Modeling Messaging Activities in a Network: Enron Case Study

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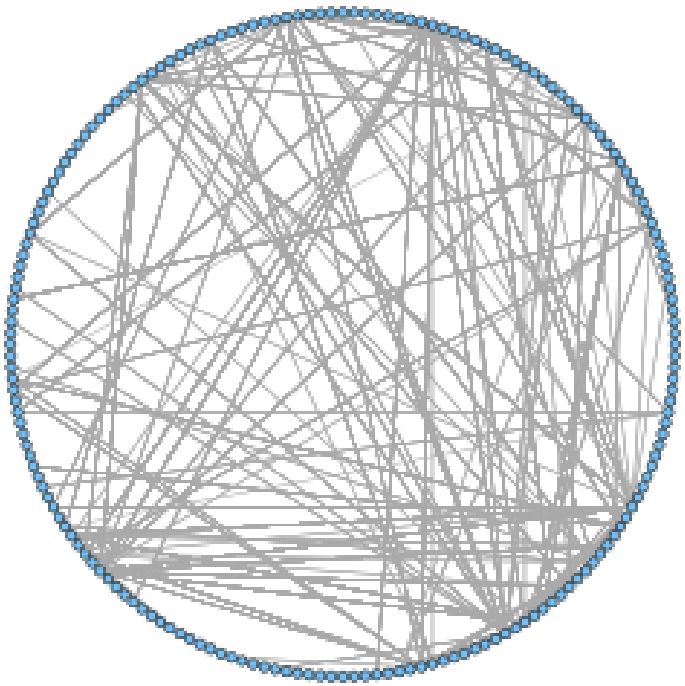
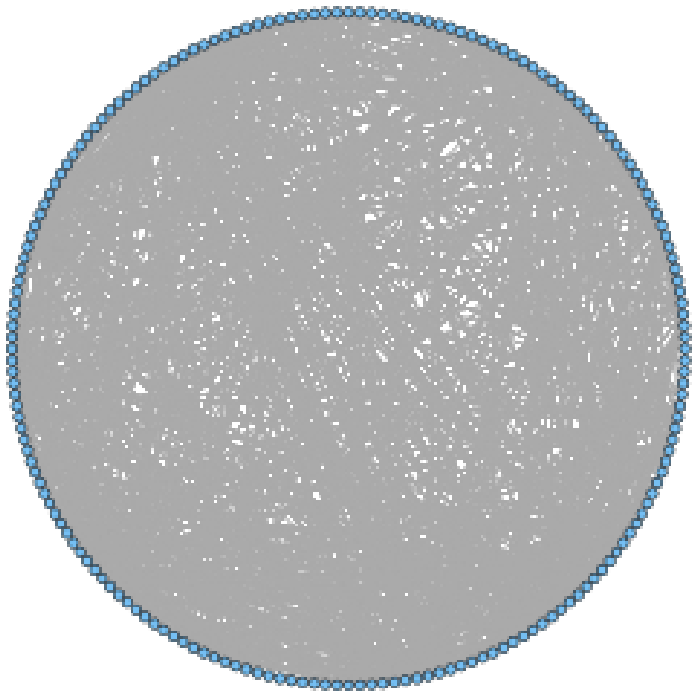
Enron Dataset

- 189 weeks from 1998 to 2002 (Nov, 13 1998 to Jun 21, 2002).
- 184 users, 125k messages.
- Data format: time t, sender i, receiver j.

1998 - 2000 year



2000 - 2001 year



Research Tasks and Solving Strategies

Task:

- Understand the structure of interactions and the communication patterns in a corporate network.
- Detect abnormal activities and identify communities within a corporate network.

Approach:

- Construct random graphs that model messaging activities in a network via:
  - homogeneous Poisson process (messaging rate $\lambda_{ij} = const$);
  - inhomogeneous Poisson process ($\lambda_{ij}(t)$ is different for Δt).
- Compare simulated and true Enron graphs using different statistics.

Simulation: Homogeneous vs. Inhomogeneous PP

Definition (Modeling with HPP)

Each edge (i, j) from the set of all $\binom{n}{2}$ edges is included in the graph G with constant probability p.

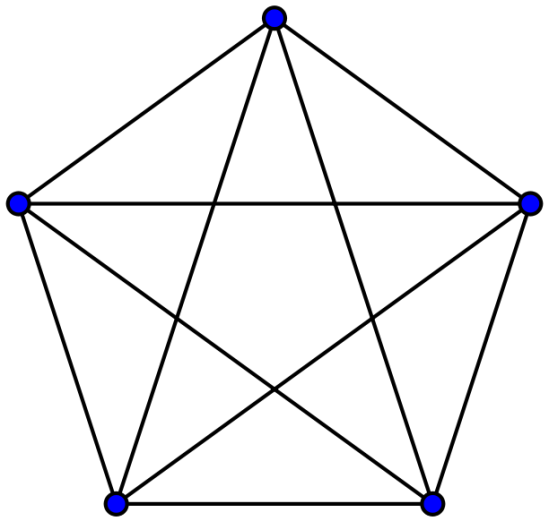
Definition (Modeling with IHPP)

An edge (i, j) from the subset of $\binom{k}{2}$ edges is drawn between two vertices $i, j \in K$ with probability $s \geq p$ and each of the remaining edges $\binom{n}{2} - \binom{k}{2}$ are drawn with probability p.

Simulation of Messaging Activities

Switching to a movie...

