This assignment is due by the start of lecture on Wednesday, October 24.

1. (Sipser 5.14) Consider the problem of determining whether a Turing machine \( M \) on an input \( w \) ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

2. Let \( FIN = \{ \langle M \rangle \mid M \text{ accepts only a finite number of strings} \} \). Prove the following results about \( FIN \). (Hint: use mapping reducibility.)
   
   (a) \( FIN \) is not Turing-recognizable.
   
   (b) \( \overline{FIN} \) is not Turing-recognizable (i.e., \( FIN \) is not co-Turing-recognizable.)

3. Read the description of Rice's Theorem in problem 5.28. Does Rice's Theorem apply to \( FIN \)? Briefly explain why or why not.

4. (Sipser 5.23) Show that \( A \) is decidable if and only if \( A \leq_m 0^*1^* \).

5. (Sipser 6.1) Give an example in the spirit of the recursion theorem of a program in a real programming language (or a reasonable approximation thereof) that prints itself out.

6. **Bonus**: Consider the following Turing machine:

   \[
   M = "\text{On input } w,} \\
   \text{1. Obtain, by the recursion theorem, } \langle M \rangle. \\
   \text{2. Simulate } \langle M \rangle \text{ on } w. \\
   \text{3. If } \langle M \rangle \text{ accepts } w, \text{ reject.} \\
   \text{4. If } \langle M \rangle \text{ rejects } w, \text{ accept.}"
   \]

   Can such a machine exist? If so, what is its language? If not, why not? (What is the contradiction?)