600.471 Theory of Computation October 17, 2007

Handout 9: Homework 4

Instructor: Susan Hohenberger *TA: Omar Zaidan*

This assignment is due by the start of lecture on Wednesday, October 24.

1. (Sipser 5.14) Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

2. Let $FIN = \{\langle M \rangle | \ M$ accepts only a finite number of strings$\}$. Prove the following results about FIN. (Hint: use mapping reducibility.)

   (a) FIN is not Turing-recognizable.

   (b) $\overline{FIN}$ is not Turing-recognizable (i.e., FIN is not co-Turing-recognizable.)

3. Read the description of Rice's Theorem in problem 5.28. Does Rice's Theorem apply to FIN? Briefly explain why or why not.

4. (Sipser 5.23) Show that A is decidable if and only if $A \leq_m 0^*1^*$.

5. (Sipser 6.1) Give an example in the spirit of the recursion theorem of a program in a real programming language (or a reasonable approximation thereof) that prints itself out.

6. *Bonus*: Consider the following Turing machine:

```
M = "On input w,
            1. Obtain, by the recursion theorem, <M>.
            2. Simulate <M> on w.
            3. If <M> accepts w, reject.
            4. If <M> rejects w, accept."
```

   Can such a machine exist? If so, what is its language? If not, why not? (What is the contradiction?)