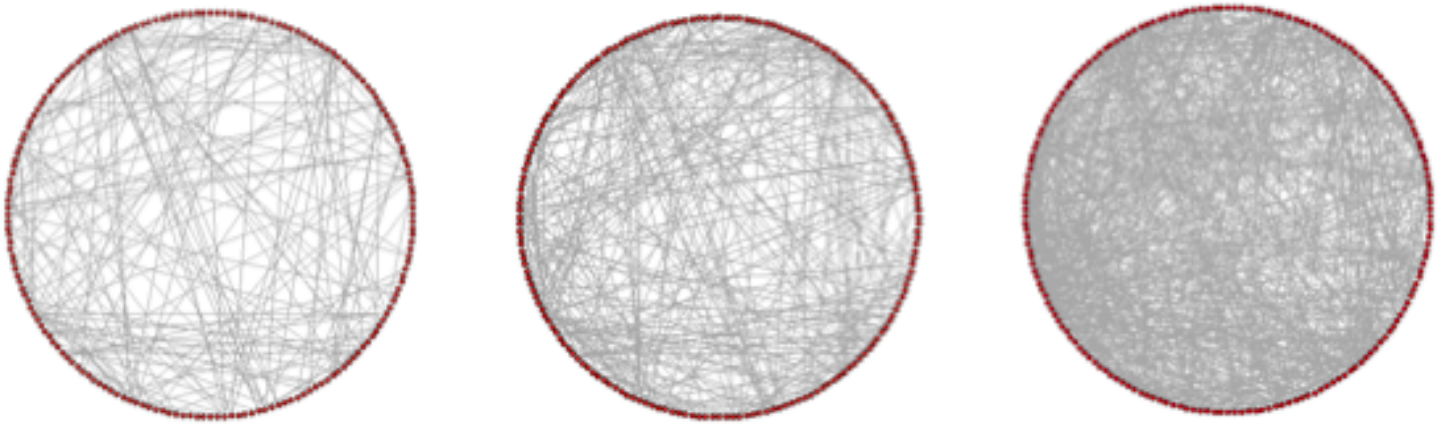
Evaluation: Summary Statistics for Graph Comparison



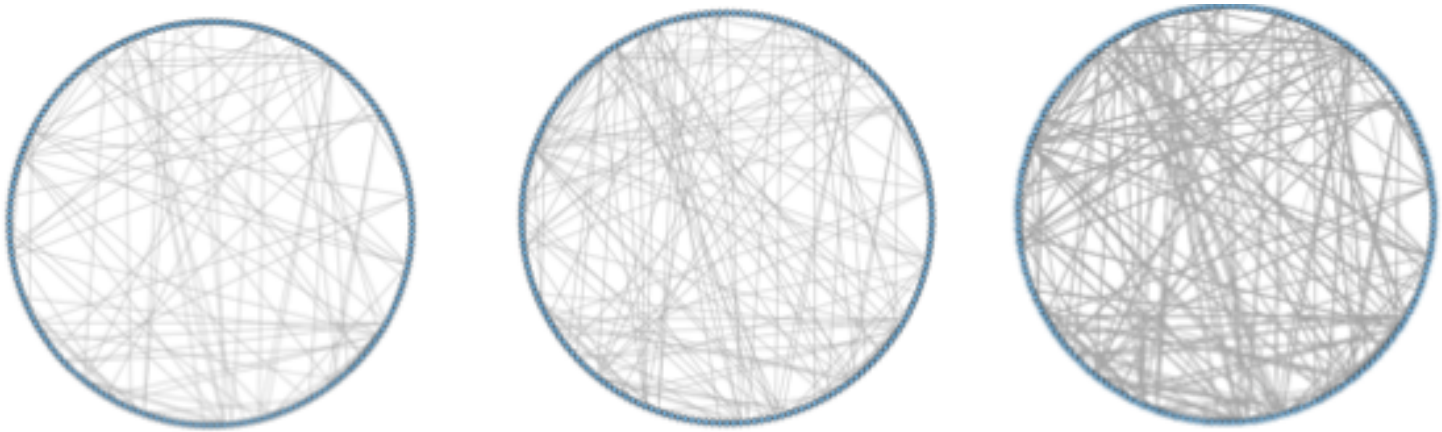
- graph size: $|E| = |E(G)| = 10$.
- graph maximum degree: $|\Delta| = |\Delta(G)| = 4$.
- Kolmogorov-Smirnov Test fits two degree distributions: p-value.

Thresholding The Number Of Messages N_{ij}

Simulated graphs with thresholds 5, 3, 1 (from left to right):



True graphs with thresholds 5, 3, 1 (from left to right):



Estimating Message Rates $\hat{\lambda}$ for HPP Simulation

- Estimate a “bulk" message rate $\hat{\lambda}$ using MLE estimator:

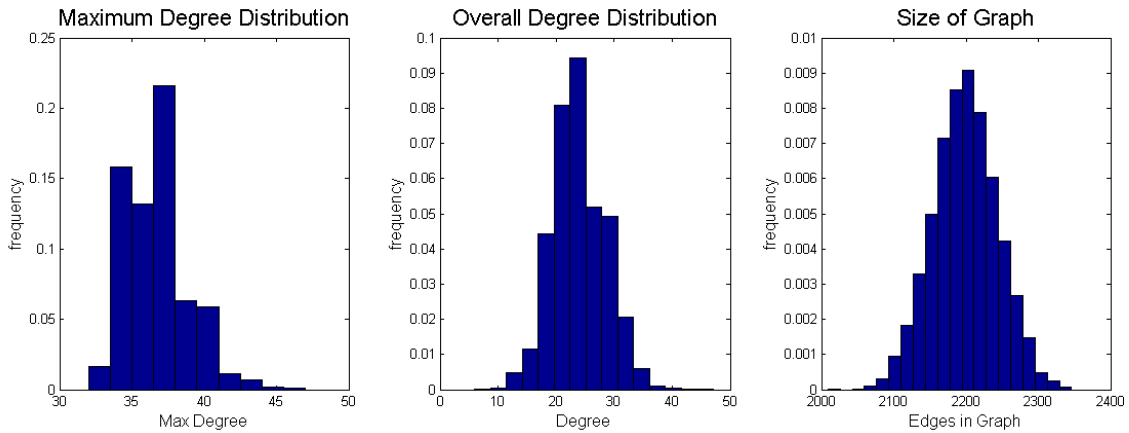
$$\hat{\lambda} = \frac{[N(t_{k+n}) - N(t_k)]}{(t_{k+n} - t_k)}$$

- Estimate a constant message rate $\hat{\lambda}_{ij}$ for a pair of vertices i, j using MLE estimator:

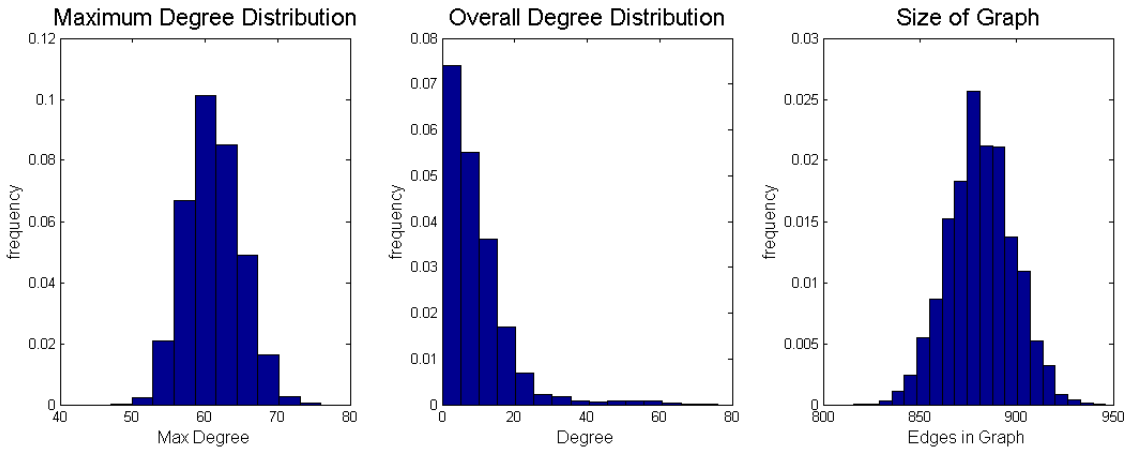
$$\hat{\lambda}_{ij} = \frac{[N_{ij}(t_{k+n}) - N_{ij}(t_k)]}{(t_{k+n} - t_k)}$$

Summary Statistics For “Bulk" and Vertex Pair Dependent λ in HPP case

Summary statistics for “Bulk" $\hat{\lambda}$ message rate, $\theta = 1$:

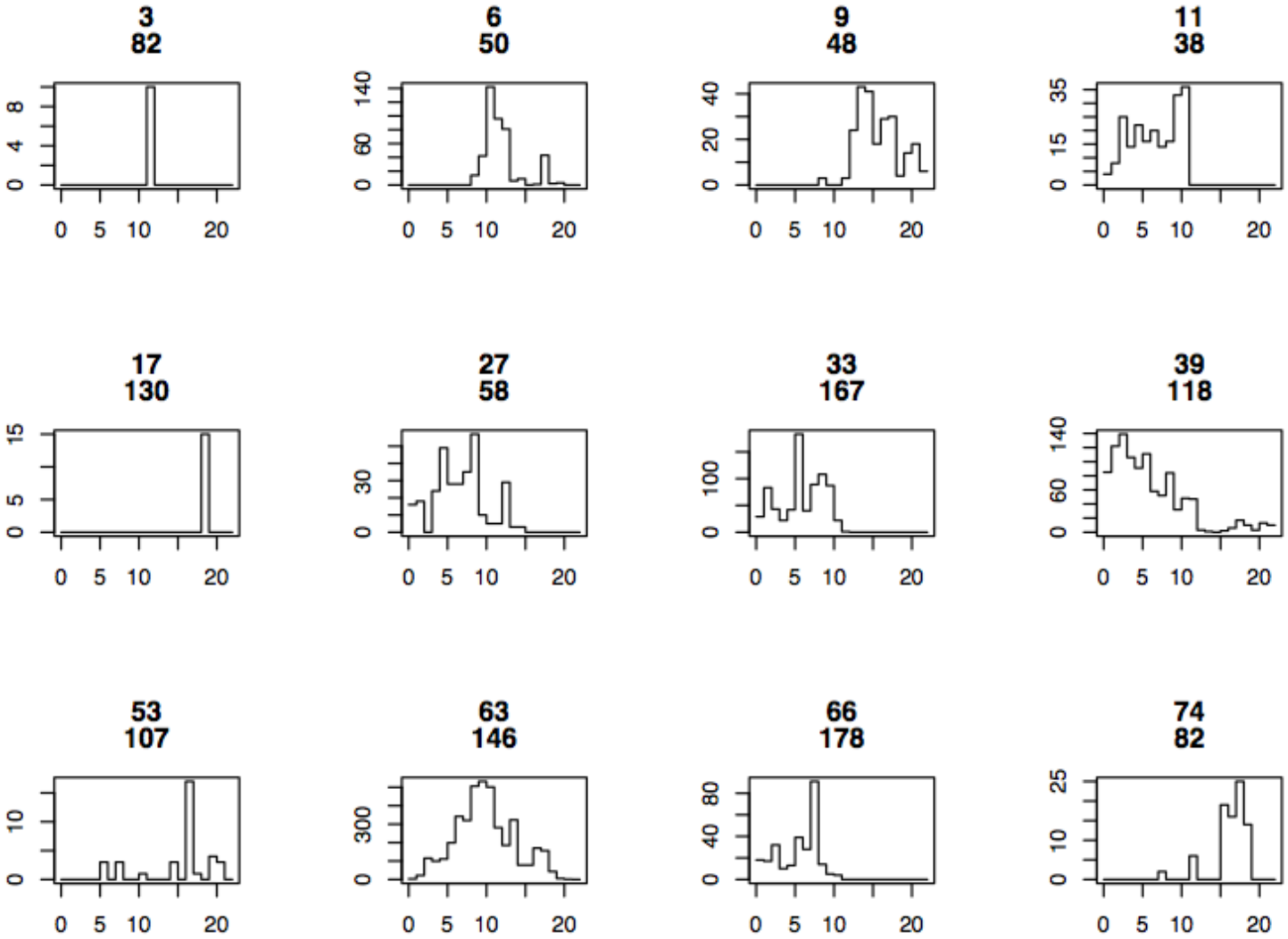


Summary statistics for vertex pair dependent $\hat{\lambda}_{ij}$, $\theta = 1$:



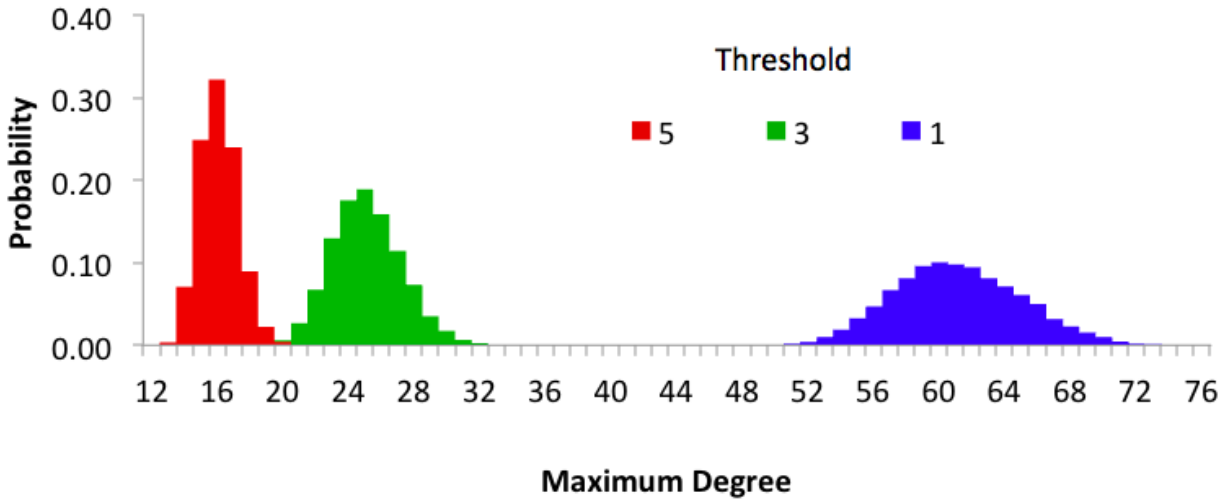
Estimating Intensities l_{ij} for IHPP Simulation

- Estimate the intensity functions l_{ij} for a pair of vertices i, j for 88 weeks (x: 4-week periods, y: number of messages).

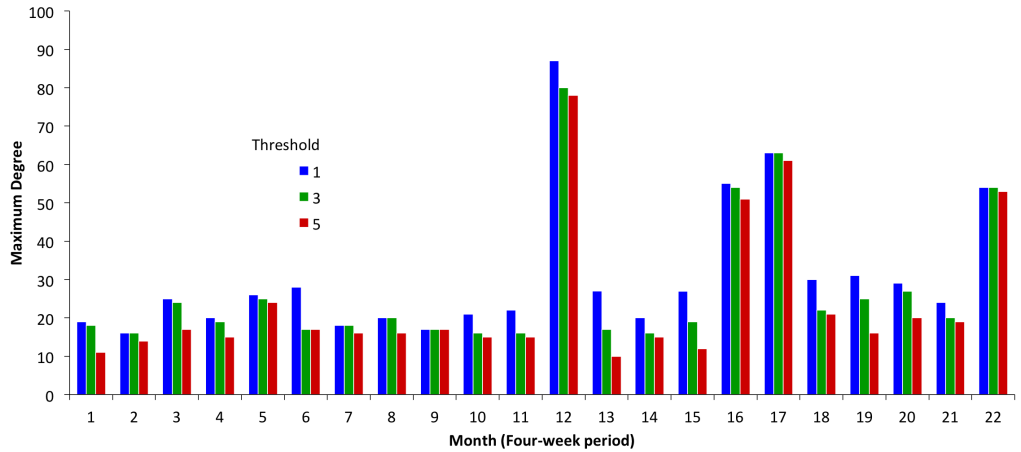


HPP Simulation: Maximum Degree Comparison

Maximum Degree Statistic for HPP simulated graphs:



Maximum Degree Statistic for Enron graphs:

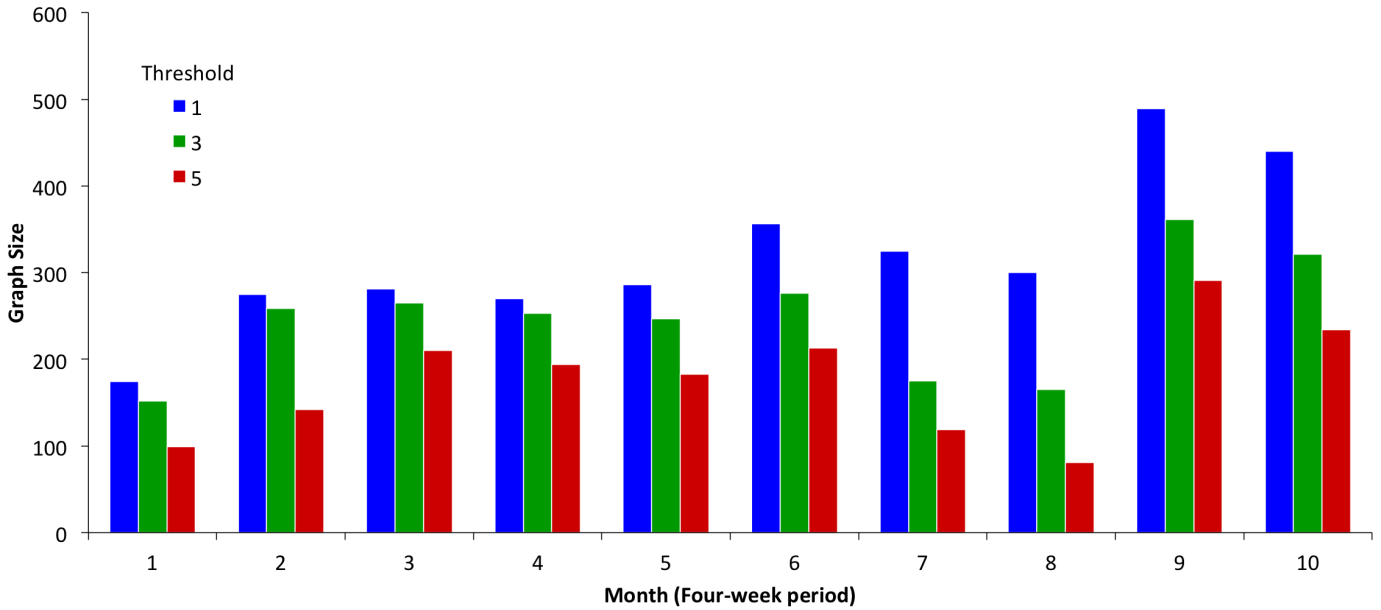


HPP Simulation: Graph Size Comparison

Percentiles for the Graph Size Statistic from Simulated Graphs:

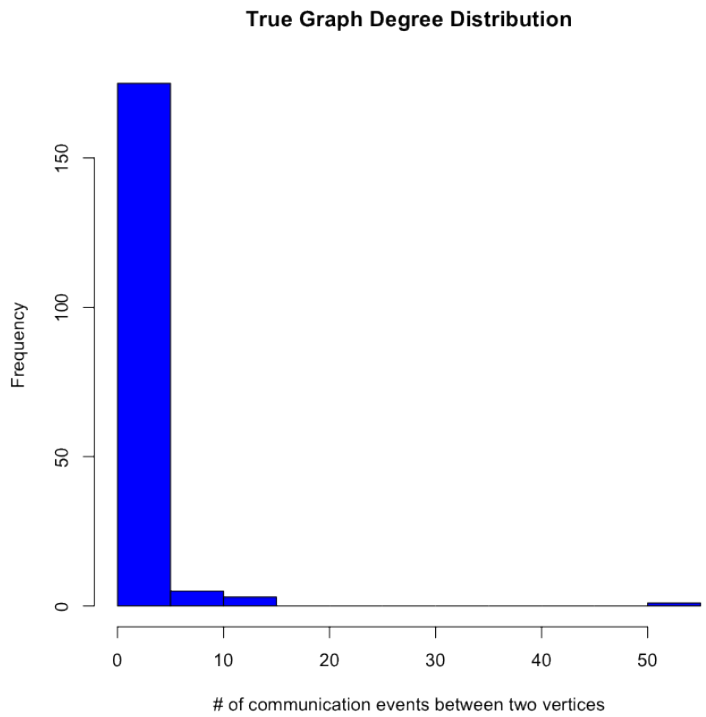
Percentile:	1	2.5	5	25	50	75	95	97.5	99
Threshold 1:	843	849	854	871	881	893	910	915	920
Threshold 3:	301	305	308	317	324	331	341	344	347
Threshold 5:	166	168	171	178	182	187	194	196	199

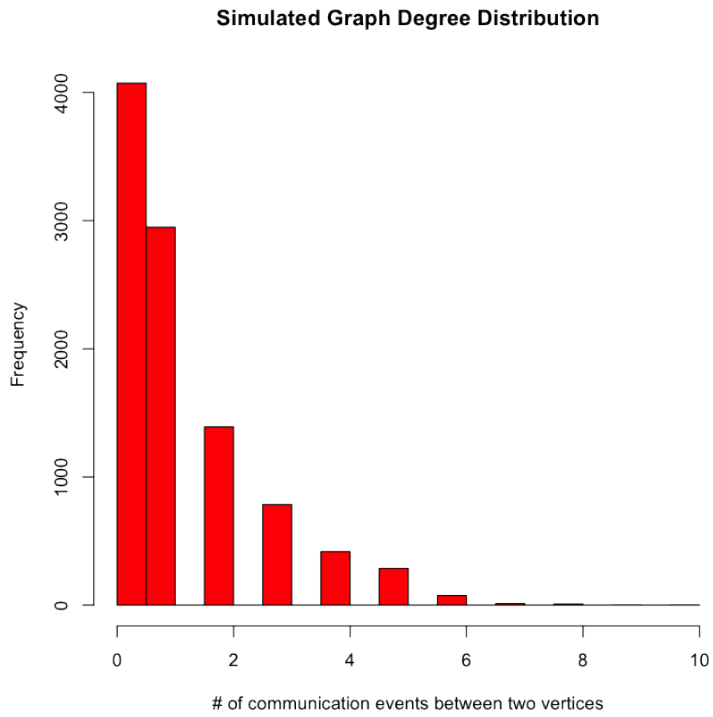
Graph Size Statistic for Enron graphs:



HPP Simulation: Kolmogorov-Smirnov Test

- Compare true Enron graph degree distribution $F_t(x)$ with simulated graph degree distributions $F_s(x)$.
- H_0: the results of the simulation come from the true Enron graph distribution $F_t(x)$.
- H_0 is rejected if at level $\alpha = 5\%$ if $p - value \leq 0.05$.

