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Dotplot: A Program for Exploring Self-Similarity in Millions of Lines of Text and Code

KENNETH WARD CHURCH* AND JONATHAN ISAAC HELFMAN†

An interactive program, dotplot, has been developed for browsing millions of lines of text and source code, using an approach borrowed from biology for studying homology (self-similarity) in DNA sequences. With conventional browsing tools such as a screen editor, it is difficult to identify structures that are too big to fit on the screen. In contrast, with dotplots we find that many of these structures show up as diagonals, squares, textures, and other visually recognizable features, as will be illustrated in examples selected from biology and two new application domains, text (AP news, Canadian Hansards) and source code (5ESS®). In an attempt to isolate the mechanisms that produce these features, we have synthesized similar features in dotplots of artificial sequences. We also introduce an approximation that makes the calculation of dotplots practical for use in an interactive browser.

Key Words: Biology; Corpora; Duplication; Scatterplot; Software engineering; String matching.

1. INTRODUCTION

We describe a graphical tool for browsing millions of lines of text and source code. It is hard to use a screen editor to conceptualize input that is much larger than the size of a screen. Following Eick (1992), who advocated the use of interactive graphical tools to help understand large software systems, we have developed a browser that can display millions of lines of input using a *dotplot*, a plot very much like those used in molecular biology for studying homology. Dotplots (not to be confused with Tukey's "dot plot" [1977, p. 50]) are constructed by first *tokenizing* a sequence (i.e., splitting it into lines, words, characters, etc.) and then placing a dot in position i, j if the ith input token is the same as the jth. We believe that the dotplot browser may be useful for discovering large-scale structures that may be hard to spot with conventional tools such as a screen editor: Conventional tools may be too myopic to show the big picture.

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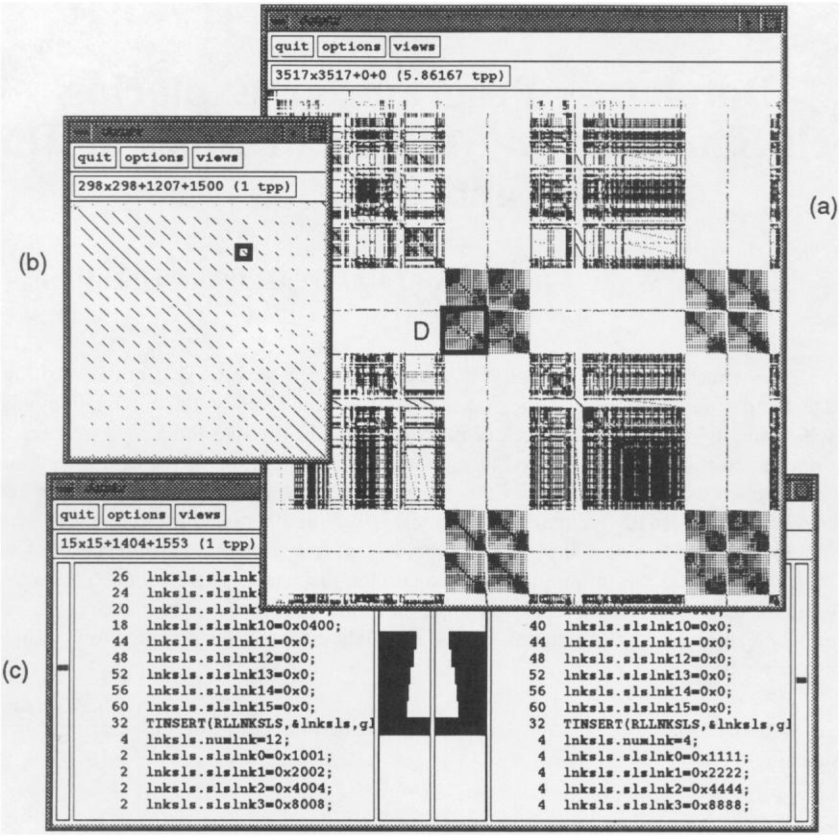

```
26  lnksls.slslnk
24  lnksls.slslnk
20  lnksls.slslnk
18  lnksls.slslnk10=0x0400;
44  lnksls.slslnk11=0x0;
48  lnksls.slslnk12=0x0;
52  lnksls.slslnk13=0x0;
56  lnksls.slslnk14=0x0;
60  lnksls.slslnk15=0x0;
32  TINSERT(RLLNKSLS,&lnksls,gl
 4  lnksls.numlnk=12;
 2  lnksls.slslnk0=0x1001;
 2  lnksls.slslnk1=0x2002;
 2  lnksls.slslnk2=0x4004;
 2  lnksls.slslnk3=0x8008;
```

```
40  lnksls.slslnk10=0x0;
44  lnksls.slslnk11=0x0;
48  lnksls.slslnk12=0x0;
52  lnksls.slslnk13=0x0;
56  lnksls.slslnk14=0x0;
60  lnksls.slslnk15=0x0;
32  TINSERT(RLLNKSLS,&lnksls,gl
 4  lnksls.numlnk=4;
 4  lnksls.slslnk0=0x1111;
 4  lnksls.slslnk1=0x2222;
 4  lnksls.slslnk2=0x4444;
 4  lnksls.slslnk3=0x8888;
```

Figure 1. Dotplot Browser.

Figure 1 shows the browser in action. Three views of a source-code file are presented —(a) a global overview of the file in the upper right, (b) a magnified view of a small portion of the file in the upper left, and (c) a text view along the bottom. The views are linked together so that clicking and scrolling in one view updates the others appropriately.

Notice the fascinating *diagonals, squares*, and *textures* in Figure 1. The texture labeled D will be discussed in more detail in Subsection 4.2. What mechanisms could be responsible for these *features*? What do the features tell us about the input sequence? This article uses two approaches to investigate such questions. In addition to the browser, which allows us to analyze naturally occurring sequences, we also synthesize artificial sequences in an attempt to replicate features found with the browser. Both methods, analysis and synthesis, are used to study the mechanisms that might be responsible for the features.

Figure 2 shows several synthesized dotplots. Figure 2a, for example, was generated from the artificial sequence: " zyxwvutsrqponmlkji." In this case, dots appear along the main diagonal and nowhere else, because all of the input tokens are distinct. In contrast with Figure 2a, there are two interesting diagonals in Figure 2b, indicating that the subsequence " abcdefghi " is repeated. We have found that diagonals and other features are often symptomatic of certain potentially important patterns in the input sequence.

