| 1E | 10 | |
| 1F | 10 | |
| 1G | 15 | 67 (50 max) |
| 2A | 5 | |
| 2B | 10 | |
| 2C | 10 | |
| 2D | 10 | |
| 2E | 10 | |
| 2F | 10 | 55 (50 max) |
| Total | | 122 (100 max) |

**Note: Students may receive at most 50 points on Question 1 and 50 points on Question 2**

1. Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.
2. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
3. It is otherwise open book, notes, and web. But you should cite any references you consult.
4. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
5. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
6. Sign and hand in the score sheet as the first sheet of your assignment.
7. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment

# Question 1




Consider the two-armed robot shown above. The left arm holds a cutting tool with a ball cutter at the end. The right arm holds a stereo video camera. The displacement $\mathbf{F}_{LR}$ of the base of the right arm is fixed and known only approximately, so that the true value is $\mathbf{F}^*_{LR}=\mathbf{F}_{LR}\Delta\mathbf{F}_{LR}$. The two robot arms move in space, but they may have some error, so that at time $t$, the true value of the position of the arms relative to its base are $\mathbf{F}^*_L[t]=\Delta\mathbf{F}_L[t]\mathbf{F}_L[t]$ and $\mathbf{F}^*_R[t]=\Delta\mathbf{F}_R[t]\mathbf{F}_R[t]$, respectively. The errors are small enough so that they can be approximated:

$$\Delta\mathbf{F}_L[t]\approx[\mathbf{I}+sk(\vec{\alpha}_L[t]),\vec{\varepsilon}_L[t]]$$
$$\Delta\mathbf{F}_R[t]\approx[\mathbf{I}+sk(\vec{\alpha}_R[t]),\vec{\varepsilon}_R[t]]$$
$$\Delta\mathbf{F}_{LR}\approx[\mathbf{I}+sk(\vec{\alpha}_{LR}),\vec{\varepsilon}_{LR}]$$

Where the context is obvious, we may omit the [$t$]. The displacement $\vec{\mathbf{p}}_{Lb}$ of the ball cutter relative to the tooling attachment plate of the left arm is fixed but unknown. Similarly, the pose $\mathbf{F}_{RC}$ of the camera system relative to the tooling attachment plate of the right arm is fixed but unknown.

The system has image processing software that is capable of locating the position $\vec{\mathbf{b}}[t]$ of the ball relative to the camera system so long as $\left\|\vec{\mathbf{b}}_c\right\|\ge 2f$ and $\mathrm{b}^2_{C,x}+\mathrm{b}^2_{C,y}\le\mathrm{b}^2_{C,z}$. Again, there is some error, so that the true position is given by $\vec{\mathbf{b}}^*_C[t]=\vec{\mathbf{b}}_C[t]+\vec{\varepsilon}_{Cb}[t]$. We know constraints on the accuracy of this measurement: $\vec{\varepsilon}^2_{C,x}+\vec{\varepsilon}^2_{C,y}\le\nu^2\vec{\mathbf{b}}_c\bullet\vec{\mathbf{b}}_c$ and $\left|\vec{\varepsilon}_{C.z}\right|\le\eta\vec{\mathbf{b}}_C\bullet\vec{\mathbf{b}}_C$. Note that can approximate the first constraint by a set of linear inequalities of the form: $\mathbf{d}_i\bullet\vec{\mathbf{b}}_c\le\nu\left\|\vec{\mathbf{b}}_c\right\|_2$ where $\vec{\mathbf{d}}_i=\left[\cos\theta_i,\sin\theta_i,0\right]^T$.

A. Give expressions for the position $\vec{\mathbf{b}}_L[t]$ of the ball and the pose $\vec{\mathbf{F}}_{LC}[t]$ of the camera relative to the base of the left arm, assuming nominal values.

B. Give an expression for the expected value $\vec{\mathbf{b}}_{C,ex}[t]$ of $\vec{\mathbf{b}}_{C}[t]$ given the nominal values of the expressions that you produced in Question 1.A.

