Review

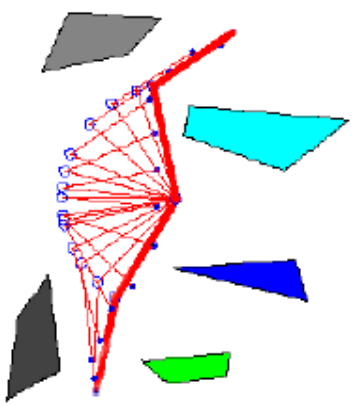
- Rigid body motion
  - Position/Orientation
  - Linear/Angular velocities (spatial/body)
- Kinematics:
  - Forward: configuration space to SE(2)/SE(3)
  - Inverse: not really
  - Singularities
- Velocities:
  - Manipulator Jacobian (singularities)

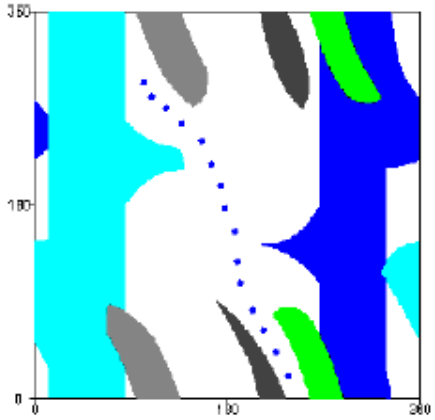
Review

- Hand-Eye calibration
  - AX=XB is ubiquitous
  - How to formulate the problem and how to solve it
  - Difference between closed form (2 “A” and 2 “B”) and least squares solutions (N “A” and N “B”)
  - Not just for robots: “X” relates two coordinate frames

Potential Fields

- Configuration spaces
  - Robot arms
    - Revolute joints $q \in \mathbb{S}^1$
    - Prismatic joint $q \in \mathbb{R}^1$
  - Mobile robot
    - R^2, R^3, SE(2), SE3(3), …
- Workspace SE(2), SE(3)
- Potential Fields
  - Explicit Attractive/Repulsive potential functions
  - Gradient descent
  - Robot arms use implicit potential in the workspace and the Jacobian to relate workspace potential to configuration space potential



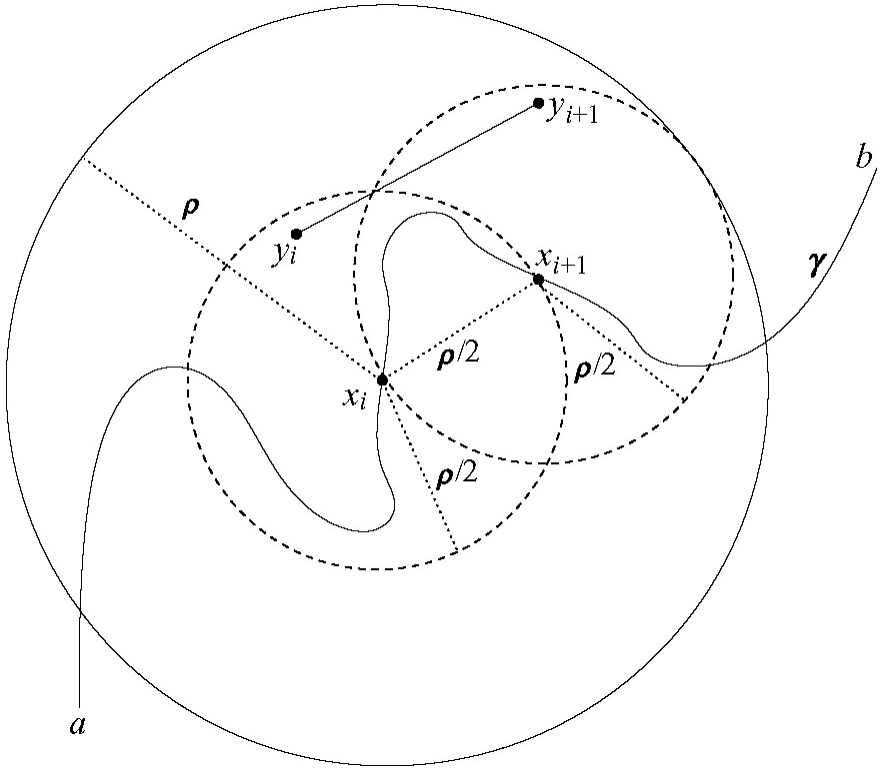


Roadmaps

- General idea:
  - Focus on finding paths on a small subset of the free space
- Efficient algorithms in 2D
  - Visibility graph
    - Connect vertices of polygons and use the edges as roads
  - ~~Trapezoid cell decomposition~~
    - ~~Decompose the free space in convex cells and make edges between midpoints of shared segments~~