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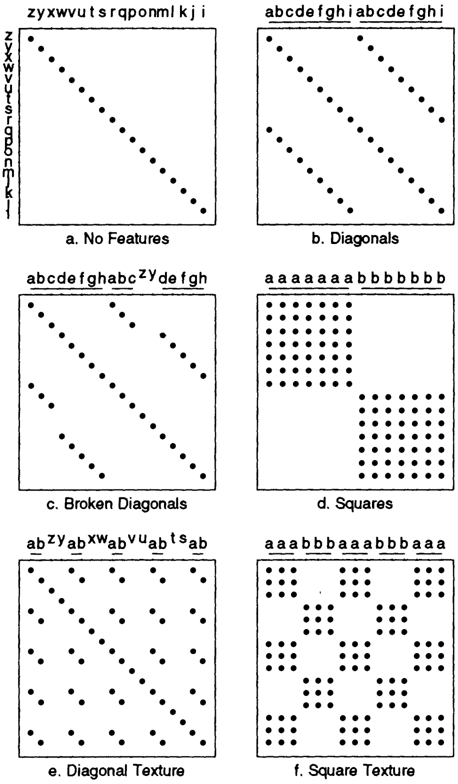

a. No Features

b. Diagonals

c. Broken Diagonals

d. Squares

e. Diagonal Texture

f. Square Texture

Figure 2. Features in Synthesized Dotplots.

Several conventions will be used throughout this article. Underlining (as in Figure 2b) is used to emphasize tokens that repeat; raising the baseline (as in Figure 2c) is used to emphasize tokens that do not repeat. In general, letters from the beginning of the alphabet are used to denote repeating tokens, and letters from the end of the alphabet are used to denote nonrepeating tokens. Labels along the left margin are often omitted. Finally, the somewhat unusual convention of placing the origin in the upper left corner was chosen to conform to the fact that English text is read left to right and top to bottom. The interaction of the text views and dotplot views is more natural when the location of the origin is consistent.

2. DOTPLOTS OF DNA SEQUENCES

The features in Figure 2b-f can also be found in dotplots of real sequences. For example, diagonals are found in Figure 3, a dotplot of two concatenated DNA sequences, (a) the plasmid pBR322 (Balbas et al. 1986), which we will refer to as sequence A and (b) the plasmid pUC19 (Yanisch-Perron, Vierira, and Messing 1985), which we will refer to as sequence B. Dotplots are a well-known technique in biology for studying homology (e.g., Maizel and Lenk 1981; Pustell and Kafatos 1982). Biologists are very interested in diagonals, which indicate, in this case, that both pBR322 and pUC19 carry the β-

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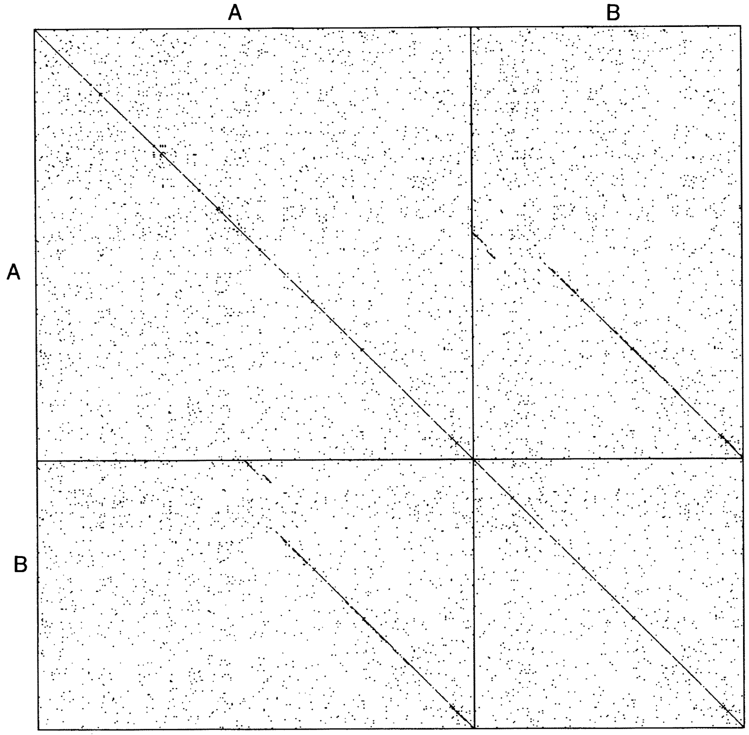

Figure 3. Dotplot of Two DNA Sequences (7,000 Nucleotides).

lactamase gene that confers ampicillin resistance. Biologists have also used dotplots to look at how sequences fold into three-dimensional structures (e.g., Blundell, Sibanda, Sternberg, and Thornton 1987; Carrington and Morris 1987; Quax-Jeuken, Quax, and Bloemendal 1983) and to investigate evolutionary questions (e.g., Doolittle 1981; Lake, de la Cruz, Ferreira, Morel, and Simpson 1988; Laver, Air, Dopheide, and Ward 1980). A brief description of the value of this approach can be found in Argos (1987). See Vingron (1991) for a recent thesis on genetic sequence alignment.

A few additional conventions are introduced in Figure 3. Grid lines are used to indicate the boundaries between sequence A and sequence B. In addition, the grid box in the upper left corner is called " AA ," the grid box in the upper right is called " AB ," and so on. Grid box AA compares sequence A with itself, whereas grid box AB compares sequence A with sequence B . In general, the grid boxes along the main diagonal compare two identical sequences, and off-diagonal grid boxes compare two different sequences.

Note that the diagonals are broken. What does this mean? Figure 2c shows, by synthesis, that broken diagonals are caused by the insertion of nonrepeating tokens (zy) into an otherwise matching subsequence (abcdefgh). In Figure 3, the breaks probably indicate that some nonrepeating nucleotides have been inserted near the beginning of B, interrupting the match between the second half of A and most of B .

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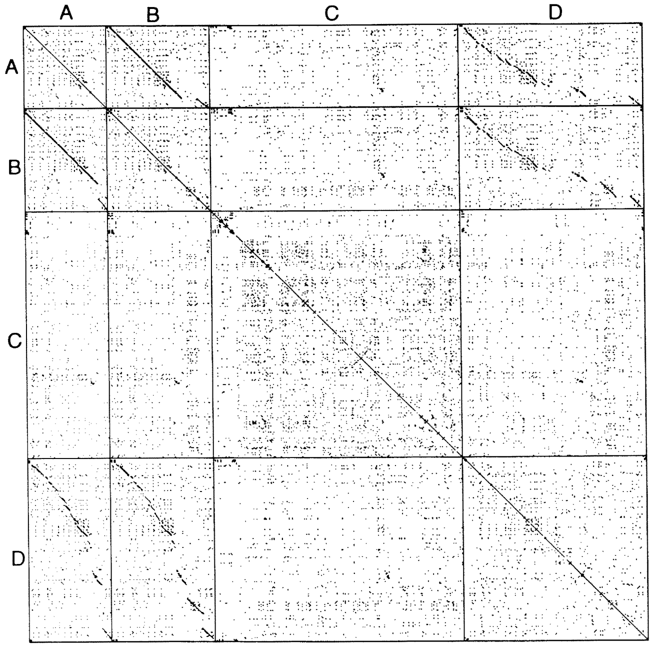

Figure 4. Four AP News Stories (3,000 words).