C. The actual value $\vec{\mathbf{b}}^*_{C,ex} = \vec{\mathbf{b}}_{C,ex} + \vec{\varepsilon}_{C,ex}$ will have some error. Produce an expression estimating the value of $\vec{\varepsilon}_{C,ex}$ based on the various uncertainty parameters given in the Question 1 scenario. In this case you may also assume that you have estimated values for $\vec{\mathbf{p}}^*_{Lb} = \vec{\mathbf{p}}_{Lb} + \vec{\varepsilon}_{Lb}$ and $\mathbf{F}^*_{RC} = \mathbf{F}_{RC}\Delta\mathbf{F}_{RC}$, where $\Delta\mathbf{F}_{RC} \approx [\mathbf{I} + sk(\vec{\alpha}_{RC}), \vec{\varepsilon}_{RC}]$. Express your answer as a standard linearized expression in terms of the $\vec{\alpha}$ and $\vec{\varepsilon}$ variables.

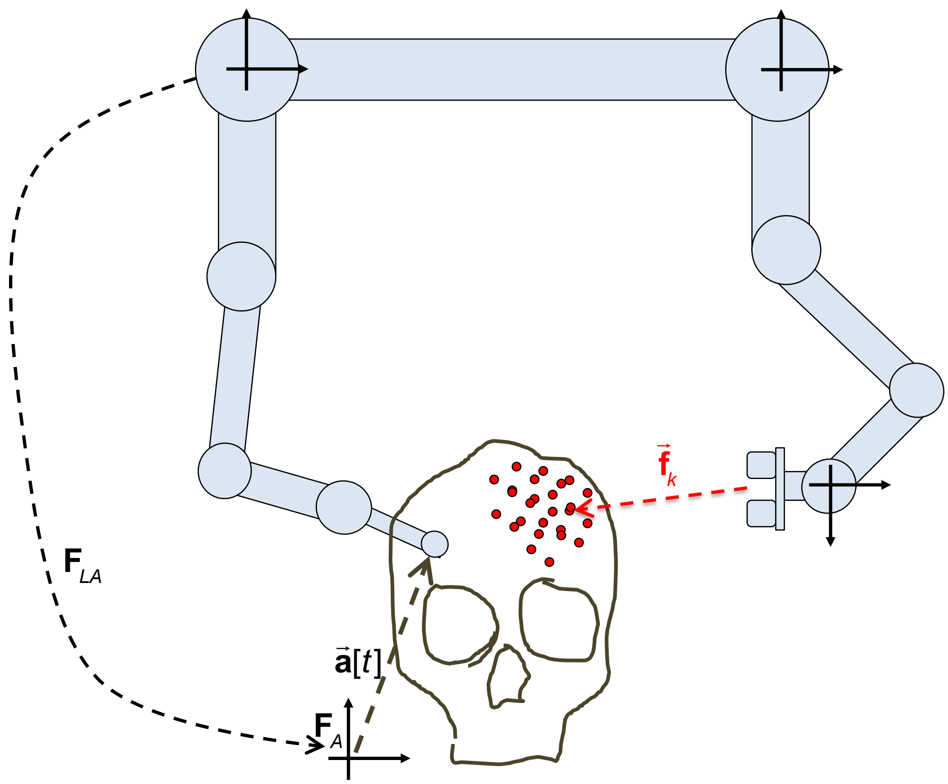
D. Assume for the moment that the camera system is extremely accurate, i.e., that $\nu = \eta = 0$. Also, assume that the uncertainty values $\Delta\mathbf{F}_L$, $\Delta\mathbf{F}_{LR}$, and $\Delta\mathbf{F}_R$ are also extremely small, but that the value of $\mathbf{F}_{RC}$ is unknown. Describe a procedure for estimating the values of $\vec{\mathbf{p}}_{Lb}$ and $\mathbf{F}_{RC}$. Explain the workflow and all the data that you will take. Then also explain the mathematical process that you will follow in order to estimate the requested value. This process may involve moving one or both arms to multiple poses and acquiring and processing image data for each pose. If there are important considerations for how these poses are chosen, you should explain briefly what those considerations are. **Hint:** you may want to start out by finding $\mathbf{R}_{LC}$. You may also want to adopt a notational convention like $\mathbf{F}_L[t] = \left[\mathbf{R}_{Lt}, \vec{\mathbf{p}}_{Lt}\right]$.

E. Suppose that something has happened to the camera mount, so that there is now a small error in your value for $\mathbf{F}_{RC}$. You have taken additional data with multiple poses of the left and right arms to recalibrate the camera mount. The data set consists of many sets of observations $\left\{\cdots\left[\mathbf{F}_L[k], \mathbf{F}_R[k], \vec{\mathbf{f}}[k]\right]\cdots\right\}$ where the $\vec{\mathbf{f}}[k]$ represent the observed position of the ball relative to the camera system. Describe a procedure for computing a more accurate value for $\mathbf{F}_{RC}$ using this additional data. If there are important considerations for how the $\left\{\cdots\left[\mathbf{F}_L[k], \mathbf{F}_R[k], \vec{\mathbf{f}}[k]\right]\cdots\right\}$ observations are chosen, you should briefly explain what those considerations are.