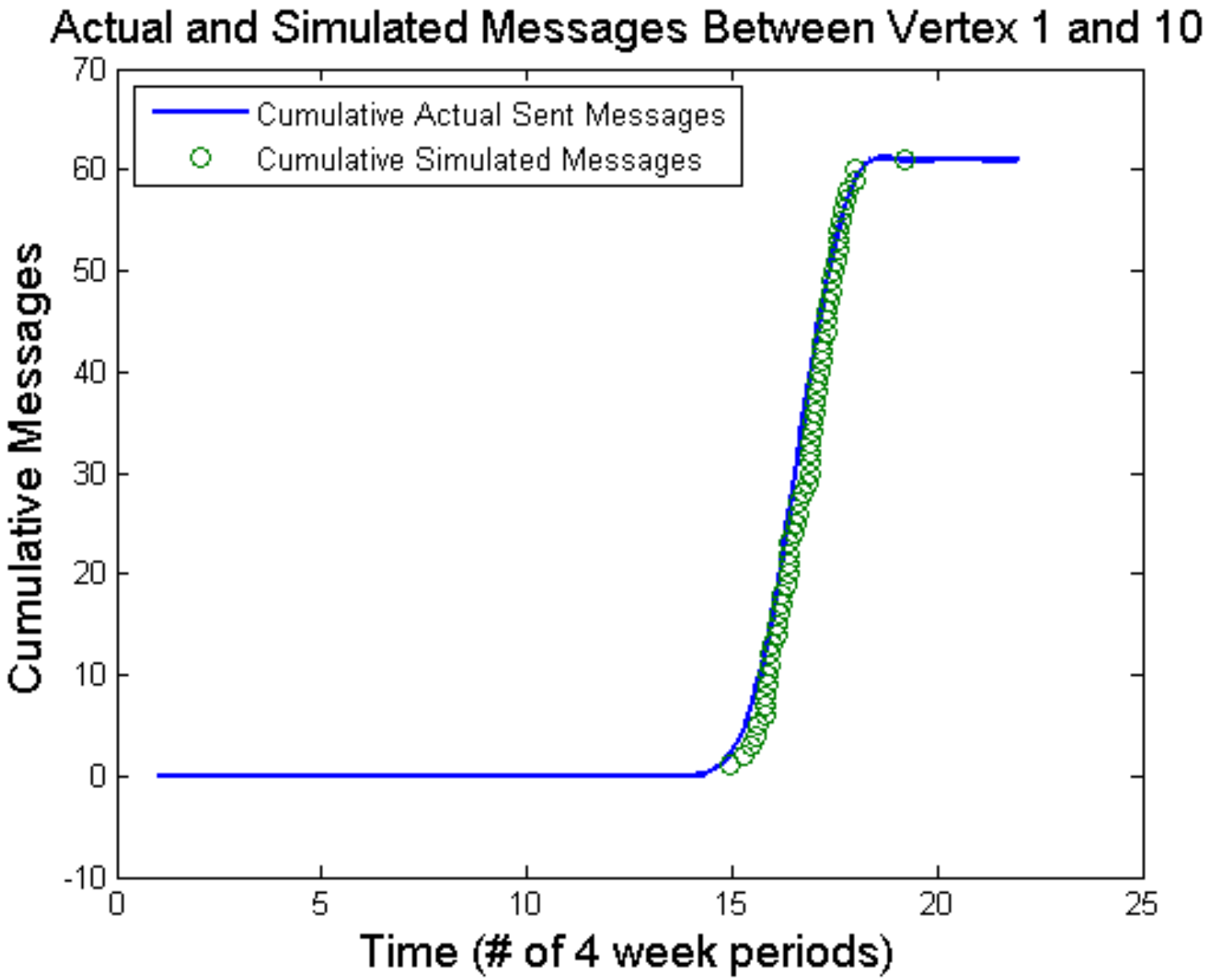




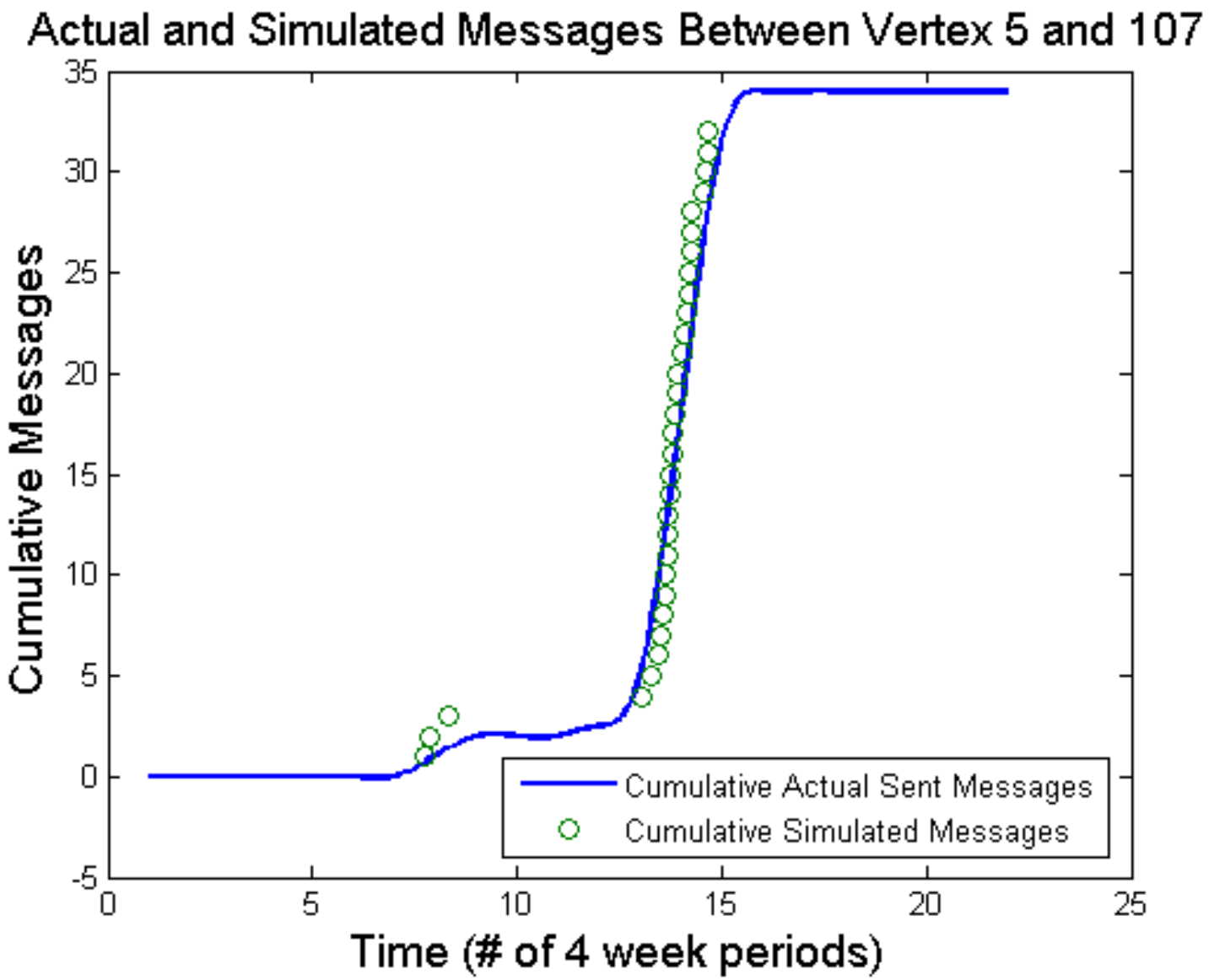
Limitations of HPP

- HPP model consistently predicts an excessively high rate of messaging due to inhomogeneous bursts of messaging activity from a few highly active vertices.
- Each summary statistics does not yield useful measures of aberrant or normal messaging behavior.
- Calculating vertex-depending messaging rates and setting higher graph thresholds does not sufficiently improve the model.

