600.471 Theory of Computation November 11, 2009

Handout 10: Homework 5

Instructor: Susan Hohenberger *CA: Venkata Ponnam*

This assignment is due by the start of lecture on November 23.

Try to solve all five problems, but turn in solutions to only four of them of your choice. If you turn in five solutions, we will grade four of them of our choice.

1. **(25 points)** (Sipser 8.9) [LADDER$_{DFA}$ $\in$ PSPACE]
2. **(25 points)** Let Z be the language of all strings w in $\{0,1\}^*$, where for any prefix of w the number of zeros is greater than or equal to the number of ones. Show that Z is in L.
3. **(25 points)** (Sipser 8.27) [strongly-connected is NL-complete]
4. **(25 points)** (Sipser 9.22) [tale of two oracles]
5. **(25 points)** To solve this, it will be helpful to read ahead to Section 10.2 in Sipser. Below we summarize the requirements on the behavior of machines for languages in several probabilistic classes. A language L is in a specified class if and only if there exists a probabilistic polynomial time Turing machine that accepts words in L with the given probability, and rejects words not in L with the given probability.

Class	Probability of accepting $w \in L$	Probability of rejecting $w \notin L$
P	1	1
NP	> 0	1
coNP	1	> 0
RP	$> \frac{2}{3}$	1
coRP	1	$> \frac{2}{3}$

Define a CP-machine ("Confused Polynomial-time") to be a probabilistic Turing machine which is permitted three types of output on each of its branches: accept, reject and *ido* (stands for "I don't know.") A CP-machine M decides a language A if M outputs the correct answer on every input string w (accept if $w \in A$ and reject if $w \notin A$) with probability at least $\frac{3}{4}$, and M never outputs the wrong answer. On every input, M may output *ido* with probability at most $\frac{1}{4}$. Furthermore, the average running time over all branches of M on w must be bounded by a polynomial in the length of w. Show that RP $\cap$ coRP = CP.