3. DOTPLOTS OF TEXT

3.1 AP NEWS: BROKEN DIAGONALS IN TEXT

Broken diagonals can also be found in dotplots of text, as illustrated in Figure 4, a dotplot of four Associated Press (AP) news stories, labeled A-D . The four stories, about Ryan White's death from AIDS, were sent over the AP wire within a few days of each other in early April of 1990. For text applications, we usually choose to tokenize the input into words.

	Date	Time	Lead	Title
A	040390	19:54	RyanWhite-Chro	
B	040990	02:47	White-Chronolo	White's Struggle Wi...
C	040390	14:03	RyanWhite	Ryan White, AIDS Be...
D	040390	04:18	RyanWhiteChron	

What do the broken diagonals tell us about these AP news stories? We suspect that the diagonals are broken when a news story is updated with a few additional facts. Stories A, B, and D appear to be related in this way, as evidenced by the broken diagonals in grid boxes AB, BA, AD, DA, BD, DB . In contrast, story C is probably not a rewrite of the others, as evidenced by the absence of the broken diagonals in AC, CA, BC, CB, DC, CD.

Dotplots may have practical ramifications for Information Retrieval (IR) (Salton 1989). There are several IR systems that provide rapid access to documents in large

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electronic libraries. Experience with such systems has shown that users find it difficult to construct queries that cover most of the documents of interest and not too many others. This problem could be alleviated, in part, if document retrieval systems had a more effective way of handling rewrites. In particular, the user is probably only interested in one of the rewrites—for example, story D. To date, most retrieval systems consider each document one at a time, and consequently they would usually return either all of the rewrites or none of them. There is generally no easy way to retrieve just one of the rewrites, along with an indication that there are a few more that are nearly the same.

Thus far we have seen two examples of broken diagonals. The next section shows how diagonals can be combined with squares.

3.2 HANSARDS: COMBINATIONS OF SPARSE FEATURES

Figure 5 is a dotplot of 37 million words of Canadian *Hansards*, parliamentary debates, which are available in both English and French. The input is constructed by concatenating three years of debates in English (37/2 million words) followed by the French equivalent (the remaining 37/2 million words). Consequently, there is a lag of approximately 37/2 million words between an English sentence and its French translation.

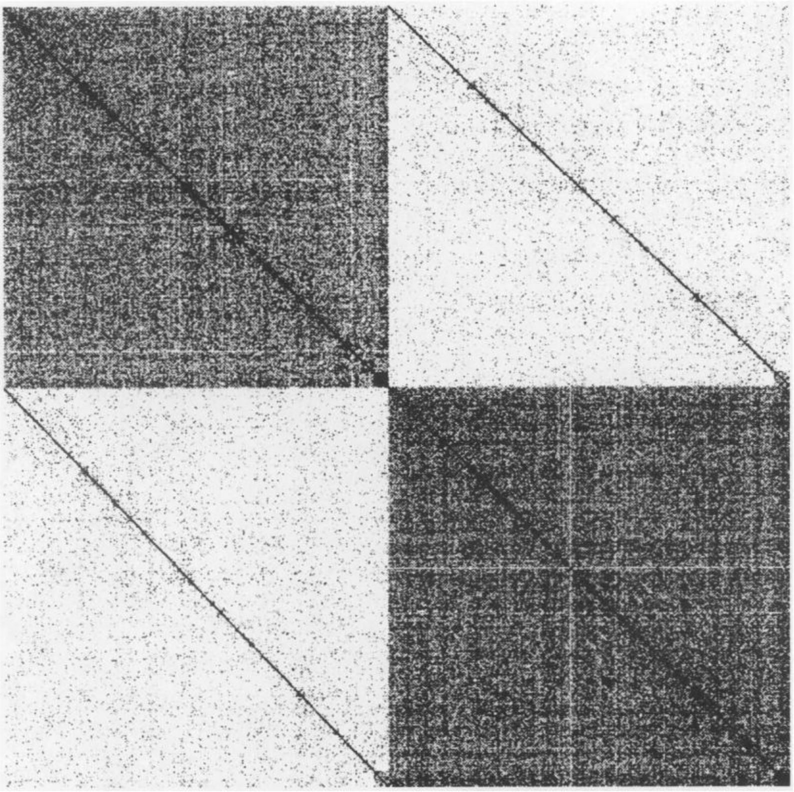

Figure 5. Three Years of Hansards (37 million words).

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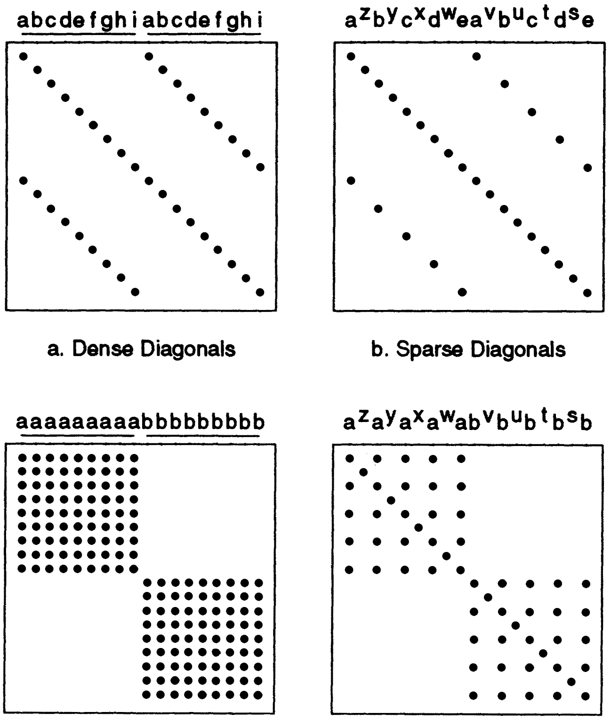

a. Dense Diagonals

b. Sparse Diagonals

c. Dense Squares

d. Sparse Squares

Figure 6. Dense Versus Sparse Features.

Thirty-seven million is such a large amount of data that the dots in Figure 5 represent the relative number of matches per pixel rather than the existence or nonexistence of a particular match.

Note the diagonals and large dark squares in Figure 5. We have seen examples of these features in isolation, but what mechanism could explain the combination? Figures 6 and 7 present a two-step solution: First, sparse versions of the diagonals and squares are synthesized, as illustrated in Figure 6, b and d, and then the interesting halves of Figure 6, b and d, are interleaved to produce the desired combination in Figure 7.

How does the synthetic sequence in Figure 7 relate to the Hansards? Let the a's denote English words—for example, *government*—the b's denote French words—for example, *gouvernement*—and the c-k's denote words that are the same in both English and French—for example, proper nouns, dates, times, numbers, and so forth. We hypothesize that the square in the upper left is formed because there are many a's matching a's or English words matching English words. Similarly, the square in the lower right is probably formed because there are many b's matching b's or French words matching French words. The diagonals indicate how the English text should be aligned with the French. There is a good chance of a dot contributing to the diagonal when the two texts are so aligned because there are a fair number of proper nouns, dates, times, numbers, and so forth that will match when text is compared with its translation.

