On the Analysis of Frequency-Hopped Multiple-Access Communication with Noncoherent OFDM-ASK in AWGN Channels

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Abstract

In this work, a new analysis approach is used to analyze the performance of orthogonal frequency division multiplexing-noncoherent amplitude shift keying systems for frequency hopping multiple access networks with additive white Gaussian noise. A simple, yet accurate, expression was derived to describe the system bit error rate (BER). Montecarlo simulations were used to verify the accuracy of the derived expression.

Key Words
Frequency hopping, OFDM, ASK.

1. Introduction

In frequency hopping multiple access (FHMA) systems, the total spread spectrum bandwidth (BW_{SS}) is divided into q subbands called frequency slots, with one carrier frequency available in each of these slots. The bandwidth of each subband (BW_{slot}) equals the bandwidth of the modulated signal. Due to the difficulty of maintaining phase coherence in FH networks, noncoherent M-ary frequency shift keying (NCMFSK) is the most common modulation scheme used [1-3]. The main disadvantage of MFSK is its poor bandwidth efficiency that leads to wide BW_{slot} and small q. This problem becomes more severe when error control coding is used. For a constant bit rate, coding requires higher values of M. In MFSK schemes, the BW_{slot} increases exponentially when the number of bits per symbol (k) increases linearly. The minimum tone spacing required for noncoherently detected orthogonal signals is $1/T_s$, where T_s is the symbol duration. For $1/T_s$ tone spacing between the MFSK frequencies, $BW_{slot} \cong (2^k+1)R_s$, where $R_s = R_b/k$ and R_b is the bit rate. The large bandwidth expansion decreases q, hence increases the chance of collision between different users, and consequently erodes the coding gain. High data rate transmission is another case where higher order modulation schemes are required. In this case, higher order modulation schemes are used to avoid frequency-selective fading by decreasing the instantaneous MFSK signal bandwidth, which is the transmitted tone bandwidth. In addition, M-ary schemes can reduce the system complexity by decreasing the required hardware speed. In the literature, little work has been done to solve the FSK bandwidth problem. For example, [2] has presented a FH system with binary FSK (BFSK) and Reed-Solomon codes. To reduce the effect of bandwidth expansion resulted from coding nonorthogonal BFSK was proposed. The tone spacing used was less than $1/T_s$. Therefore, a tradeoff has to be made between the degradation resulted from the nonorthogonality and degradation resulted from reducing q. A system with trellis-coded modulation (TCM) with MFSK has been proposed in [4]. TCM requires no bandwidth expansion in coherent phase shift keying systems (PSK). However, this is not the case with MFSK. The same technique used in [2] was used in [4] to reduce the effect of q reduction. Lately, [5] has proposed to combine orthogonal frequency division multiplexing (OFDM) and NCASK as an efficient alternative for NCMFSK systems, the system performance was evaluated in Rayleigh fading channels. The fact that reference signal and the interferers amplitudes have Rayleigh distribution has made the output of the inphase and quadrature correlators to be Gaussian which enabled for a closed-form solution, Fig.1. When the channel is Gaussian with no fading it will be rather tedious to drive the probability distribution function at the output of the correlators. Using M-ary ASK combined with OFDM and FH was proposed by [6] where the binary case can be considered as a special case of the M-ary case.

In this work, a new approach is used to analyze the performance of FHMA system using noncoherent amplitude shift keying (ASK) and orthogonal frequency division multiplexing (OFDM) in AWGN channels.

2. OFDM/ ASK System Description

The OFDM-ASK system is shown in Fig. 1. The serial bit stream with bit rate of R_b is converted into k parallel streams. Each of the parallel streams has a symbol rate

$R_S = R_b / k$. The subcarrier frequencies $\omega_1, \dots, \omega_k$ have frequency separation of 1/ T_s. The transmitted signal during any symbol duration can be expressed as

$$s(t) = \sqrt{2P} \sum_{i=1}^{k} x_i \cos(\omega_i t + \theta_i) \qquad (1)$$

where P is the signal power and the data bit $x_i \in \{1, 0\}$ for ASK, and θ_i is an arbitrary initial phase. The bandwidth of the signal $s(t)$ will be denoted as the slot bandwidth (BW_{slot}), where $BW_{slot} = (1+k)R_S$. The second stage in the transmitter is the frequency hopping. The signal $s(t)$ is upconverted by one of the available q frequencies that are generated pseudo-randomly by the frequency synthesizer.

