Homework Assignment 2 – 600.445 Fall 2003

Instructions and Score Sheet (hand in with answers)

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1. Remember that this is a graded homework assignment.
2. You are to work alone and are not to discuss the problems with anyone other than the TAs or the instructor.
3. It is otherwise open book, notes, and web. But you should cite any references you consult.
4. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
5. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
6. Sign and hand in the score sheet as the first sheet of your assignment.
7. Remember to include a sealable 8½ by 11 inch self-addressed envelope if you want your assignment
Scenario

Consider the pelvic osteotomy situation illustrated in Figure 1. Here we assume that three locating pins (similar to those used in robodoc) have been inserted into the patient’s pelvis, and a CT scan of the pelvis with the pins inserted has been produced. The patient has been placed onto the operating table. Also, a magnetic navigation system (here, the Northern Digital Aurora) is present in the room. Two surgical tools are available:

- A probe/pointer device
- An osteotome (essentially a fancy chisel) that will be used to cut the pelvis.

6 DOF Aurora sensors have been attached to the handle of each tool. The aurora is capable of determining the position and orientation of each sensor relative to the Aurora base unit. Initially, you can assume that the Aurora base unit will not move relative to the operating table and that the patient is also fixed on the operating table. Let:
\( \mathbf{F}_j \) = coordinate system associated with base unit
\( \mathbf{F}_f \) = coordinate system associated with operating table
\( \mathbf{F}_s \) = coordinate system associated with pelvis (and CT of pelvis)
\( \mathbf{F}_s \) = coordinate system associated with sensor on osteotome
\( \mathbf{F}_p \) = coordinate system associated with sensor on pointer
\( \mathbf{F}_k \) = coordinate system associated with the top of the \( k \)th pin

Further, assume that we have the following relationships:

\( \mathbf{F}_{TA} = \) transformation from table to base coordinates (unknown)
\( \mathbf{F}_{TI} = \) transformation from table to pelvis/CT coordinates (unknown)
\( \mathbf{F}_{IC} = \) desired position and orientation of osteotome tip relative to \( \mathbf{F}_f \)
\( \mathbf{F}_{ST} = \) coordinate transformation from osteotome sensor to osteotome tip
\( \mathbf{F}_{PT} = \) coordinate transformation from pointer sensor to pointer tip
\( \mathbf{F}_{AS} = \) coordinate transformation from base to osteotome sensor (measured)
\( \mathbf{F}_{AP} = \) coordinate transformation from base to pointer sensor (measured)
\( \mathbf{F}_{i,k} = \begin{bmatrix} \mathbf{R}_{i,k} \mathbf{p}_{i,k} \end{bmatrix} = \) coordinate transformation from CT to \( k \)th pin coordinate system
\( \mathbf{F}_{AC} = \) desired position and orientation of osteotome sensor relative to base (to compute)

**Question 1**

Initially, we will assume that the registration process is performed as in “classic” Robodoc. I.e., the tip of the pointer probe is placed in a small dimple on the top of the pin.

A. Write down an expression for \( \mathbf{F}_{AC} \) in terms of the other quantities defined above. I.e., give an expression that will give the position and orientation of the osteotome sensor with respect to the base unit that will place the osteotome tip into the desired pose \( \mathbf{F}_{IC} \) relative to \( \mathbf{F}_f \).

B. Assuming that all measurements are perfectly accurate, write down a series of equations that may be solved to determine \( \mathbf{F}_{AC} \). (Hint: This is essentially a “registration” step. You may find it useful to consider combining \( \mathbf{F}_{TA} \) and \( \mathbf{F}_{TI} \) into a single (unknown) transformation \( \mathbf{F}_{AI} \).)