IHPP Simulation Example: Vertex 1 and 10

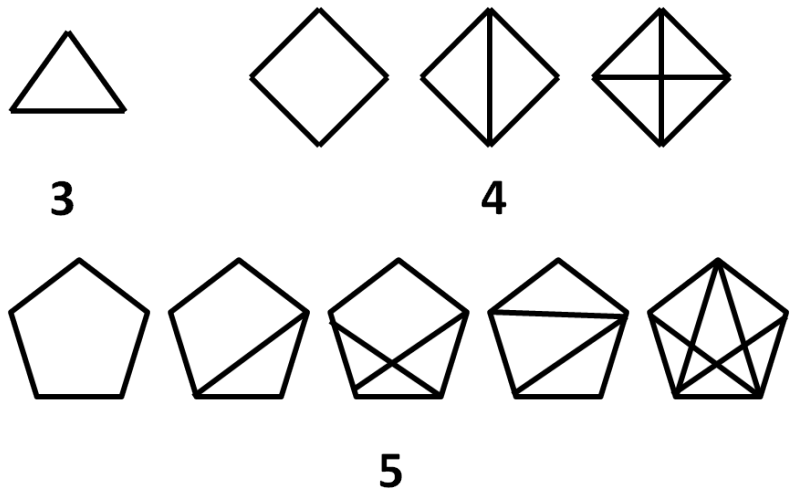


IHPP Simulation Example: Vertex 5 and 107

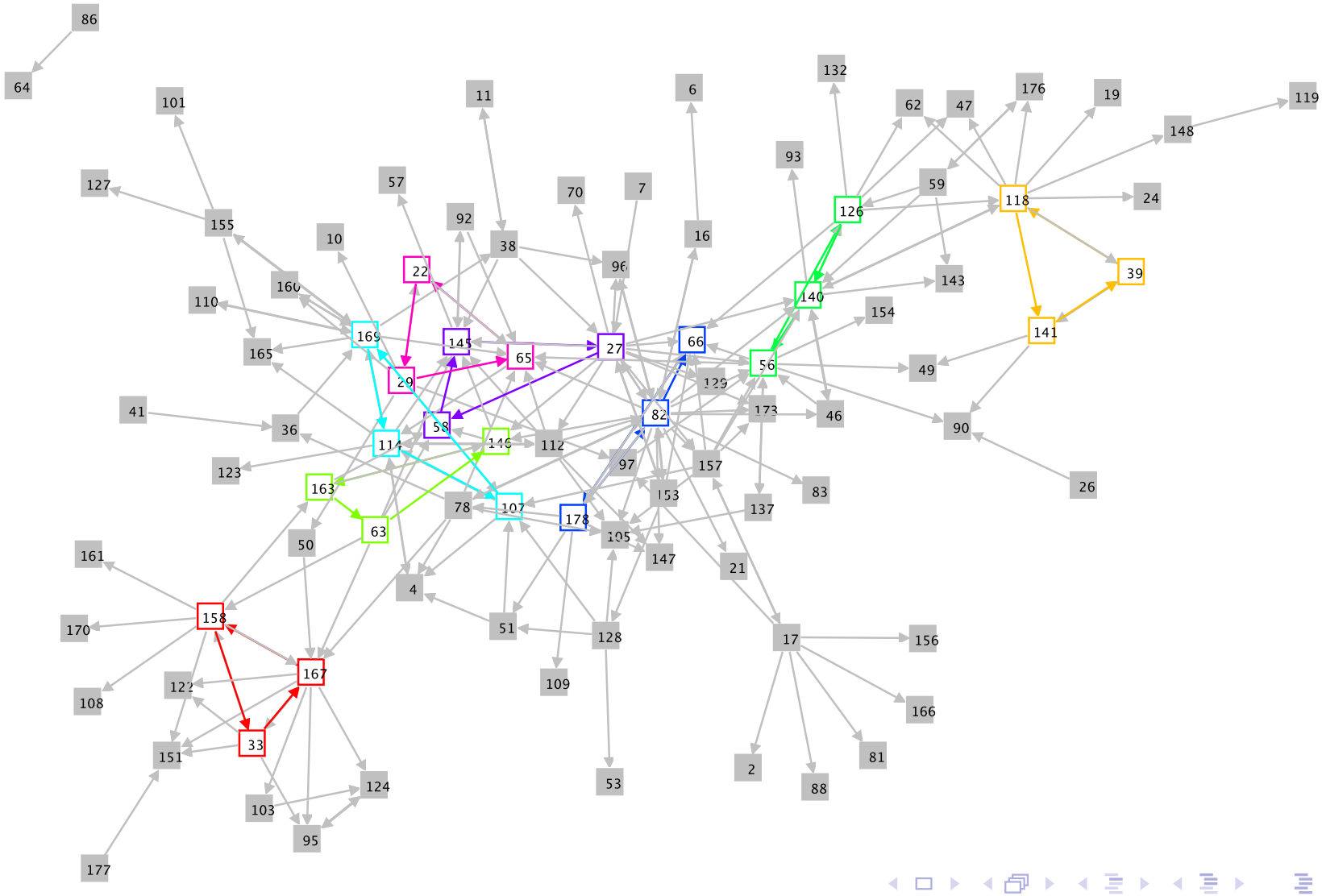


Anomaly Detection using Motifs Count

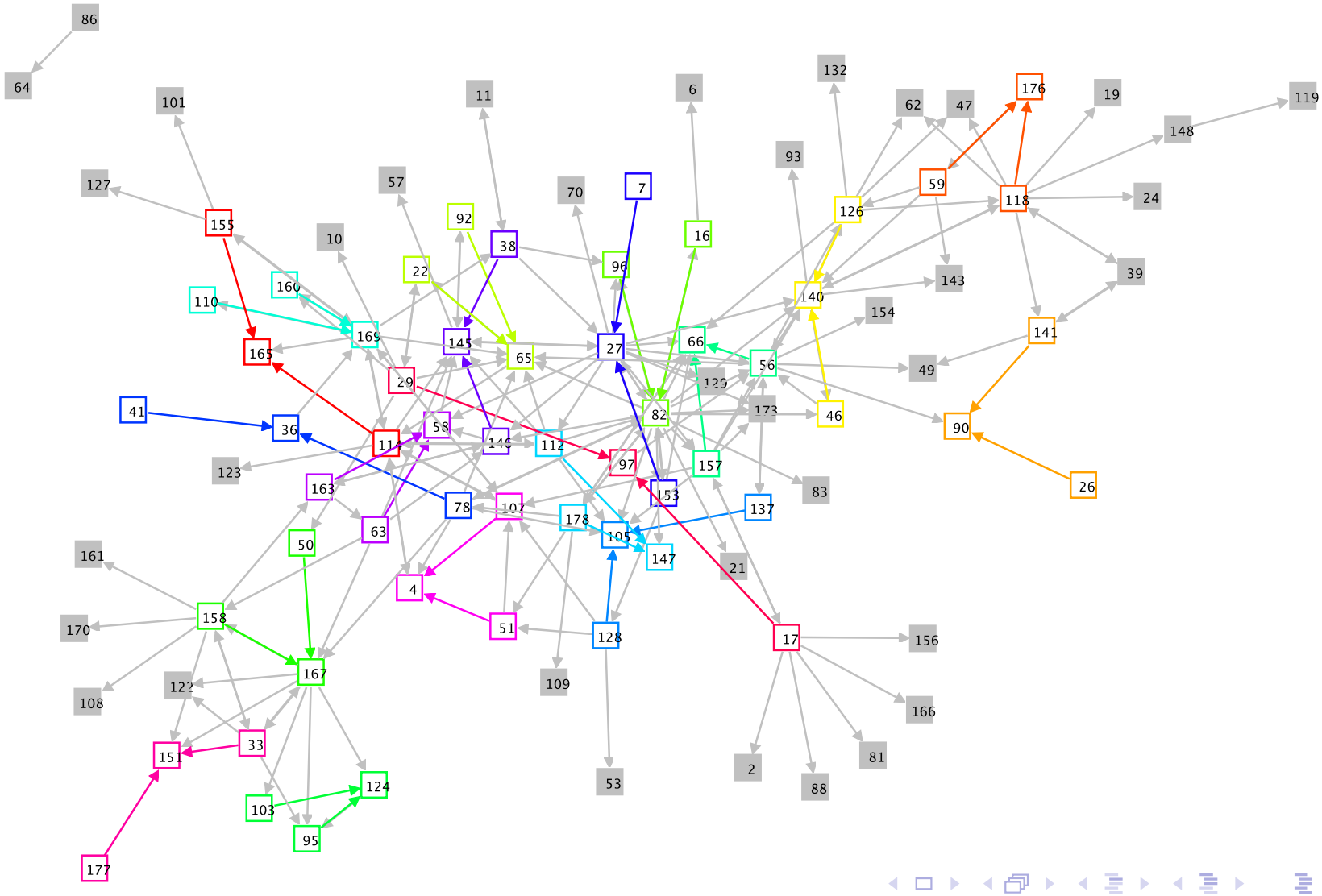
- Goal: detect small subgraphs occurring in a network more often than would be expected in a random graph.



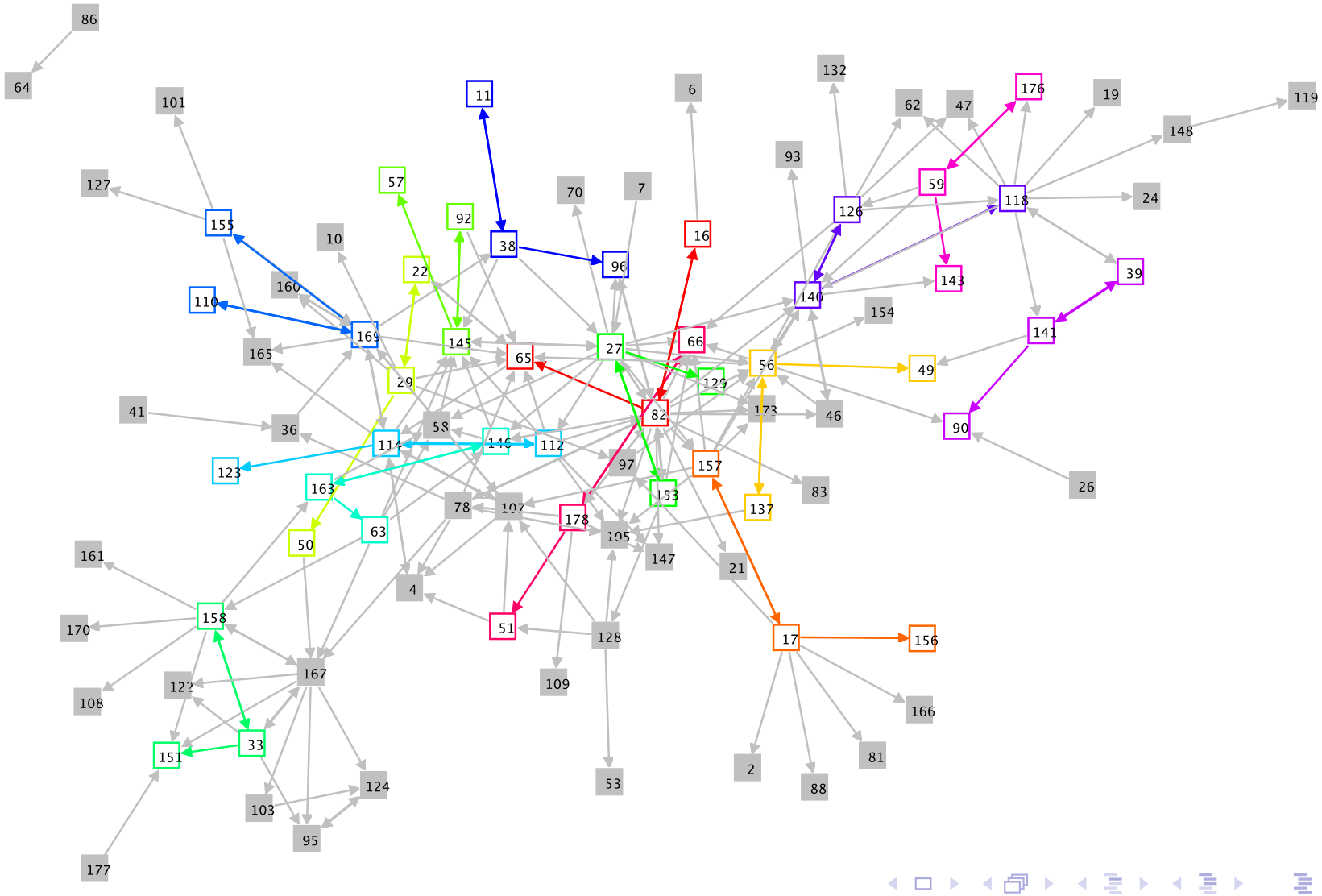
Motifs Count Statistic: GOX tuples in 4 week period



Motifs Count Statistic: GCR tuples in 4 week period



Motifs Count Statistic: F8X tuples in 4 week period



Future Work

- Model messaging behavior using self-exciting Poisson process.
- Take into account additional parameters during the simulation, e.g., message content, communicant gender, age.
- Explore more other statistics, e.g., motif count analysis.
- Apply similar approach of modeling messaging behavior to other datasets, e.g., Twitter.

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