The receiver consists of three stages. The first stage is the frequency dehopping. This stage has the same structure as the transmitter hopping section. In this work it is assumed that there is no loses due to the hopping and the dehopping process. The second stage consists of k sections of noncoherent quadrature receiver with structure shown in Fig. 1, where $i= 1 \dots k$. The third stage is the parallel to serial conversion.

The channel is assumed to be AWGN channel with the noise $n(t)$ having two sided power spectral density $N_0/2$. At any given time, we will assume that there are K active users transmitting in the channel. Let us call one of the active users as the reference user. The reference user signal will be jammed (hit) when any of the $K-1$ active users transmits on the same frequency as the reference user. We call the active user who is transmitting on the same frequency as an interferer. The probability of having n interferers can be expressed as [7],

$$P(n) = \binom{K-1}{n} \left[\frac{1}{q}\right]^n \left[1 - \frac{1}{q}\right]^{K-1-n} \qquad (2)$$

which corresponds to a binomial distribution. In this model, the FH system is assumed to be synchronous, i.e., only full hits will be considered.

3. System Performance

The optimum receiver for signals with unknown phases can be implemented as a quadrature receiver [8,9]. The OFDM/ASK receiver consists of k quadrature receiver sections, Fig. 1. The center frequency of each section is centered at one of the k subcarrier frequencies. Due to the subcarriers' orthogonality, each subchannel can be considered as an independent channel. Hence, it is sufficient to consider only one of the k channels. All other channels will have the same performance. At the receiver, the dehopped signal can be expresses as

$$r(t) = \sqrt{2P} \sum_{i=1}^{k} x_i \cos(\omega_i t + \theta_i) + I(t) + n(t) \qquad (1)$$

where $n(t)$ is the AWGN, and the interference $I(t)$ can be expressed as

$$I(t) = \sum_{j=1}^{n} s_j(t) \qquad (2)$$

where, $s_j(t)$ is the j*th* interference signal and has the form of (1), and n is the number of interferers. The probability of error can be calculated as the mean of several situations corresponding to the symbol transmitted and the hit pattern ($h_j^{(n)}$) produced by the MA process. That is,

$$P_e = \sum_{n=0}^{K-1} \sum_{j=1}^{H_{total}^{(n)}} P(e \mid n, h_j^{(n)}) P(h_j^{(n)}) P(n) \qquad (3)$$

where K is the total number of active users in the channel, $H_{total}^{(n)}$ is the total number of possible hit patterns for a given n.

To simplify the discussion we will start with an example. Assume that the symbol k was hit by another user signal (n=1), then all the possible combinations of reference signal and the interferer can be summarized as the following, Table 1,

n	j	x_0	x_1	$P(h_j^{(1)})$	$P(e \mid h_j^{(1)})$
1	1	0	0	¼	P_{ASK}
	2	1	0	¼	1-P_{ASK}
	3	0	1	¼	P_{ASK}
	4	1	1	¼	$P_{ASK,R=2}$

Table 1. Hit patterns made by one interferer.

All the possible hit patterns for this case will be, $\{h_1^1$=(0,0), h_2^1=(1,0), h_3^1=(0,1), h_4^1=(1,1)$\}$. To drive the system performance for this case we have to notice that when $n = 0$, the situation is just noncoherent detection of ASK signal in AWGN channel, and the system performance is given by [10],

$$P(e \mid n=0) = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right) + \frac{1}{2}Q\left(\sqrt{\frac{E_b}{N_0}}\right) = P_{ASK} \qquad (4)$$

where E_b is the average energy per bit. When n=1, we will have the following conditional probabilities, $P(e \mid h_1^1) = P_{ASK}$, $P(e \mid h_2^1) = P_{ASK}$, in both cases the interferer has no effect, and can be ignored. The $P(e \mid h_3^1) = 1 - P_{ASK}$, in this case, the interferer has a significant effect, the symbol will be received correctly if the noise pushed the signal below the threshold. Therefore, the probability of correct decision ($P(c)$) is equal to P_{ASK} and the probability of error is equal to 1-P_{ASK}. The last case will be the $P(e \mid h_4^1) = P_{ASK,R=2}$, where