F. Suppose that your estimates of $\vec{\mathbf{p}}_{Lb}$ and $\mathbf{F}_{RC}$ are known accurately and that the camera system is still extremely accurate. Assume that the kinematic errors associated with the two arms are known to high accuracy, but that the value of $\mathbf{F}_{LR}$ is known only approximately, so that the true value is $\mathbf{F}^*_{LR} = \mathbf{F}_{LR}\Delta\mathbf{F}_{LR}$. Describe a procedure for estimating $\mathbf{F}^*_{LR}$. Explain the workflow and all the data that you will take. Then also explain the mathematical process that you will follow in order to estimate the requested values. This process may involve moving one or both arms to multiple poses and acquiring and processing image data for each pose. If there are important considerations for how these poses are chosen, you should explain briefly what those considerations are.

G. Recall that our overall goal is to enable the left and right arms to operate in a common coordinate system. The answer to Question 1F provides a means to correct for any misalignment of the left and right arms, but it assumes that the arms can be placed accurately in Cartesian space relative to their own bases. Suppose that your estimates of $\vec{\mathbf{p}}_{Lb}$ and $\mathbf{F}_{RC}$ are known accurately and that the camera system is still extremely accurate. Assume that $\vec{\alpha}_L[t]$ and $\vec{\alpha}_R[t]$ are still extremely small, but that the other kinematic uncertainties are not negligible. However, the robot is highly repeatable. If the robot is commanded to go to the same place from different initial positions, it will do so with negligible random variation. However, other kinematic errors cause the apparent workspace to be distorted, so that $\vec{\varepsilon}_L$ and $\vec{\varepsilon}_R$ will vary slowly over the work volume. Further, the distortion varies smoothly. Describe a procedure for producing functions $\vec{\varepsilon}_L(\vec{\mathbf{p}}_L)$ and $\vec{\varepsilon}_R(\vec{\mathbf{p}}_\mathbf{R})$ that correct for these distortions. For example, given a nominal pose $\mathbf{F}_L = [\mathbf{R}_L, \vec{\mathbf{p}}_L]$, the undistorted pose will be $\mathbf{F}_L^{cart} = [\mathbf{R}_L, \vec{\mathbf{p}}_L + \vec{\varepsilon}_L]$. For the present purposes, it is fine if these functions cause a shift in the apparent transformation between the left and right arm coordinate systems, since the answer to Question 1F will correct for that transformation. You can assume that you have an almost correct approximate value for $\mathbf{F}_{LR}$ current purposes. Explain the workflow and all the data that you will take. Then also explain the mathematical process that you will follow in order to estimate the requested values. **HINT:** You can separate the problem into first dewarping the left arm and then dewarping the right arm.

# Question 2




Suppose that all the calibration steps called for in Question 1 have been performed successfully. Assume also that we have an anatomic object that is fixedly secured at some unknown pose $\mathbf{F}_{LA}$ relative to the left arm. For specificity, we can assume that the object is a skull. Suppose also that we have defined a desired tool path $\mathbf{F}_{path}[t] = \left[\mathbf{R}_{path}[t], \vec{\mathbf{a}}[t]\right]$ relative to the anatomic coordinate system $\mathbf{F}_A$ so that the desired path of the cutter is $\mathbf{F}_{LA}\vec{\mathbf{a}}[t]$. Assume, also, that the computer has available a model $M_A$ of the surface of the anatomic object, which you can think of as a very dense cloud of oriented points or as a triangular mesh model. For any given point $\vec{\mathbf{s}}_k$ relative to the anatomic coordinate system $\mathbf{F}_A$, softwre is availabe to compute $\vec{\mathbf{n}}_k$ are outward facing unit vector normal to the surface at $\vec{\mathbf{s}}_k$. Suppose also that the camera system in the right arm is able to identify dense clouds of points $\vec{\mathbf{f}}_k[t]$ relative to the camera system coordinates, but it is not able to detect specific anatomic features. The user has the ability to use some form of hand-over-hand guidance to move the robots and has the ability to observe the video images captured by the video camera system.