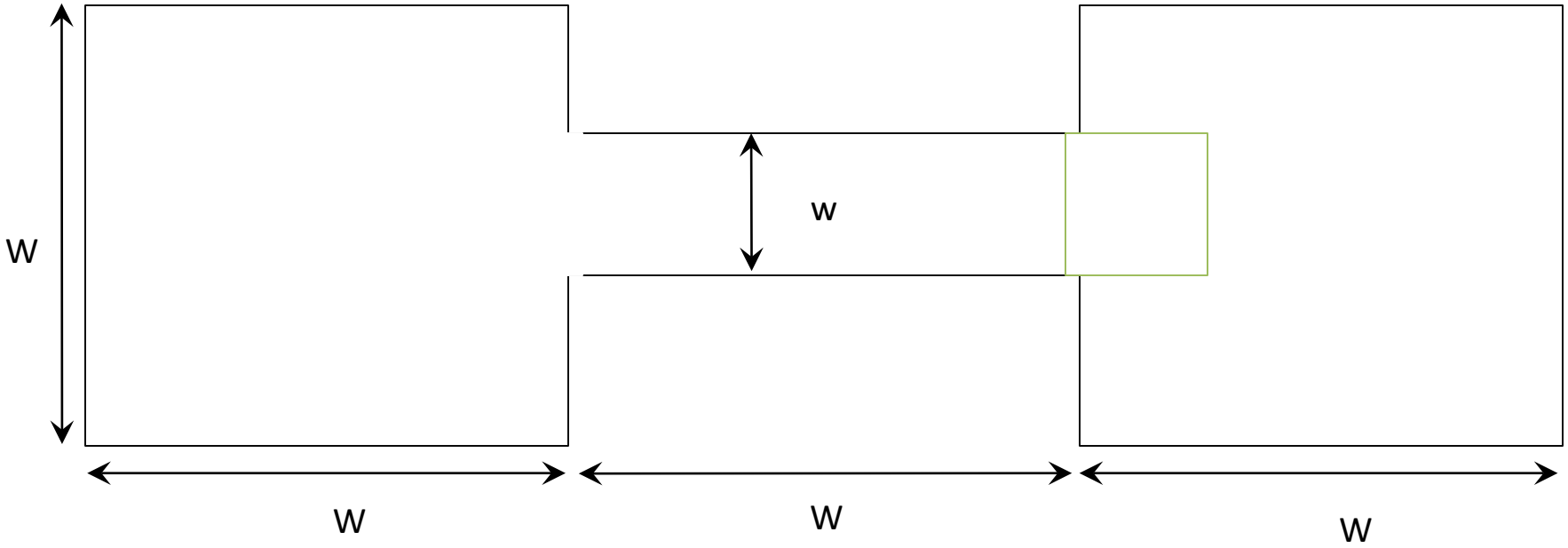
Sampling-Based Algorithms

- PRM
  - Randomly sample configuration space
  - Connect some or all pairs of configurations
  - Use the roadmap to find a path between two *configurations*
- Many variants and "flavor-of-the-months"
  - Build and connect trees
  - Multiple query: Plain, obstacle sampling, Gaussian sampling, visibility sampling, bridge sampling, importance sampling
  - Single query: RRT, EST
- Completeness
- (ε, α, β) expansiveness



$$\Pr[(a,b)\text{SUCCESS}] \geq 1 - \left\lceil \frac{2L}{\sigma} \right\rceil \bar{e}^{\sigma \rho^d n},$$

An Example



Given n random samples, what are the odds the resulting graph is connected?