$P_{ASK,R=\hat{R}+1}$ represents the probability that the envelope (η) of the received signal ($r(t)$) will be less than the threshold ($\sqrt{E_s}/2$) given that the received signal contains $\hat{R}$ interferers with nonzero amplitudes. Therefore, the conditional probability of error given that n=1 will be,

$$P(e \mid n=1) = \frac{1}{4}(1 + P_{ASK} + P_{ASK,R=2}) \tag{5}$$

To evaluate (3), we can divide each hit pattern into two terms, the reference signal, and the interfering signals, where each interfering signal is different from others by the number of interfering symbols with nonzero amplitude. Since the reference symbol can take only two values {0, 1}, equation (3) can be expressed as,

$$P_e = \sum_{n=0}^{K-1}\sum_{i=0}^{1}\sum_{j=0}^{n} P(e \mid n, x_{r,i}, \hat{R}=j) P(x_{0,i}) P(\hat{R}=j) P(n)$$

If we defined the probability of having an interferer with nonzero amplitude as the success event, then the probability that $\hat{R}$ takes a specific value equals the probability of having i successes out of n trials. Hence;

$$P(\hat{R}=j) = P(n,j) = \binom{n}{j} p^{j}(1-p)^{n-j} \tag{6}$$

where,

$$\binom{n}{j} = \frac{n!}{j! \cdot (n-j)!} \tag{7}$$

Noticing that, $P(e \mid n, x_r = 0, \hat{R}=j) = 1 - P_{ASK,R=j}$, $P(e \mid n, x_r = 1, \hat{R}=j) = P_{ASK,R=j+1}$, and p=0.5, then (3) can be expressed as

$$P(e \mid n) = \frac{1}{2^{n+1}} \left[\begin{array}{l} 2P_{ASK} + 2^{n}. \\ \sum_{i=1}^{n} \left(\left(\begin{array}{l} (1 - P_{ASK,R=i}) \\ + (P_{ASK,R=i+1}) \end{array} \right) \binom{n}{i} \cdot \left(\frac{1}{2}\right)^{n} \right) \end{array} \right]$$

which can be simplified to

$$= \frac{1}{2^{n+1}} \left[2P_{ASK} + (1 - P_{ASK,R=1} + P_{ASK,R=n+1})(2^{n} - 1) \right]$$

Since $P_{ASK,R=1} = P_{ASK}$, then

$$P(e \mid n) = \frac{1}{2^{n+1}} \left[\begin{array}{l} 2P_{ASK} + \\ (1 - P_{ASK} + P_{ASK,R=n+1})(2^{n} - 1) \end{array} \right] \tag{8}$$

where $P_{ASK,R=n+1}$ will be calculated in the following paragraph.

When x_r=1 and $\hat{R}$ =n, the input to the quadrature receiver excluding the noise can be expressed as

$$f(t) = \sum_{i=1}^{n+1} A\cos(\omega_x t + \theta_i) \tag{9}$$

The signal $f(t)$ is a sinusoidal signal with random phase and random amplitude (peak). Since noncoherent demodulation will be used, the phase has no effect. Assume that the average value of the amplitude is equal to ($\alpha_{n+1} \cdot A$), $f(t)$ can be expressed as

$$f(t) = \alpha_{n+1} \cdot A \cdot \cos(\omega_x t + \phi) \tag{10}$$

In this case, the outputs of the I and Q correlators are $\sqrt{\alpha_{n+1}^2 E_s}\cos(\phi)$ and $\sqrt{\alpha_{n+1}^2 E_s}\sin(\phi)$ respectively. Hence, $P_{ASK,R=n+1}$ can be expressed as

$$P_{ASK,R=n+1} = Q\left(\sqrt{\frac{\alpha_{n+1}^2 \cdot E_s}{N_0}} \right) \tag{11}$$

As an example, we will consider the case for n=1. The signal $f(t)$ in this case can be expressed as;

$$f(t) = A\cos(\omega_x t + \theta_1) + A\cos(\omega_x t + \theta_2) \tag{12}$$

where θ_1 and θ_2 are random variables with uniform distribution in the period [0, 2π]. Using trigonometric identities (12) cane be rewritten as

$$f(t) = 2A\cos\left(\omega_x t + \frac{\theta_2 + \theta_1}{2}\right) \cdot \cos\left(\frac{\theta_2 - \theta_1}{2}\right) \tag{13}$$

$$= 2A\cos(\omega_x t + \phi) \cdot \cos\left(\frac{\theta_2 - \theta_1}{2}\right) \tag{14}$$