A. For the moment, let us continue to assume that the camera system is extremely accurate (i.e., $\nu = \eta = 0$). Describe a procedure for determining $\mathbf{F}_L[t]$ so that the cutter ball traverses the desired path $\vec{\mathbf{a}}[t]$. Explain the workflow and all the data that you will take. Then also explain the mathematical process that you will follow in order to compute the requested values. You do not need to recite the steps of known algorithms, but you should explain what the inputs and outputs of those algorithms are. **Hint:** This will require you to compute $\mathbf{F}_{LA}$.

B. For now, continue to assume that the camera is very accurate. Suppose that the matching phase of a registration algorithm has suggested a match between surface point $[\vec{\mathbf{s}}_k, \vec{\mathbf{n}}_k]$ and observed camera point $\vec{\mathbf{f}}_k$. This observation will constrain the possible values of $\vec{\alpha}_{LA}$ and $\vec{\varepsilon}_{LA}$, where $\mathbf{F}^*_{LA} = \mathbf{F}_{LA}\Delta\mathbf{F}_{LA}$ and $\Delta\mathbf{F}_{LA} \approx \left[\mathbf{I} + sk(\vec{\alpha}_{LA}), \vec{\varepsilon}_{LA}\right]$. Provide an equation expressing this constraint..

C. Suppose now that the camera system is less accurate, so that $\nu$ and $\eta$ cannot be ignored, so that $\vec{\mathbf{f}}^*_k[t] = \vec{\mathbf{f}}_k[t] + \vec{\varepsilon}_k[t]$, where $\left|\vec{\varepsilon}_{k,z}\right| \le \eta \vec{\mathbf{f}}_k[t] \bullet \vec{\mathbf{f}}_k[t]$ and $\vec{\mathbf{d}}_i \bullet \vec{\varepsilon}_k[t] \le \nu \left\|\vec{\mathbf{f}}_k[t]\right\|$ for a suitable set of $\vec{\mathbf{d}}_i$ chosen to approximate the disc constraint described in the problem scenario. How would this affect the constraints on possible values for $\vec{\alpha}_{LA}$ and $\vec{\varepsilon}_{LA}$?

D. Assume that the robot calibration is perfect but that there is some image processing error with non-negligible values of $\nu$ and $\eta$ and that we have an accurate value for $\mathbf{F}_{LA}$. Given a desired tolerance $\delta$, how should the camera system be positioned relative to so that one can guarantee that $\left\|\vec{\varepsilon}_k[t]\right\|_2 \le \delta$. Here, I am looking for some constraint(s) on the positioning of the camera system relative to surface points $\vec{\mathbf{s}}_k$, i.e. on the expected values of the $\vec{\mathbf{f}}_k$

E. Assume again that we have a perfectly calibrated robot and an accurate value for $\mathbf{F}_{LA}$, but that the image processing error parameters $\nu$ and $\eta$ are non-negligible. Assume that the image processing software has identified a spot on a smooth portion of the anatomy at a displacement $\vec{\mathbf{f}}_k$ from the camera system. Although there is sufficient information to produce a predicted position $\vec{\mathbf{s}}_k$ relative to the anatomic coordinate system $\mathbf{F}_A$, the true position will differ from this position. I.e., $\vec{\mathbf{s}}^*_k = \vec{\mathbf{s}}_k + \Delta\vec{\mathbf{s}}_k$. Provide an estimate for a bound $\sigma$ on $\left\|\Delta\vec{\mathbf{s}}_k\right\|_2$, so that $\left\|\Delta\vec{\mathbf{s}}_k\right\|_2 \le \sigma$. **Hint:** This will involve use of the surface normal $\vec{\mathbf{n}}_k$ at $\vec{\mathbf{s}}_k$.

F. Suppose that we have an approximate value for $\mathbf{F}_{LA}$ but there is some uncertainty, so that the true value is $\mathbf{F}^*_{LA} = \mathbf{F}_{LA}\Delta\mathbf{F}_{LA}$ with $\Delta\mathbf{F}_{LA} \approx \left[\mathbf{I} + sk(\vec{\alpha}_{LA}), \vec{\varepsilon}_{LA}\right]$. Provide some constraints on the values of $\vec{\alpha}_{LA}$ and $\vec{\varepsilon}_{LA}$ that will ensure that the cutter stays within some desired displacement $\rho$ of its desired position along the path $\vec{\mathbf{a}}[t]$ relative to $\mathbf{F}_A$.