Homework Assignment 4 – 601.455/655 Fall 2019

Instructions and Score Sheet (hand in with answers)

Name	
Email	
Other contact information (optional)	
Signature (required)	I have followed the rules in completing this assignment ______________________________
Name	
Email	
Other contact information (optional)	
Signature (required)	I have followed the rules in completing this assignment ______________________________

Please indicate whether you are taking 600.445 or 600.645 (Circle one)

Question	Points 601.455/655	Points
1	10	
2	10	
3	5	
4	10	
5	10	
6	5	
7	15	
8	15	
9	15	
10	10	
11	15	
12	15	
13	15	
Total	**150**	

Although the total number of points exceeds 100, at most 100 points will be counted toward your grade.

The availability of extra credit means that you do not need to attempt each question in order to reach 100 points.

You may want to consider this when approaching this assignment.

- Remember that this is a **graded** homework assignment. **It is effectively an exam.**
- You are to work alone or in a team of two people and are not to discuss the problems with anyone other than the TAs or the instructor.
- Use a separate page to start each question
- You are encouraged to make free use of any published materials, the web, etc. in developing your answer but a) you must give full and proper citations to any references consulted and b) you may not consult, discuss, or otherwise communicate about this assignment with any human being except your lab partner, the course instructor, or the TAs. The one exception is that you should not refer to previous years' homework.
- Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
- Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
- You will be submitting on GradeScope. **By submitting, you both certify that you have both followed all the rules for this assignment.** However, it would still be good for you both to sign the grade sheet and include that with your submission.

