Road Map Methods

Including material from Howie Choset
The Basic Idea

• Capture the connectivity of $Q_{\text{free}}$ by a graph or network of paths.
RoadMap Definition

- A roadmap, RM, is a set of trajectories (i.e. \( f(t, q_A, q_B) \)) such that for all \( q_{\text{start}} \in Q_{\text{free}} \) and \( q_{\text{goal}} \in Q_{\text{free}} \) can be connected by a path:

- The three ingredients of a roadmap
  - **Accessibility:** There is a path from \( q_{\text{start}} \in Q_{\text{free}} \) to some \( q' \in \text{RM} \)
  
  - **Departability:** There is a path from some \( q'' \in \text{RM} \) to \( q_{\text{goal}} \in Q_{\text{free}} \)
  
  - **Connectivity:** there exists a path in RM between \( q' \) and \( q'' \)
RoadMap Path Planning

1. Build the roadmap
   a) nodes are points in $Q_{\text{free}}$ or its boundary
   b) two nodes are connected by an edge if there is a free path between them (i.e. $f(t, q_A, q_B)$)

2. Connect $q_{\text{start}}$ and $q_{\text{goal}}$ points to the road map at point $q'$ and $q''$, respectively

3. Find a path on the roadmap between $q'$ and $q''$. The result is a path in $Q_{\text{free}}$ from start to goal
Overview

• Deterministic methods
  ✓ Some need to represent $Q_{\text{free}}$, and some don’t.
  ✓ are complete
  ✓ are complexity-limited to simple (e.g. low-dimensional) problems
    • example: Canny’s Silhouette method (5.5)
      – applies to general problems
      – is singly exponential in dimension of the problem
Visibility Graph methods

- Defined for polygonal obstacles
- Nodes correspond to vertices of obstacles
- Nodes are connected if
  - they are already connected by an edge on an obstacle
  - the line segment joining them is in free space
- Not only is there a path on this roadmap, but it is the shortest path
- If we include the start and goal nodes, they are automatically connected
- Algorithms for constructing them can be efficient
  - $O(n^3)$ brute force
Visibility Graph in Action

1. Draw lines of sight from the start and goal to all “visible” vertices and corners of the world.

\[ e_{ij} \neq \emptyset \iff sv_i + (1 - s)v_j \in \text{cl}(Q_{\text{free}}) \quad \forall s \in (0, 1) \]
Visibility Graph in Action

2. Second, draw lines of sight from every vertex of every obstacle like before. Remember lines along edges are also lines of sight.

\[ e_{ij} \neq \emptyset \iff sv_i + (1 - s)v_j \in \text{cl}(Q_{\text{free}}) \quad \forall s \in (0, 1) \]
Visibility Graph in Action
The Visibility Graph in Action (Part 4)
Visibility Graph in Action

- Repeat until you’re done.
Visibility Graphs

• Find a path in the graph.
  – Bread first, depth first, Dijkstra’s, etc.