Therefore,

$$\alpha_{n+1} = \alpha_2 = 2\cos\left(\frac{\theta_2 - \theta_1}{2}\right) = 2\cos(\psi) \tag{15}$$

It is straight forward to show that the pdf of ψ can be expressed as, Fig. 2

$$f_{\Psi}(\psi) = -\frac{|\psi|}{\pi^2} + \frac{1}{\pi} \tag{16}$$

We should notice that $\cos(\psi)$ can take positive or negative signs. However, the negative sign is just a phase shift that can be combined with ϕ. Therefore, we will consider making $y = |\cos(\psi)|$, the pdf of y, can be expressed as, Fig. 3,

$$f_Y(y) = \frac{2}{\pi\sqrt{1-y^2}} \cdot \left[2 - \frac{1}{\pi} \left(\begin{array}{l} |\arccos(y)| \\ + |\arccos(-y)| \end{array} \right) \right] \tag{17}$$

the mean of y can be obtained by evaluating the following integration,

$$E\{y\} = \int_{-\infty}^{\infty} y \cdot f_Y(y)\, dy \qquad (18)$$

$$= 0.636619772\ 2 \approx \frac{2}{\pi}$$

Therefore; $\alpha_2 = 1.27323954 \approx 4/\pi$. It is clear that $P_{ASK,R=2} << P_{ASK} << 1$ since the difference between E_s and $\alpha_2^2 \cdot E_s$ is approximately 2.1 dB. As a conclusion, $P_{ASK,R=2}$ can be ignored without affecting the accuracy of the derived expression. Calculating α for all possible values of R Following the same approach is rather tedious, using simulation we can see that (Table 2)

$$P_{ASK,R=1} > P_{ASK,R=2} > P_{ASK,R=3} > \cdots > P_{ASK,R=K} \qquad (19)$$

where K is the maximum possible value of R. Therefore, all values of $P_{ASK,R=n+1}$ can be ignored, Fig. 4 shows the $P_{ASK,R}$ for different values of R.

$R+1$	1	2	3	4	5	6	7	8	9	10
α_{R+1}	1	1.27	1.56	1.99	1.99	2.34	2.51	2.66	2.80	2.93

Table(2): The Average peak of *f(t)* for different values of *n*.

Therefore, (3) will be reduced to:

$$P(e \mid n) \approx \frac{2^n - 1}{2^{n+1}} + \left[\frac{3}{2^{n+1}} - \frac{1}{2}\right] . P_{ASK} \qquad (20)$$

we can see from (20) that BER is mainly determined by SNR. For a given BW_{SS}, q is given by:

$$q = \frac{k}{(k+1)} \cdot \frac{BW_{SS}}{R_b} \qquad (21)$$

for the FH-OFDM/ASK system and by

$$q = \frac{k}{(2^k + 1)} \cdot \frac{BW_{SS}}{R_b} \qquad (22)$$

for the FH-MFSK system.

4. Conclusion and Results

To investigate the accuracy of the proposed approach, the system was simulated for K=31 and q=1024, Fig. 5 shows the system performance for different values of k. It is clear that the simulation and the theoretical results are too close which emphasizes the accuracy of the new approach.

In conclusion, we can see that the proposed approach has enabled simple, yet accurate estimation of the system performance. In addition, the new approach can be applied to any channel as long as we used the suitable P_{ASK} values.

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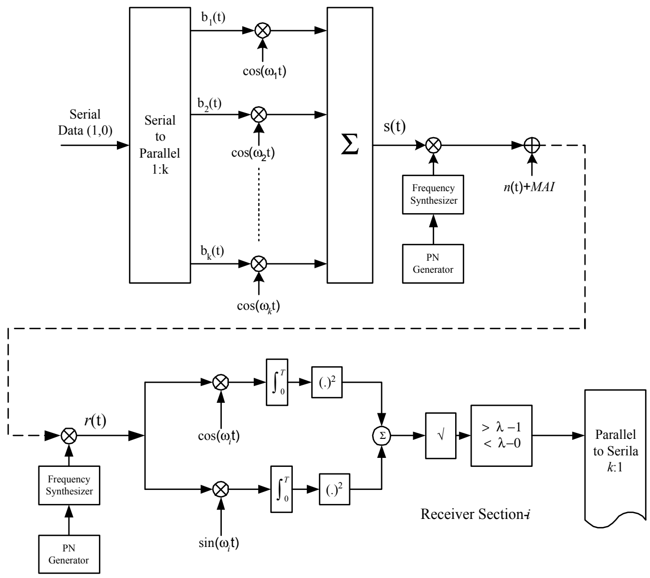