Scenario

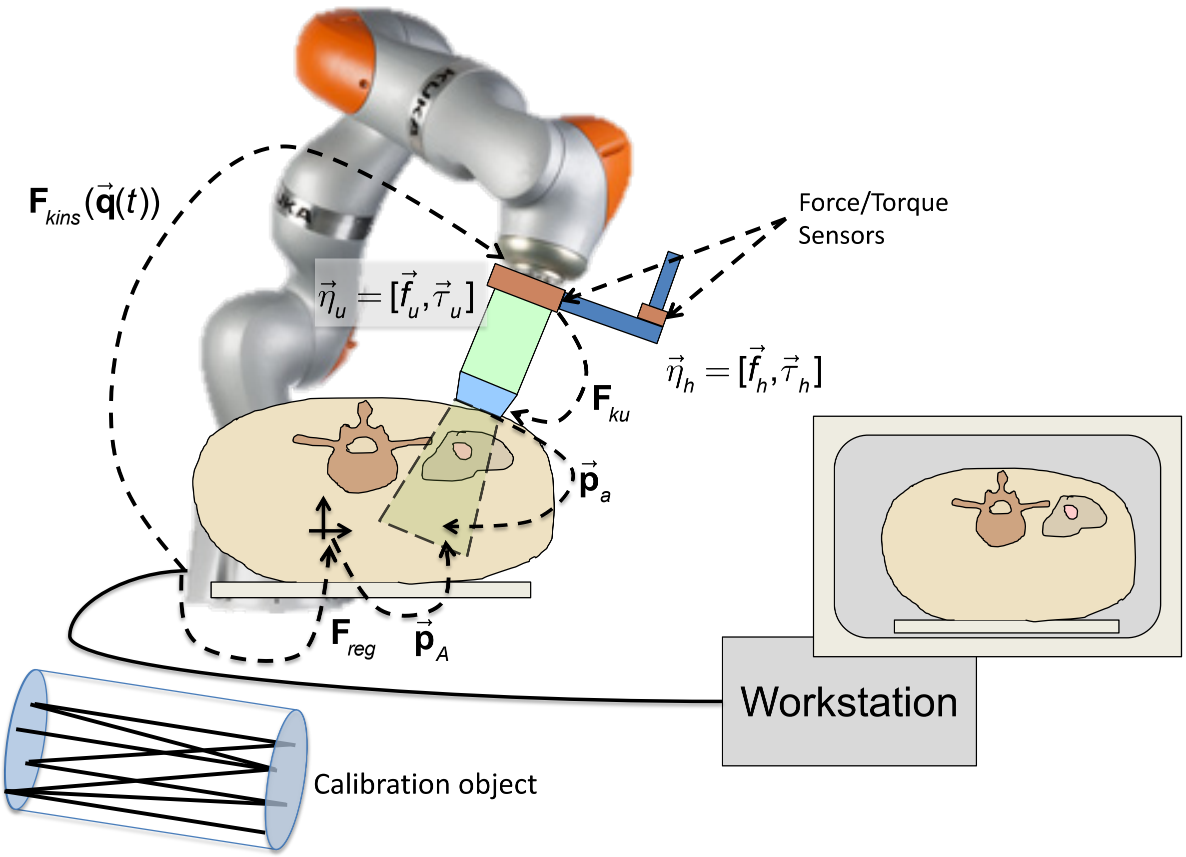

Fig. 1: Robotically-Assisted Ultrasound System

Note that, in general, we will adopt the notation $\vec{\xi} = [\vec{\alpha}^T, \vec{\varepsilon}^T]^T$ to indicate a set of small orientation and position variables. We will use $\Delta\mathbf{F}(\vec{\xi}) \approx [\mathbf{I} + sk(\vec{\alpha}), \vec{\varepsilon}]$ to indicate the corresponding pose change.

Consider the robotically-assisted ultrasound system shown in Fig. 1. This system has a workstation, an ultrasound system, and a robot and two force/torque (F/T) sensors. One of these sensors is attached to a handle, which, in turn is attached to the tooling attachment plate of the robot. When the human user exerts forces or torques on this handle, the F/T sensor senses these values and the workstation computes a corresponding F/T vector $\vec{\eta}_h = [\vec{\tau}_h, \vec{f}_h]$ resolved in the coordinate system of the robot's tooling attachment plate, where τ represents force and $\vec{\varepsilon}$ represents force. The robot has another F/T sensor that is also attached to the tooling plate and to an ultrasound probe sensor. When forces or torques are applied to the ultrasound probe (e.g., when the probe is pressed against the patient) this sensor measures them and the workstation computes an F/T vector $\vec{\eta}_u = [\vec{\tau}_u, \vec{f}_u]$, also resolved in the coordinate system of the robot's tooling attachment plate.

The workstation has a motion control loop that runs at a sample interval time ΔT (i.e., every ΔT seconds). Typical values for ΔT are usually somewhere between 2 and 10 ms. For the purposes of this assignment, you can assume that the computer workstation is fast enough to finish whatever it needs to do in the assigned time.

The workstation is able to read the joint values $\vec{\mathbf{q}}(t)$ of the robot and has a function $\mathbf{F}_{kins}(\vec{\mathbf{q}})$ that computes the pose of the tooling plate relative to the base coordinate system of the robot. It can output a new set of position goals at every sample interval. The workstation also has a function

$$\mathbf{J}_{kins}(\vec{\mathbf{q}}) = \begin{bmatrix} \mathbf{J}_{\alpha}(\vec{\mathbf{q}}) \\ \mathbf{J}_{\varepsilon}(\vec{\mathbf{q}}) \end{bmatrix}$$

such that for small changes $\Delta\vec{\mathbf{q}}$, the corresponding pose of the robot's tooling plate pose is given by $\mathbf{F}_{kins}(\vec{\mathbf{q}}+\Delta\vec{\mathbf{q}}) = \mathbf{F}_{kins}(\vec{\mathbf{q}})\Delta\mathbf{F}_{kins}(\vec{\mathbf{q}},\Delta\vec{\mathbf{q}})$, where $\Delta\mathbf{F}_{kins} \approx \Delta\mathbf{F}_k(\vec{\xi}_k) = [\mathbf{I}+sk(\vec{\alpha}_k),\vec{\varepsilon}_k]$ and

$$\vec{\xi}_k = \begin{bmatrix} \vec{\alpha}_k \\ \vec{\varepsilon}_k \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{\alpha}(\vec{\mathbf{q}}) \\ \mathbf{J}_{\varepsilon}(\vec{\mathbf{q}}) \end{bmatrix} \Delta\vec{\mathbf{q}}$$

The workstation also has a model of the patient's anatomy, which has been registered to the robot, so that a position $\vec{\mathbf{p}}_A$ in the patient coordinate system corresponds to $\mathbf{F}_{reg}\vec{\mathbf{p}}_A$ in the robot coordinate system.

Similarly, a calibration has been performed so that a point at location $\vec{\mathbf{p}}_a = [x_a, 0, z_a]$ in an ultrasound image corresponds to a point $\mathbf{F}_{ku}\vec{\mathbf{p}}_a$ relative to the tooling plate of the robot. The pose of the ultrasound probe relative to the base frame of the robot is thus $\mathbf{F}_{Bu} = \mathbf{F}_{kins}\mathbf{F}_{ku}$. For the purposes of this exercise, you can assume that points $\vec{\mathbf{p}}_a = [x_a, 0, 0]$ of the ultrasound images corresponds to points on the ultrasound probe where the surface of the probe is in contact with the surface of the patient. I.e., the ultrasound probe coordinate system has its "origin" where the probe is in contact with the patient.

