1. Weighted set-covering problem

In the original set-covering problem, it assumes the cost of adding each set into the set cover is 1. Now let’s consider another case. For each set \( S \), the cost of adding it into \( C \) is \( n = 2^k \). The cost of each set is known. The optimal cover is the cover that has minimum cost. Provide an approximation algorithm of this weighted set-coving problem. How well does this algorithm work?

2. Broadcast in Radio

A radio network is composed of many radio stations. Each station is connected to some other stations. When a station delivers a message, only the stations connected to it can get this message. Now we want to broadcast a message to all the stations. We want to minimize the number of deliveries required to make all stations get this message. Give an approximation algorithm that can solve this problem. (Hint: use the greedy-set-cover algorithm.)

3. 2-approximation Steiner Tree (35.2-3 in text)

Consider the following closest-point heuristic for building an approximate traveling-salesman tour. Begin with a trivial cycle consisting of a single arbitrarily chosen vertex. At each step, identify the vertex \( u \) that is not on the cycle but whose distance to any vertex on the cycle is minimum. Suppose that the vertex on the cycle that is nearest \( u \) is vertex \( v \). Extend the cycle to include \( u \) by inserting \( u \) just after \( v \). Repeat until all vertices are on the cycle. Prove that this heuristic returns a tour whose total cost is not more than twice the cost of an optimal tour.
Describe how the Generalized Steiner Forest algorithm is executed.