1. (20 points) Define a two-dimensional Turing machine to be a TM where each of its tapes is an infinite grid (and the machine can move not only Left and Right but also Up and Down). Show that for every function $T : \mathbb{N} \rightarrow \mathbb{N}$ and every Boolean function $f$, if $f$ can be computed in time $T(n)$ using a two-dimensional TM then $f$ can be computed in time $O(T(n)^2)$ by a standard one-dimensional TM. **Note:** You may assume that the tapes of the two-dimensional TM start at $(0,0)$ and can only access points with non-negative integer coordinates.

2. (20 points) Define a RAM Turing machine to be a Turing machine that has random access memory. We formalize this as follows: The machine has an infinite array $A$ that is initialized to all blanks. It accesses this array as follows. One of the machine’s work tapes is designated as the address tape. Also the machine has two special alphabet symbols denoted by $R$ and $W$ and an additional state we denote by $q_{\text{access}}$. Whenever the machine enters $q_{\text{access}}$, if its address tape contains $[i]R$ (where $[i]$ denotes the binary representation of $i$) then the value $A[i]$ is written in the cell next to the $R$ symbol. If its tape contains $[i]W\sigma$ (where $\sigma$ is some symbol in the machine’s alphabet) then $A[i]$ is set to the value $\sigma$.

Show that if a Boolean function $f$ is computable within time $T(n)$ by a RAM TM, then it can be computed in time $O(T(n)^2)$ by a standard TM.

3. (20 points) Show that a language can be decided by a TM (i.e., can be recognized by some TM that halts on all inputs), if and only if some machine can enumerate the language (i.e., enumerate all strings in this language) in lexicographic order. You may assume the alphabet is $\{0,1\}$.

4. (20 points) Let $T = \{ \langle M \rangle | M$ is a TM that accepts $\alpha^R$ whenever it accepts $\alpha \}$. Show that $T$ is undecidable. **Note:** $\alpha^R$ is the reverse string of $\alpha$. You may assume the alphabet is $\{0,1\}$.

5. (20 points) Define the language

$$C_{TM} = \{ \langle M_1, M_2 \rangle | M_1, M_2 \text{ are two Turing machines such that } L(M_1) \subseteq L(M_2) \}$$

Show that $C_{TM}$ is undecidable.