The combination of diagonals and squares is also apparent in Figure 8, a dotplot used

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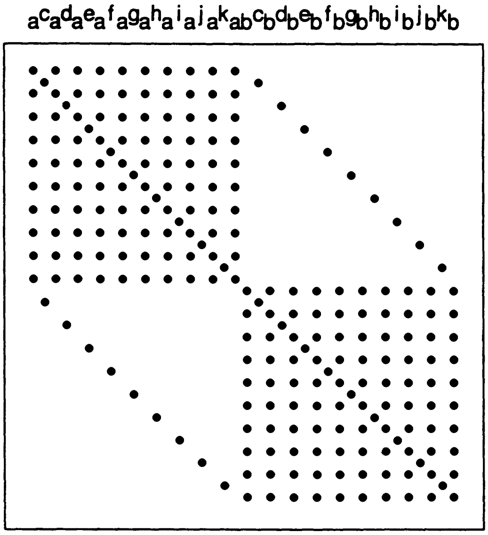
$a^c a^d a^e a^f a^g a^h a^i a^j a^k a b^c b^d b^e b^f b^g b^h b^i b^j b^k b$

Figure 7. Combination of Diagonals and Squares.

by translators at AT&T Language Line Services to help maintain six chapters of Microsoft manuals in seven languages (Dutch, French, German, Italian, Spanish, Swedish). When a file is compared with its translation, there is a good chance of finding a diagonal, whereas when a file is compared with another file in the same language, there is a good chance of finding a dark square.

With seven languages instead of just two, there is an opportunity to see matches between similar languages. Color makes it easier to see these matches. The relative number of matches per pixel is assigned to a cell in a color map using a ***histogram equalization*** technique described in Subsection 5.3. Figure 8 uses a color map (shown immediately below Figure 8) in which groups of consecutive color cells all map to the same color (e.g., cells 1–16 are yellow, 17–32 light green, 33–48 blue, 49–64 medium blue, 65–80 purple, 81–96 dark red, and the rest black). Matches between two files in the same language show up in dark red (a relatively high level), matches between two files in very different languages (e.g., Spanish and German) show up in yellow (a relatively low level), and matches between two files in similar languages (e.g., Spanish and Italian) show up in an intermediate color between yellow and dark red (e.g., medium blue, purple). Note that the ninth and eleventh files are copies of the eighth and tenth as shown by dark red boxes with black diagonals.

Of all the dotplots in this article, Figure 5 has the highest data-ink ratio (Tufte 1983, p. 93). Thirty-seven million words is at least four orders of magnitude more than could be seen with a conventional text editor. The tremendous compression factor of Figure 5 increases the apparent density of features that are actually quite sparse.

To summarize, we have seen three examples of text dotplots—AP news, Hansards, and Microsoft manuals—and two types of features—diagonals and squares. Diagonals indicate regions of ordered similarity (e.g., matches, alignments, and translations), and squares indicate regions of unordered similarity (e.g., documents in similar languages).

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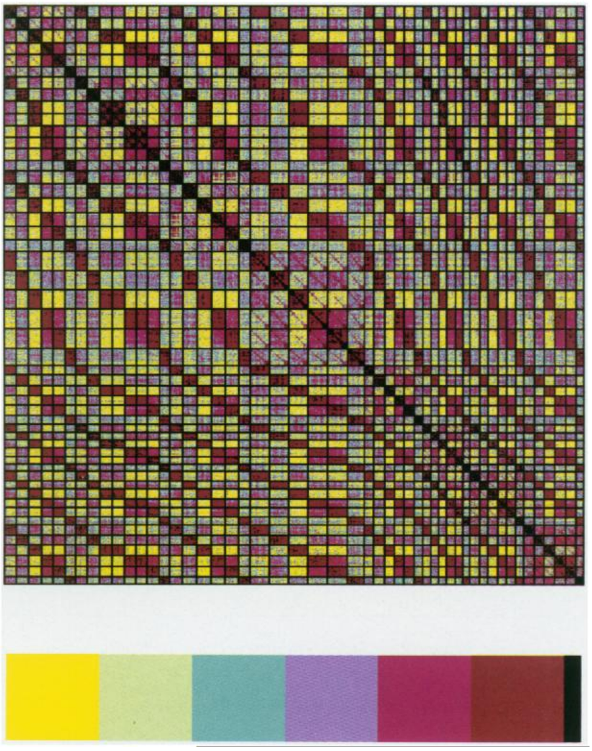

Figure 8. Six Chapters of Microsoft Manuals in Seven Languages (3.3 Million Words) With Color Map.

We have also seen how features can be preserved despite changes in density and how this property allows us to interleave combinations of sparse features. These ideas will be further developed in Section 4, which introduces the source-code application.

4. DOTPLOTS OF SOURCE CODE

4.1 Diagonals in Source Code

The source-code examples in this article are taken from the 5ESS® switch, a large program that handles much of the world's long-distance telephone service. For source-code applications, we usually choose to tokenize the input into lines of code.

Figure 9 shows a dotplot of eight source-code files labeled A-H . Two features are of interest, (1) a long broken diagonal starting at AE and extending down to DH (and, by symmetry, another long broken diagonal starting at EA and extending down to HD) and (2) 56 short diagonals, each starting in the upper left corner of a different grid box.

What could cause these features? The first feature, the long broken diagonals, indicates that files A, B, C, and D are similar to files E, F, G, and H. Perhaps these files were copied to maintain a parallel version of the software. This observation is further

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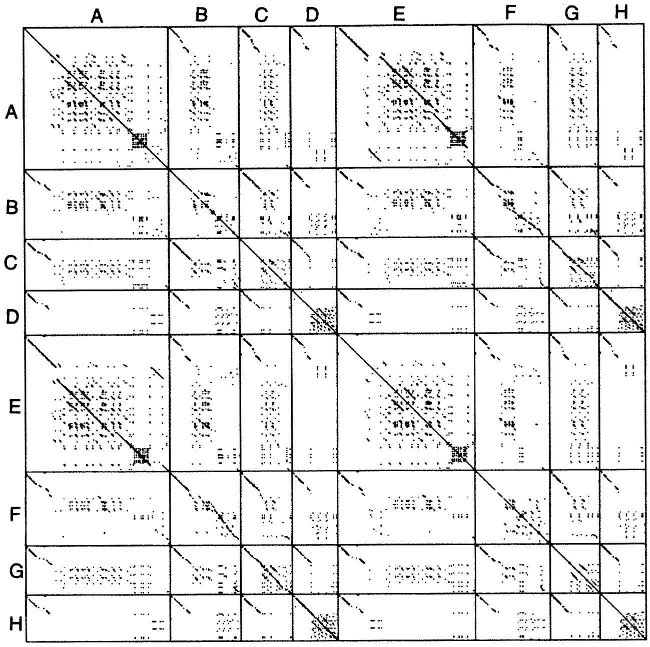

Figure 9. Three Thousand Lines of Code.

supported by the striking pattern in the names of the files.