C. Now, assume that the registration step performed above is error free, but that the Aurora has developed an error, so that each time the pose (i.e., position and orientation) of the osteotome sensor is measured as \( \mathbf{F}_{AS,j} \), the actual position is

\[ \mathbf{F}_{AS,j} = \mathbf{F}_{AS,j} \cdot \Delta \mathbf{F}_{AS,j} \]

where \( \Delta \mathbf{F}_{AS,j} \) is a small random error. Suppose that the surgeon (with hands of rock and nerves of steel) has placed the osteotome so that the measured value \( \mathbf{F}_{AS,0} \) is exactly the desired value \( \mathbf{F}_{IC} \). Give an expression giving the actual position and orientation \( \mathbf{F}_{IC} \) of the osteotome tip relative to \( \mathbf{F}_f \), in terms of \( \Delta \mathbf{F}_{AS,0} = \begin{bmatrix} \Delta \mathbf{R}_{AS,0} \mathbf{p}_{AS,0} \end{bmatrix} \)

D. Using the approximations

\[ \Delta \mathbf{R}_{AS,0} = \mathbf{I} + \text{skew}(\mathbf{\alpha}_0) \]
\[ \Delta \mathbf{p}_{AS,0} = \mathbf{\varepsilon}_0 \]

Produce a linearized expression approximating \( \Delta \mathbf{F}_{IC} = (\mathbf{F}_{IC})^{-1} \cdot \mathbf{F}_{IC} \).
E. Suppose that the Aurora began producing errors during the registration process so that each time the pose (i.e., position and orientation) of the probe sensor is measured as $F_{AP,j}$, the actual position is $F_{AP,j}^* = F_{AP,j} \cdot \Delta F_{AP,j}$ where $\Delta F_{AP,j}$ is a small random error. Assume that these errors are small enough so that you can use the approximations

$$\Delta R_{AP,j} \approx \mathbf{I} + \text{skew}(\alpha_j)$$

$$\Delta \mathbf{p}_{AP,j} = \mathbf{e}_j$$

Assume, further, that you have been told the following information about these measurement errors:

$$\alpha_{i,j,k}, \mathbf{e}_{i,j,k} \leq \gamma_k, \eta_k$$

Produce a set of linearized constraints allowing you to bound the error in the placement of the osteotome. Assume that the error measuring the osteotome sensor is as in Part D with similar limits. (i.e., $|\alpha_{0,k}| \leq \gamma_k$ and $|\mathbf{e}_{0,k}| \leq \eta_k$)

**Question 2**

Continuing the scenario from Question 1.A and B, assume that the Aurora has no measurement errors. Assume that the Aurora is located approximately 1 meter from the patient and that someone has bumped the Aurora base unit after registration, so that the new position and orientation of the Aurora base unit with respect to the operating table is given by

$$F_{TA}^* = [\Delta R(\alpha_{\text{bump}}), \Delta \mathbf{p}_{\text{bump}}] \cdot F_{TA}$$

where we know that

$$\|\alpha_{\text{bump}}\| \leq 0.01 \text{ radians}$$

$$\|\mathbf{e}_{\text{bump}}\| \leq 0.2 \text{ mm}$$

Suppose again that our rock solid, steely-nerved surgeon places the osteotome so that sensor reading $F_{AS,0}$ is exactly the desired value $F_{AC}^*$. Estimate the positional error introduced in the osteotome tip position. I.e., how far will the osteotome tip be from where it is supposed to be?
Question 3

Suppose that we have modified the surgical procedure by affixing another Aurora sensor rigidly to the patient’s pelvis before the start of the procedure (but after the CT scan has been made). The situation is now that shown in Figure 2. Let \( \mathbf{F}_b \) be the coordinate system associated with this additional sensor (which some people call a “dynamic reference body”) and let \( \mathbf{F}_{AB} \) be the measured position and orientation of the sensor relative to the Aurora base unit. Outline a procedure (include formulas) for making your surgical system insensitive to disturbances of the position of the Aurora base unit (such as in Question 2) or motions of the patient.

Question 4

Suppose that the three locating pins have been replaced by two cannulated pins inserted into the pelvis in different directions. Suppose, further, that

- The diameter of the cannulas matches the outside diameter of the probe, so that when the probe is inserted into the cannula, the axis of the probe shaft is coincident with the axis of the hole.
- The insertion depth of the probe into the hole can only be known approximately
The axis of the cannula is coincident to the z-axis of the coordinate system for the pin, as determined by the CT image processing software used in surgical planning.

A dynamic reference body similar to that described in Question 3 has been affixed to the pelvis at the start of surgery.

Outline a revised procedure and calculation method for performing the registration step. Include formulas as needed.