Figure 1. FH-OFDM/ASK system model

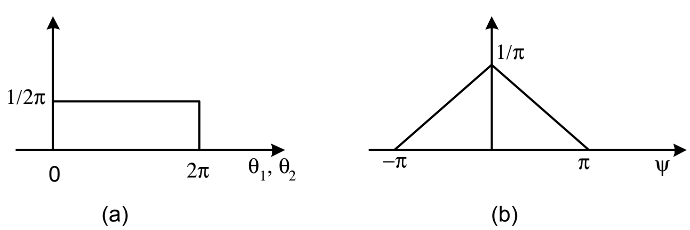

Figure 2. The pdf of ψ.

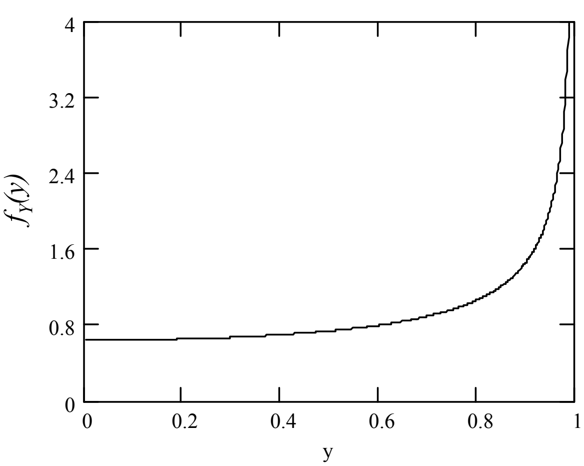

Figure 3. The pdf of y

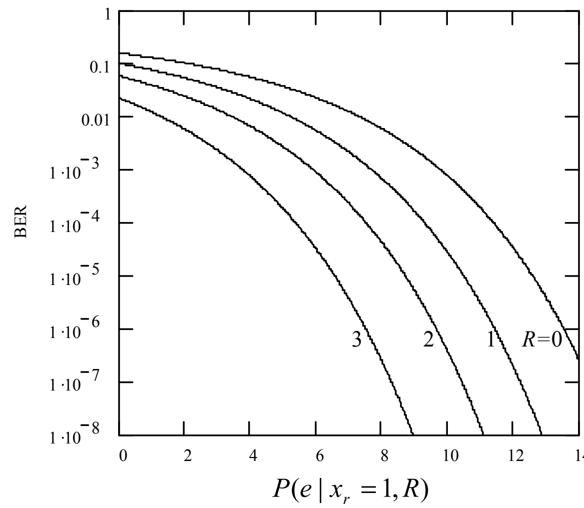

Figure 4. $P_{ASK,R}$ for R=0, 1, 2 and 3

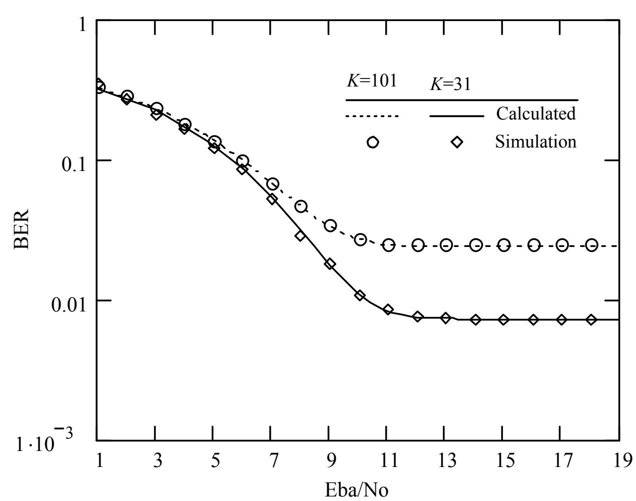

Figure 5. FH-OFDM/ASK, K=31 and 101, q=1024.