	First 4 Files		Second 4 Files
A	`P.ISqf3.c`	E	`P.ISqf4.c`
B	`P.ISqf3_hold.c`	F	`P.ISqf4_hold.c`
C	`P.ISqf3_hr.c`	G	`P.ISqf4_hr.c`
D	`P.ISqf3_rr.c`	H	`P.ISqf4_rr.c`

The second feature, the 56 short diagonals, has a different explanation. Each of the eight files starts with a highly structured comment of the following form:

```
/*
*    File:  ...
*
*    Data:  ...
*
*    Name:  ...
```

The comments also include a number of additional fields: `Abstract, Loadable Package, Usage, Parameters, Externals`, and so forth. The 56 short diagonals are caused by similarities in the eight comments.

4.2 TEXTURES IN SOURCE CODE

A relatively small number of comments in Figure 9 generate a relatively large number of diagonals. In general, n copies of a subsequence generate $n(n-1)$ diagonals.

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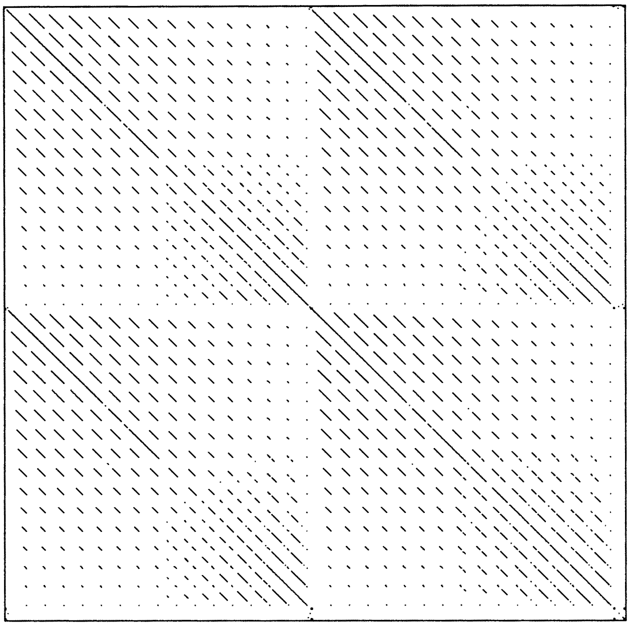

Figure 10. Six Hundred Lines of Code (detail of Figure 1).

Consequently, even a relatively small number of copies will generate such a large number of diagonals that they form a texture. A good example can be found in Figure 1a (near the label D), shown in more detail in Figure 10.

This texture consists of many diagonals of varying lengths. Consider the upper left corner where diagonals are shrinking. Figure 11 shows, by synthesis, that diagonals shrink as a repeating subsequence is diluted with increasing numbers of nonrepeating tokens.

How does the artificial sequence in Figure 11 relate to the actual input sequence for Figure 10? The texture in Figure 10 is generated by two clauses of an *if* statement. Each clause consists of 16 groups of 18 lines of code. The first group initializes all but the first field of a structure to 0, the second group initializes all but the first two fields to 0, the third group initializes all but the first three fields to 0, and so forth. The repeating sequence of initializations to 0 is diluted with increasing numbers of nonzero initializations resulting in the *shrinking diagonals* texture. There is also another pattern in the code that causes diagonals to grow longer toward the lower right corner of the texture.

This example shows that dotplots highlight a level of structure that would have been very difficult to discover using traditional tools, such as a screen editor, because the pattern extends over several hundred lines of code, much more than could possibly fit on a screen. In addition, this structure would also be difficult to appreciate with a dynamic programming approach such as the UNIX^{TM} `diff` program. Such programs attempt to find a single match and are therefore unable, in principle, to find the rich texture of multiple overlapping matches. In addition, the `diff` program would have trouble in this case because many of the matches are not exact. To handle cases like this, Baker (1992)

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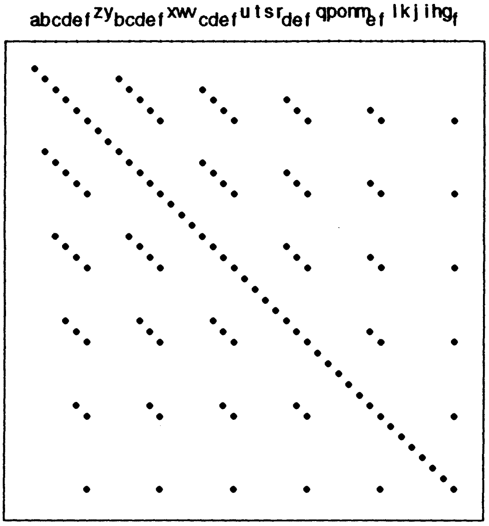

Figure 11. Shrinking Diagonals.

introduced an inexact matching criterion, ***parameterized match***, which overcomes this difficulty by equating two lines that are the same up to the names of the parameters and the values of the constants. But unfortunately this equivalence relation would also miss the pattern of shrinking diagonals, which depends crucially on the names of the parameters and the values of the constants.

In other cases, however, equivalence relations have proved to be extremely powerful. Consider Figure 12, for example, in which it appears that several large sections of code were copied verbatim (white space and all), as evidenced by the long diagonals. Suppose that we wanted to understand more about the copied code: Who copied it? When? Why?

4.3 ATTRIBUTES

Author Attributes

Author	Code
carlson	/*
carlson	* Name: RTgeninit
kedzierski	*
veach	#feature (5E2_2G)
martin	* Module: RTmain
martin	*
ahmad	#endfeature (5E2_2G)

One approach to answering these kinds of questions makes use of an equivalence relation we call ***attributes***. Large software development projects typically maintain a data base that associates each line of code with various attributes such as the author's name, modification dates, and so forth. The preceding table illustrates the author attributes for the first few lines of code that were used to generate Figure 12.

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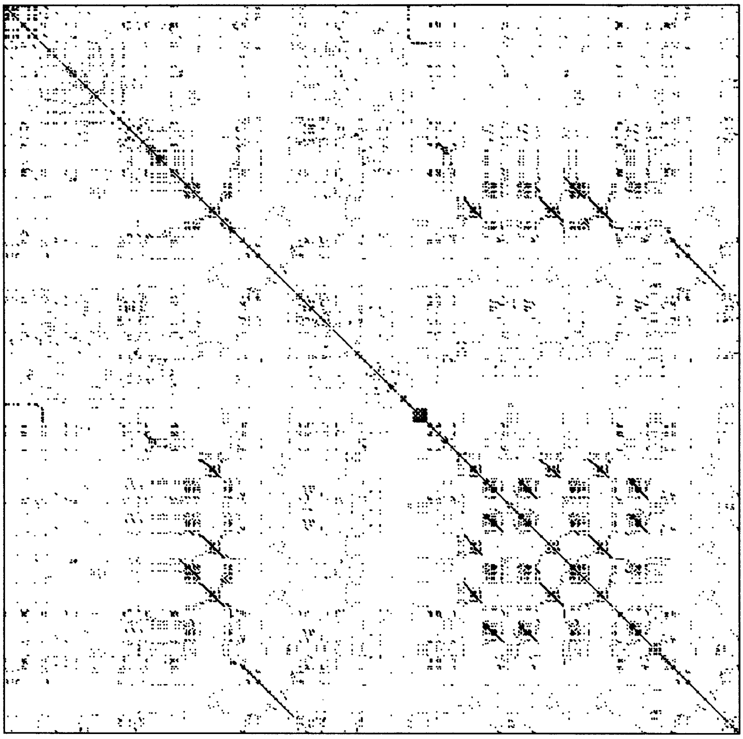

Figure 12. Three Thousand Four Hundred Lines of Code.

