

Quantity	k_e	k_{root}	Final
Expectation	$\langle p_e, p_e r_e \rangle$	$\langle Z, \bar{r} \rangle$	\bar{r}/Z
Entropy	$r_e \stackrel{\text{def}}{=} \log p_e$, so $k_e = \langle p_e, p_e \log p_e \rangle$	$\langle Z, \bar{r} \rangle$	$\log Z - \bar{r}/Z$
Cross-entropy	$\langle q_e \rangle$ $r_e \stackrel{\text{def}}{=} \log q_e$, so $k_e = \langle p_e, p_e \log q_e \rangle$	$\langle Z_q \rangle$ $\langle Z_p, \bar{r} \rangle$	$\log Z_q - \bar{r}/Z_p$
Bayes risk	$r_e \stackrel{\text{def}}{=} \mathbf{L}_e$, so $k_e = \langle p_e, p_e \mathbf{L}_e \rangle$	$\langle Z, \bar{r} \rangle$	\bar{r}/Z
First-order gradient	$\langle p_e, \nabla p_e \rangle$	$\langle Z, \nabla Z \rangle$	∇Z
Covariance matrix	$\langle p_e, p_e r_e, p_e s_e, p_e r_e s_e \rangle$	$\langle Z, \bar{r}, \bar{s}, \bar{t} \rangle$	$\frac{\bar{t}}{Z} - \frac{\bar{r} \bar{s}^{\text{T}}}{Z^2}$
Hessian matrix	$\langle p_e, \nabla p_e, \nabla p_e, \nabla^2 p_e \rangle$	$\langle Z, \nabla Z, \nabla Z, \nabla^2 Z \rangle$	$\nabla^2 Z$
Gradient of expectation	$\langle p_e, p_e r_e, \nabla p_e, (\nabla p_e) r_e + p_e (\nabla r_e) \rangle$	$\langle Z, \bar{r}, \nabla Z, \nabla \bar{r} \rangle$	$\frac{Z \nabla \bar{r} - \bar{r} \nabla Z}{Z^2}$
Gradient of entropy	$\langle p_e, p_e \log p_e, \nabla p_e, (1 + \log p_e) \nabla p_e \rangle$	$\langle Z, \bar{r}, \nabla Z, \nabla \bar{r} \rangle$	$\frac{\nabla Z}{Z} - \frac{Z \nabla \bar{r} - \bar{r} \nabla Z}{Z^2}$
Gradient of risk	$\langle p_e, p_e \mathbf{L}_e, \nabla p_e, \mathbf{L}_e \nabla p_e \rangle$	$\langle Z, \bar{r}, \nabla Z, \nabla \bar{r} \rangle$	$\frac{Z \nabla \bar{r} - \bar{r} \nabla Z}{Z^2}$