Handout 5: Homework 3

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This assignment is due by the start of lecture on October 19.

1. (30 points) On the first day of your job at SmartCompilers.com, your boss assigns you the task of adding this feature to the company's development suite:

   "Analyze the source code of a function, and verify that no array will ever be accessed out of bounds. Output TRUE if the code is safe in this respect, and FALSE if for some inputs, the function will access an array out of bounds."

   For example, the following program should fail the verification:

   ```
   void sloppy_code(int k) {
       int array[10];
       for (int i=0; i<k; i++) { array[i]=i; }
   }
   ```

   Program sloppy_code should fail, because on input $k = 15$, the array will be accessed at position 11 (i.e., array$[11] = 11$;), which is outside its declared range of 10.

   In your answers, you may assume that the behavior of the function is fully determined by its source code and its inputs (i.e., it does not make any outside calls to libraries or other functions). Also assume for simplicity that the only variables a function uses are (possibly unbounded) integers, and integer arrays.

   (a) (5 points) State your boss' request as a language decision problem. Call the language $SAFE$. (You define strings are in $SAFE$.)

   (b) (10 points) To prove to your boss that this task is impossible, show that $SAFE$ is undecidable. (Don't use Rice's Theorem here.)

   (c) (10 points) Is $SAFE$ Turing-recognizable? Co-Turing-recognizable? Neither? Both? Explain your answer.

   (d) (5 points) Read the description of Rice's Theorem in problem 5.28. Does Rice's Theorem apply to your language above? Briefly explain why or why not.

2. (20 points) Show that L is decidable iff $A \leq_m 011^*$.

3. (40 points) Let $FIN = \{\langle M \rangle |\ M$ accepts a finite number of strings$\}$. Prove the following results about FIN. (Hint: use mapping reducibility.)

   (a) (20 points) FIN is not Turing-recognizable.

   (b) (20 points) $\overline{FIN}$ is not Turing-recognizable (i.e., FIN is not co-Turing-recognizable.)

4. (10 points) (Sipser 6.1) Give an example in the spirit of the recursion theorem of a program in a real programming language (or a reasonable approximation thereof) that prints itself out.