A dotplot can be generated from any input. Figure 13, for example, is just like Figure 12 except that it uses the author attributes (column 1) as input instead of the code (column 2). It is interesting to compare Figures 12 and 13 to see if there might be a pattern between the code and the authors. In this case, at least, it appears that many of the diagonals in Figure 12 coincide with squares in Figure 13. Why? We suspect that the copies were created by the same author, at the same time, and for the same reason. This conjecture can be further tested by looking at a dotplot of modification dates, and seeing if the features in that dotplot line up with those in Figures 12 and 13.

4.4 Summary

In summary, we have explored the properties of a variety of features, primarily diagonals, but also squares and textures (Figure 2). We have seen that features can appear in many variations: They can be broken (Figure 2c), they can be dense or sparse (Figure 6), and they can appear in various combinations (Figure 7). We have also seen applications of dotplots in biology (Figure 3), as well as the two new applications, text (Figs. 4-8) and source code (Figs. 9-13). Both analysis and synthesis have been used to learn more about the relationship between features and corresponding patterns in input sequences.

Dotplots may have a number of practical ramifications for source-code applications.

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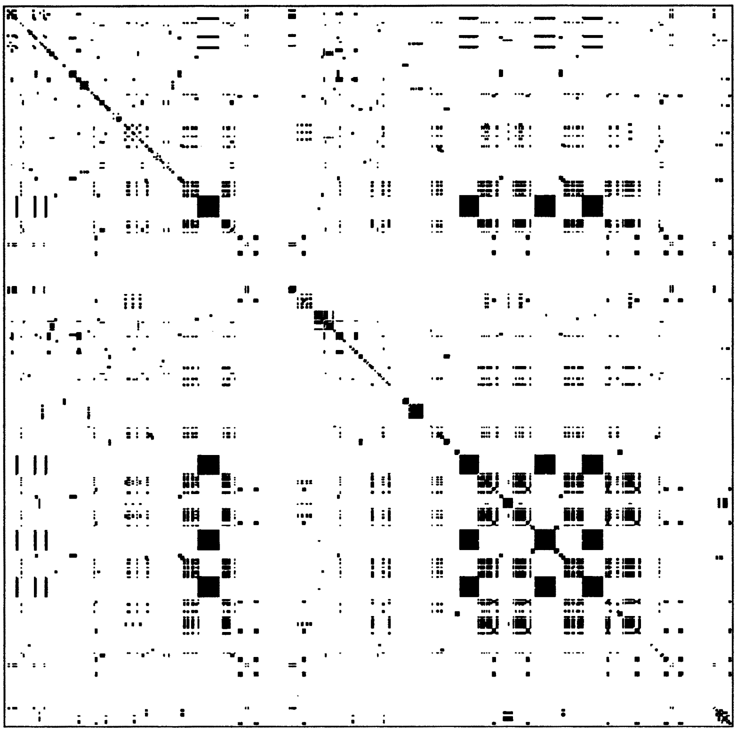

Figure 13. Three Thousand Four Hundred Author Attributes.

First, dotplots might be useful for identifying large structures in a program, especially during *discovery*, the process of reading code for the first time. Second, dotplots might help developers find undesirable duplication so that it can be removed. In some cases, for example, it is possible to replace multiple copies with a single subroutine, as suggested by Baker (1992). In other cases, the ability to identify multiple copies can be useful for maintenance. In particular, if a bug is found in one of the copies, then there is a good chance that the others might require attention as well. Thus dotplots appear to be useful for identifying large structures, removing undesirable duplication when possible, and coping more effectively with duplication that cannot be removed.

Some users might believe that redundancy is always indicative of a weakness of some kind. For example, one user has started using the browser to identify C constructions, such as `switch` statements, which are often associated with a texture generated by repeated `break` statements. In this way, dotplots have been used to help design a new programming language that avoids many of these "wordy" constructions.

Should redundancy be considered "harmful"? Following a policy like Dijkstra's stand on `goto`'s, one might suggest that redundancy should be eliminated whenever possible (Dijkstra 1968). Unfortunately, such a policy would also remove several very useful structures such as the structured comments in Figure 9. Carried to its logical extreme, such a policy would reduce a structured program to a random string, a string whose shortest description is itself. As in good writing, repetition can be a powerful rhetorical

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device for conveying emphasis, parallelism, and so forth. It would be a mistake to discourage such practices in a futile attempt to eliminate "wordiness" and other forms of "bad" writing.

5. SOFTWARE DESIGN

The next three subsections describe the implementation of the browser. First, the input data is tokenized into a sequence of N tokens. Second, this sequence is used to construct the f-image, an array of floating-point values. Finally, these values are quantized into the q-image, an array that is suitable for displaying on a color or gray-scale monitor.

5.1 TOKENIZATION

The program begins by tokenizing the input and applying the appropriate equivalence relations, if any. Equivalence relations were discussed briefly in Subsections 4.2 and 4.3; they can be used to remove white space, simulate a parameterized match, replace a token with one of its attributes, and so forth. The details of the tokenizer depend on the particular application. In the text application, for example, we have tended to tokenize the input text into words, whereas in the source-code application, we have tended to tokenize the input code into lines.

Before discussing the next topic, the calculation of the f-image, it might be worthwhile to clarify a potential source of confusion between the terms *type* and *token*. Consider, for example, the English phrase "to be or not to be," which contains six words, of which only four are distinct. We say that the sentence contains six tokens but only four types. By convention, we denote the number of tokens in the input data with the variable N, and we denote the number of types in the input data with the variable V (for *vocabulary size*).

One might normally choose to represent types as strings. That is, it would be natural to represent the word *to* as the string "to" and the line of code, "`for(i=1; i<N; i++)`" as the string "`for(i=1; i<N; i++)`". For computational convenience, we have decided not to represent types as strings but rather as contiguous integers in the range of 0 to $V - 1$. The strings are converted to numbers using standard hashing techniques. Representing tokens as integers has several advantages. In particular, it makes it easy to test whether or not the type of the ith token is the same as the type of the jth token: `if(tokens[i] == tokens[j])`. If we had used strings instead of integers, then we would have had to use `strcmp` instead of ==, which would have been much less efficient.