Note: We sometimes describe haptic interfaces in which the human pushes on the robot and the robot moves accordingly as "admittance-type" interfaces. Similarly, we refer to interfaces where the robot pushes back on the human in response to motion by the human as "impedance-type" interfaces. The "steady hand" robot virtual fixtures described in class were of the admittance type.

Suppose also that you also have available a calibration object with a number of features (such as line fiducials) visible in ultrasound images. Further, the workstation has available software that will determine the pose $\mathbf{F}_{Cu}$ of the ultrasound probe relative to this calibration object.

Questions

1. If an anatomic feature is at position $\vec{\mathbf{p}}_0$ in the ultrasound image coordinate system when the robot's joints are at position $\vec{\mathbf{q}}$, what will be the position $\vec{\mathbf{p}}_1 = \vec{\mathbf{p}}_0 + \Delta\mathbf{p}_0$ of the same feature in ultrasound coordinates when the robot's joints are at position $\vec{\mathbf{q}} + \Delta\vec{\mathbf{q}}$? Here, I am looking for a linearized estimate of $\Delta\vec{\mathbf{p}}_0$ expressed as matrix product $\Delta\vec{\mathbf{p}}_0 \approx \mathbf{M}(\vec{\mathbf{q}}) \bullet \Delta\vec{\mathbf{q}}$

2. Suppose that we have an incremental motion $\Delta\mathbf{F}(\vec{\xi}_k)$ expressed in the coordinate system of the tooling plate. I.e., if the position of the tool plate in robot coordinates is $\mathbf{F}_{kins}(\vec{\mathbf{q}})$ the position of the tooling plate after the incremental motion will be given by $\mathbf{F}_{kins}(\vec{\mathbf{q}} + \Delta\vec{\mathbf{q}}) = \mathbf{F}_{kins}(\vec{\mathbf{q}})\Delta\mathbf{F}(\vec{\xi}_k)$. What is the corresponding incremental motion $\Delta\mathbf{F}(\vec{\xi}_{Bu})$ of the ultrasound probe in robot base coordinates? Express your answer in linearized matrix form as separate linearized matrix products for $\vec{\alpha}_{Bu}$ and $\vec{\varepsilon}_{Bu}$ in terms of $\vec{\alpha}_k$, $\vec{\varepsilon}_k$, and other variables.

3. Suppose that we have a gain matrix $\mathbf{K}_h$ such that handle F/T values $\vec{\eta}_h$ should produce an incremental motion $\Delta\mathbf{F}(\vec{\xi}_k = \mathbf{K}_h\vec{\eta}_h)$ expressed in the coordinate system of the tooling plate, in the absence of other constraints. I.e., if the position of the tool plate in robot coordinates is $\mathbf{F}_{kins}(\vec{\mathbf{q}})$ the position of the tooling plate after the incremental motion will be given by $\mathbf{F}_{kins}(\vec{\mathbf{q}} + \Delta\vec{\mathbf{q}}) = \mathbf{F}_{kins}(\vec{\mathbf{q}})\Delta\mathbf{F}(\xi_k)$. An "admittance style" optimization problem that will produce this desired behavior will be given by

$$\Delta\vec{\mathbf{q}} = \underset{\Delta\vec{\mathbf{q}}}{\arg\min}\left\|\vec{\xi}_k - \mathbf{K}_h\vec{\eta}_h\right\|^2$$

$$= \underset{\Delta\vec{\mathbf{q}}}{\arg\min}\left\|\vec{\alpha}_k - \mathbf{K}_h^{\alpha}\vec{\tau}_h\right\|^2 + \left\|\vec{\varepsilon}_k - \mathbf{K}_h^{\varepsilon}\vec{f}_h\right\|^2 \quad \text{(may be more convenient for later problems)}$$

$$\vec{\xi}_k = \mathbf{J}_k(\vec{\mathbf{q}})\Delta\mathbf{q}$$

   How would you modify this optimization problem to ensure that the linear velocity of the ultrasound probe in any direction never exceeds a value $v_{\max}$. Here, I am looking for a quadratic constraint.

