600.363/463 Introduction to Algorithms/Algorithms I	October 21, 2009

Handout 9: Homework 5

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This assignment is due by the start of lecture on October 28, 2009. Please clearly indicate your collaborators.

1. (30 points) *Getting Greedy.* (CLRS 16.2-7) Suppose you are given two sets A and B, each containing n positive integers. You can choose to reorder each set however you like. After reordering, let a_i be the ith element of A, and let b_i be the ith element of B. You then receive a payoff of $\prod_{i=1}^{n} a_i^{b_i}$ dollars. Give an algorithm that will maximize your payoff. Prove that your algorithm maximizes the payoff, and state its running time.

2. (30 points) *Morning Coffee.* Every morning, customers $1, \ldots, n$ show up to get their morning espresso-based concoction from Cafe Zoo. Suppose the omniscient barista actually knows all of the customers, and knows their orders, $o_1, \ldots, o_n$. Some customers are nicer people; let T_i be the baristas expected tip from customer i. Some drinks like a simple double-shot, are easy and fast, while some other orders, like a hazelnut mocha soy-milk latte take longer. Let t_i be the time it takes to make drink o_i . Assume the barista can make the drinks in any order that she wishes, and that she makes drinks back-to-back (without any breaks) until all orders are done. Let D_i be the time that the barista finishes order i. Devise an algorithm that helps the barista pick a schedule that minimizes the quantity

$$\sum_{i=1}^{n} T_i D_i.$$

   In other words, she wants to serve the high-tippers the fastest but she also wants to take the time it takes to make each drink into consideration. (Hint: think about a property that is true about an optimal solution.)

3. (20 points) *Huffman Codes.* (CLRS 16.3-3) What is the optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?

$$a:1 \quad b:1 \quad c:2 \quad d:3 \quad e:5 \quad f:8 \quad g:13 \quad h:21$$

   Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers?

4. (20 points) *Friends.* Reword the following statement as a theorem about undirected graphs, and then prove it. Assume that friendship is symmetric but not reflexive.

   (a) Any group of at least two people contains at least two people with the same number of friends in the group.

   Hint: this is a warm-up exercise about graphs, you don't need any specific knowledge from Chapter 22 to solve it.