5.2 COMPUTING THE F-IMAGE

After the input data has been parsed into a sequence of tokens, the tokens are then converted into a floating-point image, the f-image. In the simplest case, this is accomplished by placing a dot in `fimage[i][j]` if the type of the ith token is the

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same as the type of the jth token. In other words,

```
float fimage[N][N];
for(i=0; i<N; i++)
  for(j=0; j<N; j++)
     if(tokens[i] == tokens[j])
       fimage[i][j] = 1;
     else fimage[i][j] = 0;
```

This N^2 algorithm can be improved by making use of three observations: (1) Tokens should be weighted to adjust for the fact that some matches are more surprising than others, (2) dotplots with large values of N may consume too much space, and (3) dotplots with large values of N may also consume too much time. We address these issues in the following three subsections: (1) *Weighting*, (2) *Compression*, and (3) *Approximation*.

5.2.1 Weighting

The N^2 algorithm can be improved by replacing "`fimage[i][j] = 1;`" with "`fimage[i][j] = weight(tokens[i]);`", where the function `weight` returns a value between 0 and 1, depending on how surprising it is to find that `tokens[i] == tokens[j]`. There are quite a number of reasonable functions to use for `weight`. The weighting concept is illustrated in the following, using the natural suggestion of weighting each match inversely by the frequency of the type. In this way, frequent types (e.g., the English word *the* or the line of C-code "}") do not contribute very much to the f-image because matches among such frequent types are not very surprising.

```
/* Initialize freq */
float freq[V] = {0};
for(i=0; i<N; i++)
   freq[tokens[i]]++;

for(i=0; i<N; i++)
   for(j=0; j<N; j++)
     if(tokens[i] == tokens[j])
       fimage[i][j] = 1/freq[tokens[i]];
     else fimage[i][j] = 0;
```

5.2.2 Compression

If N is large, it becomes impractical to allocate N^2 storage, and therefore it becomes necessary to compress the image in some way. Suppose that we wanted to compress the f-image from N by N, down to n by n for some $N \ll N$. Then we could simply aggregate values that fall into the same n by n cell as shown in the following code. Of course, it is recommended that the signal be filtered appropriately before compression to avoid aliasing (Gonzalez and Wintz 1987, p. 94). Filtering may also be useful if there are

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too many dots in the f-image, as is well known in the biology application (Maizel and Lenk 1981; Pustell and Kafatos 1982). In general, various well-known signal-processing techniques might be useful for enhancing features of interest.

```
/* Initialize f-image */
float fimage[n][n] = {0};
/*   Map x from token coordinates
   into f-image coordinates */
#define CELL(x) (((x) * n) / N)

for(i=0; i<N; i++)
   for(j=0; j<N; j++)
     if(tokens[i] == tokens[j])
       fimage[CELL(i)][CELL(j)] +=
         weight(tokens[i]);
```

5.2.3 Approximation

In practice, if N is very large, it becomes impractical to perform the N^2 comparisons, and it is therefore useful to introduce an approximation. Recall the dotplot of the Canadian Hansards shown in Figure 5. If we had tried to compute this figure with the N^2 algorithm, the calculation would have required $37,000,000^2$ steps, which is utterly impractical. Even if each step took only a microsecond, the N^2 algorithm would require more than 40 years.

Before presenting the approximation, it is convenient to introduce the concept of a *posting*, a precomputed data structure that indicates where a particular type can be found in the input sequence. Thus for the input sequence "to be or not to be," there are two postings for the type "to," one at position 0 and the other at position 4. One can compute the dots for the type "to" in this example by placing a dot in positions (0, 0), (0, 4), (4, 0), and (4, 4). In general, for a word with frequency f there are f^2 combinations of postings that need to be considered. The following algorithm simply iterates through all f^2 combinations for each of the V types in the vocabulary:

```
for(type=0; type<V; type++) {
   w = weight(type);
   f = freq[type];
   postings = get_postings(type);
   for(p1=0; p1 < f; p1++) {
     i = postings[p1];
     for(p2=0; p2 < f; p2++) {
       j = postings[p2];
       fimage[CELL(i)][CELL(j)] += w;}}}
```

We now come to the key approximation. If we assume that types with large frequencies ($f \geq T$, for some threshold T) have vanishingly small weights, then we do not need to iterate over their postings. This approximation produces significant savings

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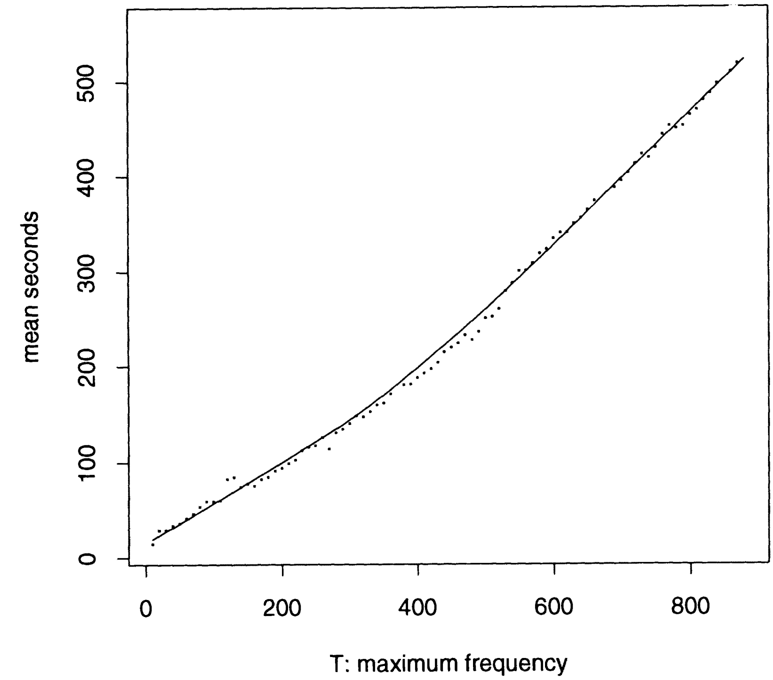

Figure 14. Computing a Hansard Plot With Different Values of T.

since it allows us to ignore just those types with large numbers of postings. In fact, the resulting computation takes less than T^2 iterations:

```
for(type=0; type<V; type++) {
   w = weight(type);
   f = freq[type];
   /* the key approximation */
   if(f < T) {
     postings = get_postings(type);
     for(p1=0; p1 < f; p1++) {
       i = postings[p1];
       for(p2=0; p2 < f; p2++) {
         j = postings[p2];
         fimage[CELL(i)][CELL(j)] += w;}}}}
```

In practice, we have found that T can often be set quite small. The Hansard dotplot shown in Figure 5, for example, was computed with $T = 20$ so that the entire calculation took less than $400V \approx 52,000,000$ steps and completed in only 25 seconds of real time on a Silicon Graphics Personal Iris workstation 4D/35.