4. Suppose that your optimizer is only able to handle linear constraints, how would you deal with this? In this case I am looking for an answer that guarantees that the linear velocity in *any* direction never exceeds $(1+\phi)v_{\max}$, where ϕ is a reasonably small positive number. Also, I am looking for a set of constraints that will not need to be updated every sample interval in the motion control loop. **Hint:** your solution will probably involve the creation of a set of unit vectors $\vec{\mathbf{n}}_j$. You don't have to provide exact formulas for how to create this set of vectors, but you should explain a condition that your set of vectors should obey.

5. Usually, however, the motion control loop can modify the constraints at every sample interval. How would you take advantage of this fact to significantly reduce the computational burden for the optimization problem?

6. Suppose that the workstation is also able to estimate the velocities $\dot{\vec{\mathbf{q}}}$ of all the joints. How can we use this to limit the accelerations of all the joints so that $-\vec{\vartheta} \leq \ddot{\vec{\mathbf{q}}} \leq \vec{\vartheta}$?

7. How would you modify the optimization problem of Question 5 so that the robot attempts to ensure that force exerted on the patient in the direction of the z-axis of the ultrasound probe stays close to some desired f_{des}? This may be important for good ultrasound image quality. Essentially, I am suggesting that you add a "virtual spring" with some spring constant κ that creates a bias force nudging the surgeon's hand in the desired direction.

8. Explain how you would further modify this optimization problem to ensure that the ultrasound probe does not exert a force greater than $f_{\max}$ in the direction into the patient. Here you can assume that the system has determined an effective "spring constant" $\mathbf{K}_{pat} = \begin{bmatrix} \mathbf{K}_{pat}^{\alpha} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{pat}^{\varepsilon} \end{bmatrix}$ for the tool-to-patient interaction.

9. Suppose that the joints are at position $\vec{\mathbf{q}}_0$ and that an interesting anatomic feature had been identified at location $\vec{\mathbf{p}}_a$ in ultrasound image coordinates. Suppose that the ultrasonographer has released the handle of the ultrasound probe. Define an optimization problem using an admittance-style virtual fixture to assist the ultrasonographer to place this feature as close as possible to position $\vec{\mathbf{p}}_c$ in image coordinates while also ensuring that the individual joint velocities do not exceed $\left|\Delta\vec{\mathbf{q}}_{\max}\right|$.

10. How would you modify the virtual fixture optimization problem of Question 9 to also ensure that ultrasound probe keeps the image in the same plane at all times? (I.e., the xz plane of the ultrasound image should remain fixed.)

11. The virtual fixture of Question 10 should also seek to ensure that the force of the ultrasound probe into the patient at does not exceed $f_{\max}$ and it should try to minimize torque exerted by the ultrasound probe on the patient. Here, you can use the patient stiffness model from Question 7.

    What is the final virtual fixture optimization problem encompassing all the behaviors from Question 9, Question 10, and this Question?

12. So far, we have assumed that $\mathbf{F}_{ku}$ is known exactly. However, suppose there is some small error so that the true value is $\mathbf{F}_{ku}{}^* = \mathbf{F}_{ku}\Delta\mathbf{F}_{ku}$. Describe a calibration procedure that will enable you to determine the true value $\mathbf{F}_{ku}{}^*$. Give sufficient mathematical and workflow detail so that one of ordinary skill can implement this method. You may assume that normal mathematical algorithms described in the lecture notes are available, but you should give sufficient details so that it is clear how you are using these methods, and you should clearly identify the mathematical algorithms used.

13. Suppose that the workstation also has available segmentation software that can locate points on the boundary of anatomic structures that are visible in ultrasound. The ultrasound probe has been equipped with an adjustable biopsy guide that can place the tip of a biopsy needle at location $\vec{\mathbf{p}}_{\text{biopsy}}$ in ultrasound image coordinates.

Suppose, further, that an MRI scan has been performed on the patient. Within this scan, the following structures are visible: 1) the skin surface of the patient; 2) a kidney; and 3) a tumor within the kidney. The kidney is visible in ultrasound, but the tumor is not. Further, an approximate initial registration has been performed, so that the approximate orientation of the skin surface in robot coordinates is known when the ultrasound probe is in contact with the patient's body. This initial registration is not accurate enough to provide a reliable location of the tumor relative to the base of the robot. Further, it has been determined that the skin surface cannot be used to produce a more accurate registration.

Describe a procedure that will enable you to place the tip of a biopsy needle within the tumor at a location corresponding to $\vec{\mathbf{p}}_t$ in MRI coordinates. Provide enough detail so that one of ordinary skill who has come to class and done the programming assignments will be able to implement the procedure. **Hint:** This will involve using the kidney somehow to produce a more accurate estimate for $\mathbf{F}_{reg}$.