Instructions and Score Sheet (hand in with answers)

Name

Email

Other contact information (optional)

Signature (required)  I have followed the rules in completing this assignment

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<th>Question</th>
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1. Remember that this is a graded homework assignment.
2. You are to work alone and are not to discuss the problems with anyone other than the TAs or the instructor.
3. It is otherwise open book, notes, and web. But you should cite any references you consult.
4. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
5. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
6. Sign and hand in the score sheet as the first sheet of your assignment
7. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment

Note changes highlighted in yellow. As mentioned in class, I had been trying to make the problem easier and inadvertently made 1.E trivial. This fixes this problem.
Scenario

Consider the probe shown in Figure 1. This probe is equipped with two magnetic position sensors located exactly on the centerline of the probe. The distance $L_1$ between these two markers is not known precisely but typically will be in the range 25 to 200 mm. The distance $L_2$ between the lower marker and the probe tip is known precisely for different probes of this design. A typical value will be about 100 mm. The tracking system reports the positions $\vec{A}$ and $\vec{B}$ of the two sensors relative to the tracking system base. In the questions below, you may assume that the probe is held generally (though not precisely) in an up-and-down direction, where “up” corresponds to the $z$-axis of the tracker.

Question 1

A. Give an expression for estimating the probe tip position $\vec{p}$ relative to the tracker base coordinate system.

\[ \vec{p} \text{ estimated} \approx \vec{P}(\vec{A}, \vec{B}) \]
B. Assume that the tracker has a small random error associated with each sensor reading. I.e., if the actual position of a marker is \( \mathbf{c} \), a corresponding sensor reading \( \tilde{C}_i \) will have an associated random error \( \Delta \tilde{C}^N \). Assume that we know bounds on these errors:

\[
-\varepsilon_N \leq \Delta C_{x,i} \leq \varepsilon_N \\
-\varepsilon_N \leq \Delta C_{y,i} \leq \varepsilon_N \\
-\varepsilon_N \leq \Delta C_{z,i} \leq \varepsilon_N 
\]

Estimate bounds for the error \( \Delta \tilde{p}_i = \tilde{P}(A_i, B_i) - \tilde{p}_i^{\text{actual}} \). You should assume that the value of \( \varepsilon_N \) is small relative to \( L_1 \) and \( L_2 \). A typical value might be 0.4 mm.

C. Assume now that the tracker has a systematic distortion, but no random noise.

\[
\tilde{C} = \mathbf{c} + \Delta \tilde{C}^{\text{distort}} (\tilde{C})
\]

where

\[
\Delta \tilde{C}^{\text{distort}} = \begin{bmatrix}
\rho \sin \left( \frac{\pi E_x}{\nu} \right) \\
\rho \sin \left( \frac{\pi E_y}{\nu} \right) \\
\rho \sin \left( \frac{\pi E_z}{\nu} \right)
\end{bmatrix}
\]

The values of \( \rho \) and \( \nu \) are not known exactly, but it is known that \(|\rho| \leq 3\) and that \( 200 \leq \nu \leq 300 \). The value of \( \tilde{C}_0 \) is not known. Estimate bounds for \( \Delta \tilde{p}_i = \tilde{P}(A_i, B_i) - \tilde{p}_i^{\text{actual}} \). **Hint:** I am looking for a formula that will involve \( \rho \) and \( \nu \).

D. Assume now that the tracker has both systematic distortion and random noise.

\[
\tilde{C} = \mathbf{c} + \Delta \tilde{C}^{\text{distort}} (\tilde{C}) + \Delta \tilde{C}^N
\]

Estimate \( \Delta \tilde{p}_i = \tilde{P}(A_i, B_i) - \tilde{p}_i^{\text{actual}} \).

E. Under the assumptions in D, how far apart should the two markers be placed to minimize the bounds on \( \Delta \tilde{p}_i \), given a design constraint that the overall distance must be more than 100 mm. I.e., what is the best value for \( L_i \) where \( L_i \leq 100 \text{ mm} \)?

**Question 2**

Suppose now that a calibration device is available for systematic distortions of the tracker, consisting of an array of sensors placed at known locations

\[
\tilde{d}_{i,j,k} = [iL_D, jL_D, kL_D]
\]

relative to the tracker base coordinate system, and that tracker measurements \( \tilde{D}_{i,j,k} \) have been made. Suppose that the designers of this calibration apparatus have also provided software for computing a distortion correction.
\[ \Delta C_{\text{correction}}(C) \]

by tri-linear interpolation, based on the known pairs \( (\tilde{d}, 
\tilde{d}, 
\tilde{D}, \tilde{D}) \).

A. Assuming a noise-free sensor and use of the interpolation software, estimate bounds on
\[ \Delta \tilde{p} = \tilde{P}(\Delta C_{\text{correction}}(A), \Delta C_{\text{correction}}(B)) - \tilde{p}_{\text{actual}} \]

B. Estimate bounds on \( \Delta \tilde{p} \) if the sensor is subject to the same noise as in question 1.B.

C. Assuming no noise and a probe for which you know \( L_1 \) and \( L_2 \), what value of \( L_D \) will be required
so that
\[ |\Delta \tilde{p}| \leq \eta \]