How much does the choice of T affect the calculation time? Figure 14 attempts to address this question for the Hansard data by plotting elapsed time as a function of T. Each point in Figure 14 shows a mean of 10 trials for each value of T. The line shows a *lowess* smooth (Cleveland 1979) of the plotted points. Over this range, it appears that

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increasing the threshold by 1 increases the computation time by less than a second; dotplots with T's up to a few hundred can be computed in a few minutes of real time.

5.3 Computing the Q-Image

After computing the f-image, the floating-point values are quantized to conform to the available display hardware. Suppose, for example, that the hardware is designed to handle at most C colors, where $C \approx 256$. An obvious quantization technique is linear interpolation. Unfortunately, we have found that the values in the f-image often belong to an extremely skewed distribution, as shown in Figure 15. Using linear interpolation on such a highly skewed distribution would introduce serious quantization errors.

We have had more success with a nonparametric approach, histogram equalization (Gonzalez and Wintz 1987, pp. 146–152), which quantizes the values in the f-image into C quantiles, one for each color. Unfortunately, even histogram equalization has difficulties when the input is highly quantized. We have found empirically that many of the f-image values are small integers and ratios of small integers. This might be expected in the text application in which Zipf's Law would predict most word frequencies to be

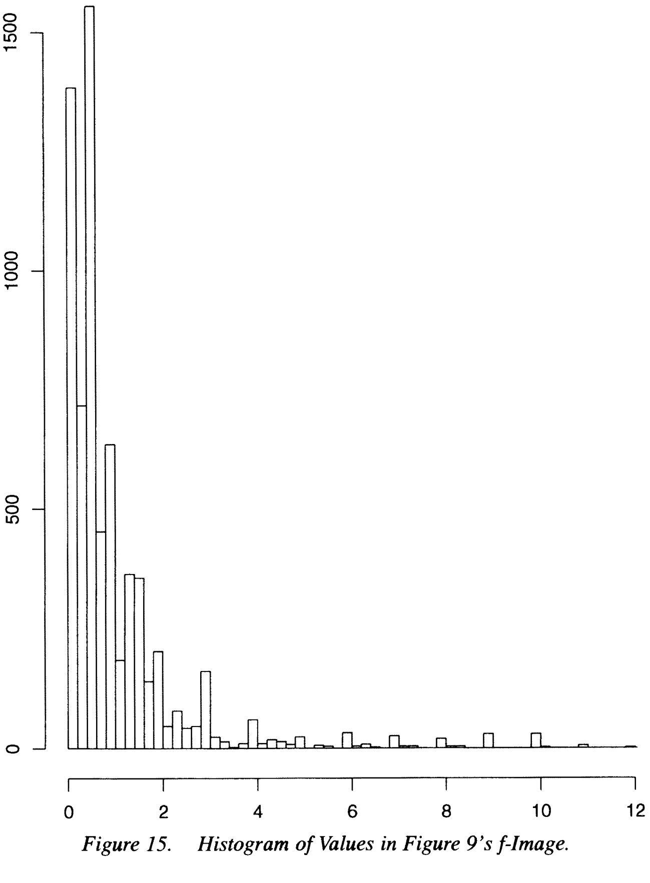

Figure 15. Histogram of Values in Figure 9's f-Image.

small integers; it also appears to hold in the other applications. To avoid assigning multiple colors to the same integer, we have found it useful to remove duplicate values before applying histogram equalization.

5.4 USER INTERFACE

Finally, the q-image is converted into an image suitable for displaying in a window as a component of the interactive dotplot browser (see Figure 1). In a color X Windows implementation (Scheifler and Gettys 1986), this final step establishes a mapping from values in the q-image to cells in the X server's default color map. We use a separate program that maintains these color cells and allows switching between various, predefined color maps. The color map typically used for black and white dotplots is *binary*: Color 0 is white (indicating relatively few matches), but all other colors are black (otherwise).

Alternatively, gray-scale color maps that vary smoothly (i.e., with perceptually even differences) from white to black indicate the total number of weighted matches per pixel. Darker pixels indicate the locations of more interesting matches, and lighter pixels indicate less interesting matches. Gray-scale is particularly useful for very dense dotplots, which may appear completely black with a binary color map.

One problem with gray-scale is that it is difficult to distinguish between adjacent values in the color map. We have found that by carefully adding color to gray-scale color maps (e.g., a range from white, through yellow and orange, to dark red), we can enhance the contrast between adjacent cells. This makes it easier to assess the relative number of weighted matches at different locations within a dotplot. We have also found that this effect is enhanced by limiting the number of colors (see Figure 8).

In a monochrome implementation, the q-image step is unnecessary since the f-image can be converted directly into black and white using various standard techniques such as thresholding, dithering, error diffusion, and so forth.

In addition to the dotplot views discussed thus far, there are also text views, as shown in Figure 1c. A text view consists of two panes so that two subsequences of the input can be presented side-by-side. The text view is linked to a dotplot view, so that clicking the mouse on a point in the dotplot corresponding to the pair of tokens x, y causes the left pane to be centered around x and the right pane to be centered around y.

6. CONCLUSION

Dotplots, which have been used to study homology in biology, are also useful for discovering potentially important patterns in text and source code. In the software application, for example, we have seen that dotplots can be used to discover large-scale structures, remove undesirable duplication when possible, and cope more effectively with duplications that cannot be removed. Similarly, there are also several practical ramifications of these patterns in the text application.

We have seen that many of these potentially important patterns are often associated with certain features in the dotplot—diagonals, squares, textures and combinations thereof. There was a considerable discussion of several mechanisms that explain some of

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these associations. Much of the discussion used the browser to analyze a feature in a real sequence and then tried to replicate the feature in a synthesized dotplot. For example, the browser was used to find broken diagonals in AP stories (Figure 4), a combination of squares and diagonals in the Hansards (Figure 5), and shrinking diagonals in a large program (Figure 10). Each of these features was replicated in a synthesized dotplot—broken diagonals in Figure 2c, the combination of squares and diagonals in Figures 6 and 7, and shrinking diagonals in Figure 11. The discussion then concluded with a speculation of the underlying mechanism. In the AP news, for example, the diagonals were probably broken by the insertion of a few extra facts into a rewrite. Similarly, the shrinking diagonals in the software example were probably caused by a repeating sequence of initializations to 0 being diluted with increasing numbers of nonzero initializations.

In many cases, the patterns are much easier to find with a dotplot than with an alternative such as a text editor or the UNIX `diff` program. A text editor, for example, is ill-suited for identifying structures that extend well beyond the size of the screen. Similarly, the `diff` program is ill-suited for identifying a texture such as the shrinking diagonals pattern discussed in Subsection 4.2, because the `diff` program attempts to find a single alignment path and therefore cannot deal effectively with the rich structure of multiple overlapping matches.

The final section of the article described the implementation of dotplots, with an emphasis on weighting, compression, and approximation. These steps make it possible to compute dotplots quickly enough for use in an interactive browser.

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