600.471 Theory of Computation October 7, 2007

Handout 5: Homework 3

Instructor: Susan Hohenberger *TA: Rebecca Shapiro*

This assignment is due by the start of lecture on Thursday, October 16.

1. (25 points) (from Sipser 5.13) A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states.

   (a) (20 points) Formulate this problem as a language and show that it is undecidable.

   (b) (5 points) Read the description of Rice's Theorem in problem 5.28. Does Rice's Theorem apply to your language above? Briefly explain why or why not.

2. (20 points) (Sipser 5.25) Give an example of an undecidable language B, where $B \leq_m \bar{B}$. Justify your answer.

3. (40 points) Let $INF = \{\langle M \rangle | \ M \text{ accepts an infinite number of strings}\}$. Prove the following results about INF. (Hint: use mapping reducibility.)

   (a) (20 points) $\overline{INF}$ is not Turing-recognizable (i.e., INF is not co-Turing-recognizable.)

   (b) (20 points) INF is not Turing-recognizable.

4. (15 points) (Sipser 6.1) Give an example in the spirit of the recursion theorem of a program in a real programming language (or a reasonable approximation thereof) that prints itself out.

5. (10 *bonus* points) Consider the following Turing machine:

```
M = "On input w,
            1. Obtain, by the recursion theorem, description of self <M>.
            2. Simulate <M> on w.
            3. If <M> accepts w, reject.
            4. If <M> rejects w, accept."
```

   Can such a machine exist? If so, what is its language? If not, why not? (What is the contradiction?)